Innovative approaches to teaching mathematics in higher education: a review and critique

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This paper provides a snapshot of emerging trends in mathematics teaching in higher education for STEM subjects (Science, Technology, Engineering and Mathematics). Overwhelmingly, papers identify a focus on conceptual understandings of mathematics in comparison to understanding that is instrumental or procedural. Calls for reform of mathematics teaching have been the basis for a range of studies; responses to these calls have embraced innovative methods for implementing changes in learning and teaching of mathematics, sometimes rooted in constructivist ideology. Observed trends have been categorised in six groups. In many studies, technology is being used as an enabler of reforms. Constraints to implementing new approaches in mathematics teaching are indicated. Discussion of contemporary research questions that could be asked as a result of the shift towards teaching mathematics in innovative ways is provided and is followed by a critique of the underlying theoretical positions, essentially that of constructivism.

Mathematics teaching (or instruction) in higher education has long embraced traditional methods: non-interactive ways of teaching mathematics (ways in which the student is the receiver of delivery from the teacher, but only minimally a participant) (Alsina, 2001; Anku, 1996; Brito et al., 2009; Hillel, 2001; Smith & Wood, 2000). Traditional approaches can be seen to be dominated by theory and not to address the needs of most students; it is even argued that these methods have not evolved much since the times of ancient Egypt and Assyria 5000 years ago (Abate & Cantone, 2005). Recently there have been calls for reforming mathematics instruction by considering more innovative pedagogical approaches, often rooted in constructivist theory, to promote students' conceptual understanding. (Abate & Cantone, 2005; Chang, 2011; Jaworski, 1994; Mokhtar, Tarmizi, Fauzi & Ayub, 2010, Orton & Roper, 2000).

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Theoretically and historically, trends in teaching and learning, have seen a shift from Behaviourism (Pavlov, 1927; Skinner, 1953; Thorndike, 1913), passing through Cognitivism (Atkinson & Shiffrin, 1968; Craik & Tulving, 1975; Kulhavy & Wager, 1993; Martin, 1993; Squire, Knowlton & Musen, 1993), towards Constructivism (Kolb, 1984; Mayer, 1999; Richardson, 2003; Steffe & Gale, 1995; Tynjälä, 1999). Constructivism is a paradigm that has been significantly influenced by cognitivism (Hergenhahn & Olson, 2004); however, it presents a more socially embracing position on pedagogy and learning as opposed to the microscopic focus of cognitivism on the internal mechanisms that underline learning processes. Constructivism perceives learning as a process of constructing knowledge by individuals themselves as opposed to the passive teacher-student model (Brown, Collins & Duguid, 1989; Kolb, 1984; von Glasersfeld, 1987b). In the process of knowledge creation, learners link new knowledge with their previous knowledge. *Social* constructivism is distinguished from *radical* constructivism by placing emphasis on social processes influencing the learners' constructions, particularly the importance of language and discourse (Ernest, 1991; Jaworski, 1994; Palincsar, 1998; Taylor & Campbell-Williams, 1993; von Glasersfeld, 1987b). Pedagogues adopting constructivism as a basis for pedagogy suggest that approaches should focus on concepts and contextualisation instead of instructing isolated facts (Brooks & Brooks, 1993). Social constructivist pedagogy emphasises the students' social interaction with peers and the teacher (Palincsar, 1998) and suggests that consideration should be given to the student's preferred learning styles (Kolb, 1984).

Some of the main pillars of (so-called) constructivist pedagogy (Doolittle, 1999; Driver, 1995; Jaworski, 1994; Richardson, 2003; Savery & Duffy, 2001; von Glasersfeld, 1987a) are: learning is a student-centred process, students' autonomy should be fostered; learning should be contextualised and associated with authentic real-world environments and examples; social interaction and discourse form an important part of learning; the taught elements should be made relevant to the learner; the taught elements should be linked with the learners' previous knowledge; it is important to facilitate continuous formative assessment mechanisms, self-esteem and motivation; teachers should act as orchestra synchronisers rather than speech givers; and teachers should consider multiple representations of their teachings.

