What characterises the heuristic approaches in mathematics textbooks used in lower secondary schools in Norway?

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In this paper I present findings of an analysis of how mathematics textbooks treat heuristic approaches. The aim of this analysis is to give a characterisation of the occurrence of nine well-known heuristic approaches by analysing 740 examples presented in six ninth grade textbook series. The findings show that many of the problems in the examples are being solved by using one or more heuristic approaches, but the characteristics of the examples and the textbooks' lack of discussion of the approaches themselves make it challenging to teach and learn these in school. The heuristic approaches seem to be used rather incidentally, which is supported by the fact that none of the textbooks explicitly treat or mention problem solving.

Textbooks have received increased attention from the international education community in recent decades, and in mathematics education this can be exemplified by the TIMSS analysis of 318 textbooks from almost 50 countries in 1995. Schmidt et al. (1996) found that the textbook plays an important role in mathematics in school, and that it is a worldwide phenomenon, but that in USA, Spain and Norway it seems to play an exceptionally strong role. The significance of mathematics textbooks in Norway is most likely alone worth an analysis. If we add the chance of the smallest improvement in a textbook and multiply it by the large number of pupils, teachers and parents using it, it indicates a potential for a much greater improvement over all.

Alseth, Breiteig and Brekke's (2003) review of L97 (KUF, 1996) and a report from the Utdanningsdirektoratet (The Norwegian directorate for education and training) (2005) in Norway about teaching materials

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and their use support Schmidt et al.'s (1996) finding by claiming that the textbooks are the foundation for teaching, legitimising and taking care of the sequencing of the teaching. Similar findings are reported from Sweden (Skolverket, 2003; Johansson, 2006), Finland (Röj-Lindberg, 1999), England, France and Germany (Pepin & Haggerty, 2001, 2002).

Teachers frequently use the textbook as their primary source for planning teaching, and therefore it plays an important role for sequencing the material, choosing content of topics and providing the teacher with ideas for instruction and pupil activities (Freeman & Porter, 1989; Apple, 1992; Robitaille & Travers, 1992; Fan & Kaeley, 2000; Schmidt et al., 2001; Fan & Zhu, 2007a; Pepin & Haggerty, 2001, 2002). Reys, Reys and Chávez (2004, p. 1) claim that "the choice of textbook often determines what teachers will teach, how they will teach it, and how their students will learn". The teaching approaches used by teachers in the classroom and those presented in the textbooks are often highly alike (Bierhoff, 1996; Fan & Kaeley, 2000). Garner (1992) claims that the textbook is the primary tool for the pupils to acquire knowledge, and makes the point that the textbook can replace the teacher as the most important source of information. Schoenfeld (1988, p. 163) states that even though good teaching practice can compensate for possible inadequacy of the textbooks "there is evidence to suggest, however, that it does not". The number of pages in the textbook that is assigned to each topic influences the amount of time the teacher spends on the topic and the corresponding level of performance (Chávez, 2003). The textbook has a function as an interpreter of mathematics for teachers, pupils and parents. In addition it has an important role in the implementation of changes in the curriculum. The textbook often defines the implemented curriculum, which may diverge considerably from the intended one.

In comparison with the international average teacher exposition to the whole class forms a proportionately small part of pupils' experience in Norway. Pupils to a great extent work alone with problems in the textbook (Grønmo et al., 2003; Danielsen, Skaar & Skaalvik, 2007). Doyle (1988) argues that problems serve as a context for thinking not only in class but also afterwards. He argues from the assumption that problems, most likely found in the textbook, to a large extent influence how pupils perceive mathematics and understand its meaning. This assumption is supported by Johansson (2006) who found that pupils were exclusively working with problems in the textbook during individual work at school, which on average was more than 50 % of the time, and that the examples and problems were mainly from the textbook. Norwegian pupils do not deviate from the international average in the amount of homework assigned, but the amount of time the teacher uses for feedback to pupils in response to their homework is substantially below the international average (Grønmo et al, 2003). In addition Norway is considerably below the international average in time spent in mathematics lessons per year in fourth grade and slightly below in eight grade (Grønmo & Ostad, 2009). The report from the Utdanningsdirektoratet (2005, p. 23) gives substantial recognition of the importance of textbooks in Norway by stating that "changes in the teaching happen through changes in the textbooks" (author's translation). This statement is of special interest because of the ongoing implementation process of the latest curriculum, LK06 (UFD, 2005a), and its pertaining new textbooks. Pehkonen (1995) claims that teaching can be influenced through new textbooks, which was practised in Finland in the 1980s.