Many investigations of reform in mathematics instruction have embraced constructivist principles; empirical findings have in general revealed enhanced outcomes and learning experiences (Alsardary & Blumberg, 2009; Aydin, 2009; Chang, 2011; Hagerty, Smith & Goodwin, 2010; Mokhtar, Tarmizi, Ayub & Tarmizi, 2010; Roddick, 2001; Ward et al., 2010). In this paper, we provide a review of recent emerging trends - many of them associated with constructivist foundations – in mathematics teaching. We focus on STEM higher education in particular: that is the teaching of Science, Technology, Engineering and Mathematics in higher education (university and college levels), with our focus being on the teaching of mathematics.[2]

Current trends of mathematics instruction in STEM higher education

We aimed to investigate current trends of mathematics teaching in STEM higher education; hence a review of the literature was conducted. A number of keywords were entered into Google Scholar and into the search engine of relevant journals such as PRIMUS: Problems, Resources, Issues in Undergraduate Mathematics Studies; the International Journal of Mathematical Education in Science and Technology; and the International Journal of Computers for Mathematical Learning. Examples of the used search keywords are: inquiry, engineering, discovery, deep, constructivist, constructive, collaborative, undergraduate, inquiry based learning in mathematics, discovery based learning in mathematics, problem based learning in mathematics, conceptual understanding in mathematics, and mathematics in engineering. The resulting relevant papers were reviewed and a number of trends have been noted as detailed below.

The use of student-centred learning methods

Constructivism is about self-construction of knowledge: student-centred approaches have been seen to play an essential role in this process. Papers promoting student centred approaches sometimes root their research explicitly in constructivist theory (e.g. Mokhtar et al., 2010; Roddick, 2001) others only by implication (e.g. Chang, 2011; Maull & Berry, 2000). Calls for embracing student-centred approaches can be traced to Piaget (e.g. Donaldson, 1978; Piaget, 1979; von Glasersfeld, 1987b) and to Dewey (1938/1998), and in the UK, to the Plowden Report (1967). Over the decades a number of student-centred pedagogies such as Inquiry/Problem/Project Based Learning (I/P/P/BL) methods have been developed and investigated; these approaches are often conducted in teams or small groups of students, but also in a solo mode.

Such methods have been more commonly used, particularly where mathematics is concerned, in school level education. For example, approaches to problem solving and associated heuristics were widely discussed in the mathematics education literature in the early 1980s, often reflecting the work of Polya (e.g. 1945) and presenting frameworks for exploratory activity in mathematics (e.g. Love, 1988; Mason, Burton & Stacey, 1982; Schoenfeld, 1985). Inquiry as an approach to teaching and learning mathematics has seen wide consideration internationally (e.g. Berg, 2009; Collins, 1986; Jaworski, 1994; Jaworski et al., 2007; Lindfors, 1999; Ponte, 1991).

In higher education, I/P/P/BL methods became popular in medicine with early starts in the 1960s in the US and Canada (Barrows & Tamblyn, 1980). Later on in the 1980s and 1990s they started to appear in engineering sciences (Hadgraft, 1998). Mathematics has lagged behind the wave (Fielding-Wells & Makar, 2008); however, in the last few years, an increased number of studies have reported the use of inquiry based learning (Chang, 2011; Roddick, 2001; Ward et al., 2010), problem/project based learning (Mokhtar et al., 2010; Niu & Shing, 2010), and discovery based learning (Hodge, 2006).

Roddick (2001), in a controlled investigation, reported that students who follow an IBL based mathematics course with Mathematica tend to follow a conceptual approach in solving problems, while students who follow traditional teaching tend to follow a procedural approach in problem solving. It has been found that PBL encourages students to search for information and that it stimulates thinking (Mokhtar et al., 2010). Utilising student-centred methods in mathematics instruction has been reported to increase students' interest in the subject and their success rate (Mokhtar et al., 2010), to increase students' appreciation of the role of mathematics in life (Ward et al., 2010), and to increase motivation to learn mathematics and realise its applicability (Chang, 2011; Mokhtar et al., 2010). Uses of student-centred approaches in mathematics instruction have been reported to result in similar or sometimes better exam scores (Alsardary & Blumberg, 2009; Jaworski & Matthews, 2011; Mokhtar et al., 2010; Roddick, 2001)

Despite the frequent reports of positive impact of student-centred approaches in mathematics teaching and learning, Ward et al. (2010) indicate a reduced attitude towards the subject in an inquiry based learning mathematics course. This indicates the need for further investigations of these approaches. It should be noted, also, that conflicting findings could be related to the highly heterogeneous nature of the study due to differences in the investigations, the pedagogical implementations used, and varied assessment instruments and evaluation approaches as well as to student attitudes and perceptions of the purposes of the programme. These cautions suggest widening the frame of research from one focusing on individuals constructing their own knowledge to a frame that is more encompassing of these sociocultural issues. We come back to this theme in our postscript.

Contextualisation of mathematics using real-world examples

An important pillar in constructivist pedagogy is contextualising learning using an authentic environment and real-world examples. A majority of students have difficulties in connecting mathematics to real world applications and this could be a reason for failure in mathematics (Chang, 2011). Making mathematics relevant (e.g. via real world examples), in particular for non-specialists, has been stressed in a number of studies (Abate & Cantone, 2005; Chang, 2011; Matthews, Adams & Goos, 2009; Pennell, 2009).

Using authentic and real-world examples is considered essential in student-centred approaches such as PBL (Mokhtar et al., 2010). Real-time data such as room temperature and humidity were used in a problem based learning approach to calculus by Niu and Shing (2010). Real data from an experimental pendulum rig for presenting a real-world context in mathematical modelling course instruction was used by Reid and King (2009).

Aydin (2009) contextualised abstract ideas from algebra and number theory, taught in a mathematics course for specialists, by using computer science and engineering examples from cryptography and coding theory. Chang (2011) utilised image processing examples from computer science to contextualise abstract ideas from linear algebra in a mathematics course for mathematics specialists. In a control theory course, an engineering subject that is mathematically intensive, remote experiments have been used in the classroom to visualise and show the applicability of the differential equations used in implementing control algorithms (Abdulwahed & Nagy, 2011).

Contextualising mathematics has been reported frequently to enhance students' experience (Abate & Cantone, 2005; Abramovich & Grinshpan, 2009; Aydin, 2009; Chang, 2011; Matthews et al., 2009; Reid & King, 2009). The most successful mathematics courses in engineering are thought to be those that have been well integrated in the engineering curriculum facilitating contextual relevance of mathematical abstracts to engineering concepts (Henderson & Broadbridge, 2007). It is thought to be important to collaborate between mathematics instructors and personnel from science and engineering domains for designing contextualised mathematical courses (Matthews et al., 2009).

Bridging the gap in previous mathematical knowledge

Another important pillar in constructivist pedagogy is to build upon previous knowledge. Many STEM higher education students enter universities with gaps in necessary prerequisite knowledge of mathematical topics; this can hinder significantly the introduction of new mathematical ideas through novel approaches. Turner (2009) designed a model of a program of three stages of predictor-corrector-refinement for supporting first year transition in a calculus course (prediction of performance in calculus, based on diagnostic testing; correction of errors based on a web-based pre-calculus course). However, it was seen not to be fully successful due to gaps in students' knowledge. The author suggests that further research is needed into models and interventions for bridging the gap in previous mathematical knowledge.

Encouraging discourse in classroom and among students

An emphasis on discourse, as in a social constructivist perspective relating to Vygotskian principles (e.g. Ernest, 1991; Vygotsky, 1978), has been seen as important to the teaching and learning process. Traditional passive teaching methods in the classroom or lecture leave little time, if any, for discussions and dialogue among students themselves and/or with the instructor. Passive lectures are criticised for many factors; for instance, Chang (2011) proposed a framework of mathematics teaching and learning in lectures that encourages lecturers to stimulate discourse in the classroom via asking thought-provoking questions; he recommends that lectures should constitute two way communication and lecturers should become better listeners. Encouraging discourse among students was an essential element in a calculus reform course (Roddick, 2001). Jaworski and Matthews (2011) report the use of small group discussion of inquiry-based mathematics problems for creating conceptual understanding among engineering students.

Enhancement of students' motivation, engagement and self-efficacy

Affective factors in students' learning include self-efficacy beliefs, motivation, engagement, and attitudes towards mathematics; these factors play an important role in success or failure of mathematics learning. Many students, along with a considerable population, consider mathematics highly abstract and boring (Fielding & Makar, 2008; Howson & Kahane, 1990; Mokhtar et al., 2010). Ward et al. (2010) enumerate a number of negative attitudes towards mathematics they observed in their students such as: mathematics capacity is genetically inherited; mathematics is not useful for most jobs and is all about memorising. Many researchers have reported correlation between beliefs about mathematics and mathematical performance (Bandalos, Yates & Thorndike-Christ, 1996; Campbell & Hackett, 1986; Lent, Lopez, Brown & Gore, 1996; Pajares

& Miller, 1997; Schoenfeld, 1989). Abate and Cantone (2005) suggest that reform in mathematics teaching should work on enhancing students' motivation towards the subject.