Nasjonalt læremiddelsenter (National centre for teaching aids) was until the year 2000 a special centre for teaching materials in Norway with authorisation to approve school textbooks. Today there is a national centre for mathematics called *Matematikksenteret* (Norwegian centre for mathematics education) without this authorisation. In theory this means that textbooks used by pupils and teachers in school can be written and published by anyone. Since the abolition of the Nasjonalt læremiddelsenter there has been little research on textbooks in mathematics besides the work by Alseth et al. (2003). By comparing the formulations in the curricula M87 (KUF, 1987) and L97 (KUF, 1996), they identified five defining areas and compared them to the textbook topics of geometry and algebra. Problem solving, which was the main new topic in M87 (KUF, 1987), was not one of the defining areas, even though problem solving ingredients like experimenting, exploration, creativity and reflection were found in M87 (ibid.). This reflects the shift within problem solving in Norway from a prominent topic of its own in M87 (ibid.) to a more implicit part in L97 (KUF, 1996). The analysis by Alseth et al. (2003) consisted of content and task analysis, both producing similar findings. Changes according to the defining areas were demonstrated in geometry but not in algebra, thus indicating that textbook authors had only partly captured the defining areas in L97 (KUF, 1996).

In Sweden more research on mathematics textbooks has been undertaken than in Norway, exemplified by Johansson's (2005, 2006) study of the use of textbooks, Jakobsson-Åhl's (2008) study of word problems in algebra and Brändström's (2005) study of differentiated tasks. In Finland Törnroos (2001) has studied mathematics textbooks and students' achievement, Röj-Lindberg (1999) studied the use of textbooks and Pehkonen (1995) has written about pupils' reaction to the use of open-ended problems in textbooks. Based on the assumption that textbooks are important for the teaching and learning of mathematics, research on problem solving in textbooks has also received attention from a range of international researchers (Nibbelink, Stockdale, Hoover & Mangru, 1987; Mayer, Sims & Tajika, 1995; Hensey, 1996; Li, 1999; Ng, 2002; Zhu, 2003; Zhu & Fan, 2006; Fan & Zhu, 2007a).

Rationale for analysing heuristic approaches

Inspired by Schoenfeld's (1985) decades of effort to understand and teach mathematical problem solving, his two major questions – what does it mean to "think mathematically"? and how can we help pupils to do it? – have constituted the foundation for my motivation to analyse heuristic approaches in textbooks. His framework for the analysis of problem solving behaviour consists of four qualitatively different aspects, cognitive resources, heuristics, control, and belief systems. His rationale for the study of heuristics and for teaching problem solving via heuristics is based on the following chain of thought. Throughout the years in school every pupil solves many mathematical problems. The solving of these is sometimes done by using a new approach and sometimes by an approach that was successful earlier. If an approach succeeds several times, the pupil may use it again when faced with a similar problem. Over time one can speak of an approach becoming a part of a strategy for solving problems. Over the years each pupil comes to rely, possibly quite unconsciously, upon those approaches that have proven useful. Each pupil will thereby develop a personal and idiosyncratic collection of problem solving approaches. But despite being idiosyncratic the approaches are also somewhat uniform, meaning that there is a substantial degree of homogeneity in ways that expert problem solvers approach new problems. By making systematic observations of experts solving large numbers of problems, it is possible to identify and describe important approaches like the ones Polya proposed (1945) in his "short dictionary of heuristic". Finally when the most important approaches have been discovered and elaborated, instruction about these approaches should be done in order to save the pupils the trouble of having to discover them on their own. In this paper I present findings of how mathematics textbooks treat approaches of this kind.

Such approaches also constitute an important part in Lester's (1996) programme for how to teach problem solving. Lester splits the teaching and learning of each of 10 approaches into two phases. The first phase includes teaching of what the approach really is about and how to use it, followed by a sequence where the pupils practise the approach themselves. The second phase is about when to use the approach. The pupils are then presented problems to be solved, but they have to decide themselves which approach is suitable. Lester (1996) states that casual attention to problem solving or occasional elements of problem solving is not enough to get pupils to become problem solvers, and that is in line with Hembree's (1992) statement that of all instructional methods, heuristics training appears to provide the largest gains in pupils' problem solving performance. Polya (1945, p. 133) states that "heuristic aims at generality, at the study of procedures which are independent of the subject-matter and apply to all sorts of problems". By learning such a set of heuristic approaches it seems reasonable to expect a more effective problem solving.