Hekimoglu and Kittrell (2010) used a video documentary in the classroom to increase students' self-efficacy beliefs towards mathematics. The evaluation indicated that using the documentary has resulted in significant enhancement in exam performance, as well as enhancement in retention rate, increased risk taking and thoughtful reflections. In a study of students' engagement with mathematics in an IBL approach to mathematics instruction versus traditional methods, Fielding-Wells and Makar (2008) found significant higher interest and demolishing of frustration towards mathematics when using the IBL method. Student-centred approaches, in general, have been reported to enhance students' motivation in learning mathematics (Mokhtar et al., 2010; Roddick, 2001).

Consideration of differences in learning styles

Scholars advocating constructivist pedagogies recognise that individuals learn with their own preferences, they emphasise taking into consideration the differences between learners when designing teaching and learning activities. Research in pedagogy and cognitive science has resulted in a number of different models of learning styles; for example, the Index of Learning Styles (ILS) model (Felder & Silverman, 1988), the 4MAT learning style model (McCarthy, 1986), the VARK learning style model (Fleming & Mills, 1992), and the multiple intelligences learning style model (Gardner, 1983). Conflicts in learning and thinking styles between mathematics teachers and students, or differences in learning style among students of mathematics courses, have been noted in many studies (Abramovich & Grinshpan, 2009; Jaworski, Robinson, Matthews & Croft, 2012; Maull & Berry, 2000; Savitz & Savitz, 2010). There could be a communication problem due to differences between the thinking styles of mathematical concepts between mathematics instructors and engineering students (Maull & Berry, 2000). Mathematics teaching in an abstract style for non-specialists has resulted in a problem of communication (Abramovich & Grinshpan, 2009). Savitz and Savitz (2010) investigated using classroom activities that can be compatible with different learning styles based on Gardner's (1983) multiple intelligences theory. The study's findings indicate that the activities were useful in addressing students' learning styles, and allowed them to learn better than they would do following conventional teaching approaches. Engineering students are thought to prefer an experiential learning style, hence student-centred experiential learning approaches with real-world problems such as PBL are more compatible to their learning style than classical abstract methods of teaching mathematics.

Technology as an enabler of innovative mathematics instruction

The use of technology for mathematics teaching and learning can be classified in two dimensions: 1 – the use of domain-specific mathematical analysis computer software packages and 2 – general use of learning technologies and online tools.

It is argued that technology evolution has been a driver for reform in mathematics teaching and learning (Chang, 2011; Roddick, 2001). Domain-specific mathematical analysis computer software such as Mathematica, together with an IBL approach, played an essential role in reforming calculus courses in the US (Roddick, 2001). Matlab has been used for in-class activities that demonstrate linear algebra concepts (Chang, 2011). Matlab is particularly popular in mathematics courses intended for engineering students (Mtenga & Spainhour, 2000; Waldvogel, 2006). Matlab/ Simulink and LabVIEW have been used for designing illustrative examples of differential equations in an engineering mathematics course (Pennell, 2009). GeoGebra has been used to promote inquiry and facilitate conceptual understanding of students in a first year university mathematics course for engineering students (Jaworski, 2010; Jaworski & Matthews, 2011).

Formative assessment can be facilitated to a great extent using computer algebra tools such as MapleTA; implementation cases have been detailed in Brito et al., (2009) and in Jones (2008). Students' experience is reported to have been positive towards providing formative assessment activities using MapleTA (Brito et al., 2009)

Advances in online tools can be used in an innovative manner for enhancing students' experience of mathematics teaching and learning and for enabling students' autonomy in the learning process. Specific online learning services provide support for mathematics instruction in higher education, such as MyMathLab (www.mymathlab.com) and ALEKS (Assessment and LEarning Knowledge Spaces, www.aleks.com). MyMathLab enables educators to design a customisable e-learning module that contains many useful features, such as interactive assignment exercises with guided solutions, personalised study plan, multimedia aids including videos of lectures and animations, assessment managers for editing tests and quizzes, and a grade book that automatically tracks students' results. ALEKS is an online assessment and learning system that utilises artificial intelligence algorithms for adaptive assessment of a student's knowledge of the course. Potocka (2010) implemented an online mathematics course that could be followed entirely without a need for an instructor. Students who followed the course have achieved similar or better exam scores than their counterparts who attended traditional lectures. Due to its success and advantages, Potocka (2010) indicates that such an approach could be a very useful addition for University classes. Hagerty and Smith (2005) utilised ALEKS for a college algebra course; evaluation indicated that students who followed ALEKS have significantly showed better performance and mastery of the subject than students who followed the traditional teaching approaches.