Another inspiration for my motivation to analyse heuristic approaches in Norwegian textbooks has been the role of heuristics in the leading country in the TIMSS rankings. Since 1990 the development of pupils' ability in mathematical problem solving has been the primary aim and problem solving has kept its place as the core in the Singaporean curriculum's framework (Fan & Zhu, 2007b).

Problem solving is placed at the core in the curriculum's framework with skills, attitudes, metacognition, processes and concepts connected to it (see figure 1). Heuristics constitute a part of the processes involved in mathematical problem solving. The role of heuristics has been substantial in Singapore with explicit lists of heuristics together with samples to illustrate their use in the syllabi, and textbooks devoting whole chapters to the instruction of problem solving with emphasis on specific heuristics

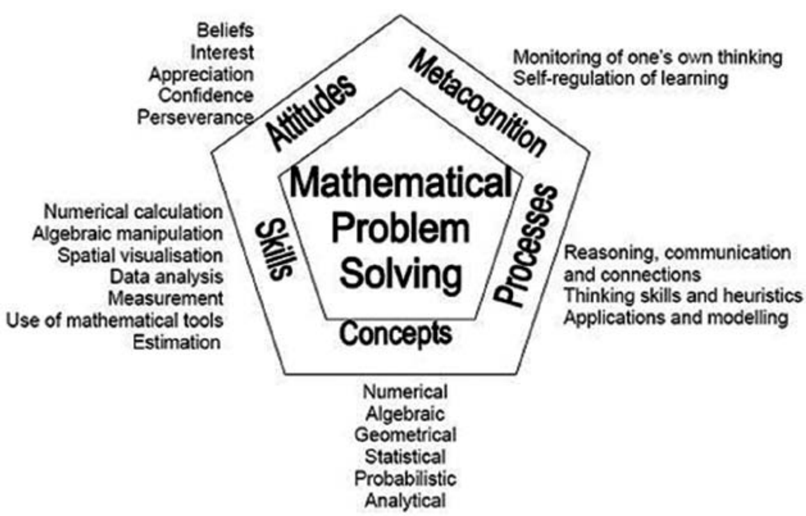


Figure 1. *The framework in the Singaporean curriculum (MOE, 2007)*

(Zhu, 2003; Fan & Zhu, 2007b). Among these heuristics ”use a model”, often called ”bar modelling”, has become a label of Singapore school mathematics (figure 2). The solution explicitly labels the approach, ”use a model”, which includes visualisations and algebraic notation together with an explanation to the solution process. Researchers within problem solving in Singapore have studied the advantages and disadvantages of how textbooks present problem solving (Ng, 2002; Zhu, 2003), and their findings have influenced the development of new textbooks (Fan & Zhu, 2007b). Internationally there has not been much research on how textbooks represent heuristic approaches (Fan & Zhu, 2007a).

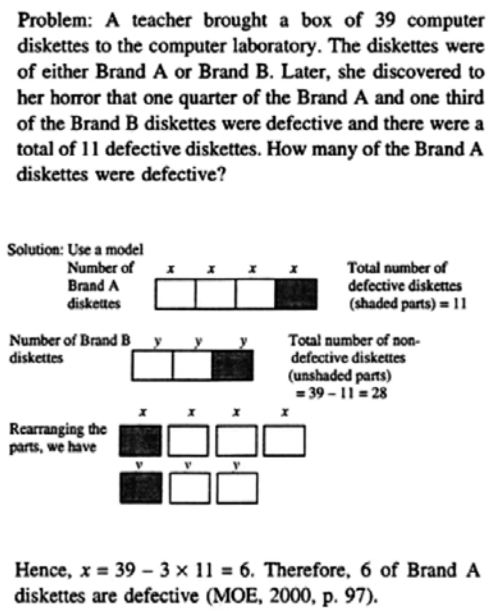


Figure 2. *Use a model (Fan & Zhu, 2007b, p. 496)*

By considering the teaching and learning of mathematics in school from both a theoretical and a pragmatic point of view, it seems reasonable to seek development and strengthening of effective approaches to solve problems such as the ”bar modelling”. Educators at different levels and textbook authors in mathematics have a responsibility to treat such approaches in various ways in order to create learning resources so that the pupils can become better problem solvers.

Based on the explanation in the two sections about rationales for the analysis my aim is to give an objective characterisation of how mathematics textbooks used in lower secondary school in Norway treat heuristic approaches, reveal some of their similarities and differences, and add some knowledge of how textbooks can contribute to the teaching and learning of heuristic approaches.

Definitions and discussion of key concepts

Problem

Both ”problem” and ”problem solving” have had different, and sometimes contradictory meanings through the history, and different understandings of what constitute these concepts still exist today (Lester, 1994; Stanic & Kilpatrick, 1989; Schoenfeld, 1992; Pehkonen, 1995; Fan & Zhu, 2000). In general there are two different definitions of problem found in the literature. The general problem solving view of what constitutes a problem is usually as a situation in which a goal is to be attained but no readily accessible approach of solution is available. Some researchers include that the problem solver should make an attempt to achieve the goal, otherwise the situation could not be considered a problem (Charles & Lester, 1984; Mason & Davis, 1991). In both variants whether or not a situation is classified as a problem is mainly determined by the individual's reactions. What is a problem for one individual might not be a problem for someone else. What is a problem for one individual at one time might not be a problem at a later time. Therefore, the property of being a problem is not inherent in a textbook. This definition is therefore problematic for textbook analysis, although researchers such as Boesen (2006) has handled this by looking at what type of reasoning is needed to solve the actual problem.

Another way of defining a problem is as a situation that requires a decision and/or an answer, no matter if the approach of solution is readily available or not to the problem solver. Whether there exists a blockage when the problem solver is attempting to find a solution is not the concern. I have like many other researchers who have conducted textbook analyses used this definition in my study (Kantowski, 1981; Nibbelink et al., 1987; Stiff, 1988; Stockdale, 1991; Li, 1999; Mayer, Sim & Tajika, 1995; Zhu, 2003; Fan & Zhu, 2007a). This definition is more operational for textbook analysis but has the result that all situations that require a decision will be defined as a problem. This is of course in contrast to the general problem solving view of what constitutes a problem. However, elementary mathematical problems can also present all the desirable variety of kinds of problems, and the study of the approaches used to solve them is particularly accessible.

Heuristic approaches

Polya (1945, p. 112–113) states that ”the aim of heuristic is to study the methods and rules of discovery and invention. [...] Heuristic, as an adjective, means 'serving to discover.' ” Inspired by Polya (1945) and Björkqvist (2003) I use the term *heuristic approaches* and try to adjust it to the level of mathematics in lower secondary school. In the definition I have tried to incorporate Schoenfeld's (1985, p. 75) claims that ”heuristics are subtle, complex and highly abstract” and ”anyone who thinks that fourth-graders, for example, can use these mathematical strategies in the way Polya described them either fails to understand the complexity of the strategies or fails to understand the lessons from Piaget's work”. I define heuristic approaches as rules of thumb for successfully solving problems, general approaches that help an individual to understand a problem better and/or to make progress toward its solution. It has distinct connections to the notion of heuristics, which Schoenfeld (1985, p. 44) defines as ”rules of thumb for successful problem solving, general suggestions that help an individual to understand a problem better or to make progress toward its solution”. My definition includes the common textbook analysis definition of a problem unlike Schoenfeld's (1985) definition which includes the common problem solving definition and is directly related to heuristics à la Polya. I think that pupils in lower secondary school cannot use such heuristics in the way Polya described them (Polya, 1945). The popularised version of Polya's ideas of problem solving that is often used in school is much less complex than Polya's original ideas. When Schoenfeld (1985, p. 74) speaks about problem solvers and heuristics in the spirit of Polya he presupposes a degree of mathematical maturity and states that ”learning to use heuristics calls for a (reasonably) firm foundation of mathematical resources and for a fair amount of sophistication as well” and sets a lower age limit to college students. By that he suggests that pupils should not first be exposed to heuristics in upper secondary school. The foundations for using such heuristics can and should be established during the whole of a pupil's mathematical education. Younger pupils can recognize, appreciate, and mimic the use of such heuristics in a less strict sense than what is generally treated in mathematical problem solving. I call heuristics in that sense heuristic approaches. An example of such a heuristic approach is ”make a systematic table” and described as ”constructing a systematic list or table containing the possibilities for a given situation”. An example of how one textbook uses this approach to solve a problem is presented in figure 3.