Wikis and online forums have been used to facilitate discourse and collaboration among students learning mathematics (Carter, 2009; Peterson, 2009). It is argued that Web 2.0 tools (e.g. Wikis and social networking websites) may facilitate cyber-social-constructivist learning of mathematics. Classroom voting systems such as TurningPoint (2011) have been used for facilitating discourse in mathematics lectures (King & Robinson, 2009).

Constraints in implementing innovations in teaching

Traditional teaching methods are familiar, and are easy to conduct or follow for both lecturers and students (Mokhtar et al., 2010); there are difficulties in overcoming traditions of mathematics instruction (Hagerty el al., 2010). A reform towards embracing student-centred approaches in mathematics instruction, by nature of being different from the norm and requiring alternative ways of thinking and resource provision, would attract conflict and resistance from students, teachers, and institutional administrators and policy makers (Hodge, 2006; Johnson et al., 2009). Student-centred methods such as discovery based learning tend to feel uncomfortable when being tried first; change requires time to take place and these methods needs lots of work by the teacher (Hodge, 2006). Many students believe that transmitter-receiver teaching approaches are the only path for successful teaching (DeLong & Winter, 1998). In some cases, students are considered as customers to be satisfied in so far as they pay high University fees so that it is important to satisfy them according to their own perceptions of need (Johnson et al., 2009). It can be argued that low rates of digital literacy, in particular among some teachers, can hinder the adoption of modern technologies into mathematics learning and teaching. As opposed to mathematics teachers at school level who often receive considerable pedagogical training, higher education mathematics' instructors are in general specialist mathematicians with little (if any) pedagogical background. This could be a factor militating against innovative methods.

Despite the many constraints and a natural resistance towards change, trends reported in the literature suggest that there is evidence of success in implementation, overcoming of constraints and enhanced learning experience. Johnson et al. (2009) enumerate a number of strategies for dissolving conflicts and constraints arising while deploying student-centred approaches, including tactics for classroom and actions to be taken with administration and policy makers of the educational institute. They argue that resistance to change can be managed and should not be a reason for giving up reform into innovative methods in mathematics teaching.

Discussion

It is worth mentioning that methods for implementing innovative teaching and learning in mathematics are highly heterogeneous and widely varied; apart from those mentioned so far, other methods include games (Gallegos & Flores. 2010), peer instruction (Alsardary & Blumberg, 2009; Lucas, 2009), competitions (Bruks, 2011), self-regulated learning (Lazakidou & Retalis, 2010) etc. This presents a rich set of methods for designing reformed mathematics courses. From our observations in the literature, the question of whether to stick with traditional passive methods or to shift towards innovative methods is clearly answered in favour of the shift. Innovative approaches have been appealing for educators interested in improving mathematics instruction. However, riding the shift implies many serious research questions such as: which innovative approach(es) to choose for particular audience(s), what is the optimal method, how to integrate multiple methods together in a coherent form balancing between emergent complexity and the aimed enhancement of this integration, what novel pedagogies to develop for meaningful utilisation of technology, how to respond to impedance of change, etc. Answering these questions opens a fruitful research field which may radically change mathematics instruction within the next two decades.

Conclusions

This review is by no means exhaustive, but it is representative to a good extent. It seems that innovative methods in learning and teaching of mathematics in STEM higher education are rather new practices. Methods used for facilitating conceptual understanding and constructivist learning include novel pedagogies (e.g. collaborative learning, inquiry/problem/project/discovery based learning), contextualising with real-world examples, the use of documentary movies for stimulating motivation and self-efficacy beliefs, mathematical software packages (e.g. GeoGebra, Matlab/Simulink, LabVIEW, Mathematica, Maple and MapleTA, etc), and online tools (Wikis and web based courses). Most of the studies in this review were from the USA, with a few papers from the UK, Australia and Malaysia. The majority of papers were published in the last few years; this could be a motivating indication for further research investment to contribute to this emerging shift in mathematics education. Collaborative research between mathematicians and personnel from science and engineering might be expected to enrich the field of study.

A post-script on theory in innovative approaches

Some of the papers considered in this review wrote about innovative approaches as "constructivist approaches". In others, it was clear that a theoretical agenda relating to the individual construction of knowledge was implicit. We have used the term "innovative", following certain authors in the review, to distinguish approaches such as IBL and PBL (designed to promote conceptual learning for the individual) from the more traditional approaches (promoting passive learning) that they replaced. We highlighted above (Section 1.1) some of the pillars of so-called constructivist pedagogy. We might ask what it is that makes such innovative approaches "constructivist".

Constructivism is a theoretical stance on knowledge and learning; it is not a pedagogy. It deals with the learning of the individual cogniser through interactions with the external world, physical and social. It is seen to derive variously from the writings of Piaget and Vygotsky. In fact one branch of constructivism, often called Radical Constructivism and seen largely in mathematics education, derives strongly from Piaget and from theoretical constructs of assimilation, accommodation and reflective abstraction (Confrey, 2000; Piaget, 1950; Steffe, 1990; von Glasersfeld, 1987b). An alternative branch, often called social constructivism and seen both in science and mathematics education, derives from Vygotsky and sees individual construction of knowledge as being strongly related to social interaction, discourse and patterns in language (Driver, 1995; Edwards & Mercer, 1987; Ernest 1991; Vygotsky, 1978). In both cases however, theory addresses the learning *of the individual*, albeit in relation to the external world.

The pedagogical positions dealt with above may be seen to be the consequences for teaching of seeing learning through a constructivist lens. Von Glasersfeld, an exponent of Radical Constructivism, has suggested

”noteworthy consequences” of the theory for the teacher, two of which are expressed as follows:

– The teacher will realise that knowledge cannot be transferred to the student by linguistic communication but that language can be used as a tool in the process of guiding the students construction.

– The teacher will try to maintain the view that students are attempting to make sense of their experiential world.

(von Glasersfeld, 1987a, cited in Jaworski, 1994, p. 27)

Thus, the actions of the teacher are seen to derive from the theoretical stance. The teaching approach only makes sense theoretically if the theoretical principles on which it is based are made explicit. This is lacking in many of the articles reviewed. We might ask whether this matters, or why it matters.

The methodology of any research project needs to explain why what is done is done in the way that it is done (Burton, 2002). If research takes a constructivist perspective, then whether it is rooted in Piagetian or Vygotskian psychology, we assume it is looking at the constructions of the *individual* learner, that it is involved with seeing insights to individual cognition. This is useful if we wish to look closely at the ways in which the *individual* construes particular mathematical ideas or concepts (see for examples, the clinical interviews of Steffe, 1983, following Piagetian traditions). Such a perspective, with its focus on the individual, has no tools to address the wider social factors that impinge on the learning context and influence its outcomes. Taking a Vygotskian focus draws attention to the ways in which social factors impinge on the individual?s consciousness. However, to look at learning and teaching in the full sociocultural contexts in which they are located requires an alternative to constructivism.

This is not the place to expound sociocultural theories. Suffice it to say that these are largely rooted in the work of Vygotsky and Vygotskian thinkers and researchers in three generations of theory (See Daniels, 2001, for a succinct overview). They are not related to social constructivism: in fact some scholars would claim the two areas of theory, constructivism and sociocultural theory, are incommensurable (See for example, Lerman, 1996, and Steffe & Thompson, 2000). However, sociocultural theories offer ways of studying learning and teaching in relation to the myriad constraints (and affordances) that impinge on them (institutional systems, societal expectations, cultural ways of being; etc.)

So, to come back to the focus of this review, we suggest that in many cases, the appellation ”constructivist pedagogy” is misplaced. It would be valuable to see a more explicit focus on the theories underpinning particular studies and an associated critique of how data is analysed according to the theories espoused.

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Notes

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2 STEM, standing for Science, Technology, Engineering and Mathematics, has been the basis of a major programme of educational development in the UK since 2002: http://www.nationalstemcentre.org.uk/stem-in-context/stem-background. Its higher education focus has developed since 2006: http://www.hestem.ac.uk/.

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