The systematic table consisting of six numbers at the right hand side of the example is the heuristic approach that is used to solve the problem. In

EKSEMPEL

Hvor mange tresifrede tall kan vi lage med sifrene 5, 7 og 9?
Du kan bruke hvert siffer en gang i hvert tall.

579	759	957
597	795	975

EXAMPLE

How many three-digit numbers can we make with the digits 5, 7 and 9?
You can use each digit once in each number.

579	759	957
597	795	975

Figure 3. *Heuristic approach (Torkildsen & Maugesten, 2007b, p. 79, author's translation)*

the study I analyse nine heuristic approaches labelled "look for a pattern", "make a systematic table" "make a visualisation", "guess and check", "solve part of the problem", "work backwards", "think of a related problem", "simplify the problem" and "change your point of view". These approaches are gathered from a literature review (Polya, 1945; Lester, 1996; Björkqvist, 2003; Fan & Zhu, 2007a) and selected after a small-scale tryout. More details regarding the selection of the approaches are presented in the methodology section later in the paper.

According to Schoenfeld (1992) the scientific status of heuristic strategies à la Polya (1945) has been problematic, especially in the 1970s, when referring to studies by Wilson (1967) and Smith (1973), which concerned the classical transfer-problem of learning. Both studies indicated that the heuristics that students were taught did not, despite their seemingly generality, transfer well into new domains. Studies by Kantowski (1977), Kilpatrick (1967) and Lucas (1974) indicated that the students' use of heuristics was relatively small but positively correlated with performance on ability tests and on specially constructed problem solving tests. Later studies have however provided further empirical evidence to back up the sense that heuristics can be used as a means to enhance performance in problem solving,

> [...] many classroom-based studies have indicated that appropriate instruction on problem-solving procedures, especially problem-solving heuristics, benefits the development of students' ability in problem-solving (e.g., Charles & Lester 1984; Hembree 1992; Higgins 1997; Lee 1982; Oladunni 1998; Schoenfeld 1979)
>
> (Fan & Zhu, 2007a, p. 62)

and thereby support Schoenfeld's (1992, p. 352) statement that "the evidence appears to have turned in Polya's favour".

Problem solving

Problems have occupied a central place in mathematics curriculum since antiquity (Stanic & Kilpatrick, 1989). In contrast, the importance of problem solving just started to be recognized in the late 1970s (Schoenfeld, 1992), and in terms of research it has been a focus for three decades (Zhu, 2003). Since the late 1970s the importance of developing pupils' competence in problem solving has been widely recognized. *Problembehandlingskompetence* (problem solving competence) is one of the eight competencies identified in the Danish Competence in Mathematics-project (Niss & Jensen, 2002, p. 200). According to Blomhøj and Jensen (2007) Niss' (2002) set of mathematical competencies has the potential of replacing the syllabus as the focus of attention due to its alternative vocabulary for what it means to master mathematics. A competence based description of mathematics is to some extent also found in the Norwegian curriculum and can be exemplified by the description of the objectives of the subject, "problem solving is part of mathematical competence" (UFD, 2005b, p. 1). In my study problem solving is interpreted as a competence similar to the one exposed in the curriculum (UFD, 2005a) which is informed, in part, by Niss' report (2002).

Whether it is used to solve mathematical problems in school or to solve problems during ones spare time, problem solving is an essential competence that every person should acquire. Although there are some gifted people who have achieved such a competence in a natural way, the vast majority of people require training to develop a competence in problem solving. Providing this training is an important responsibility resting with educators, including teachers of mathematics and textbook authors.

In *How to solve it* (Polya, 1945) Polya presented for the first time his now famous four-stage model of the problem solving process. Under those four stages, a variety of specific problem solving heuristics is available for problem solvers to use in order to understand or/and solve problems. The four-stage model is often referred to as a general strategy and the heuristics as more specific strategies, both are rules of thumb for making progress in problem solving (Borgersen, 1994).

Scientific studies of mathematical problem solving are usually dependent on how one defines the relationship between the problem and the problem solver. My own study is consistent with the majority of researchers who differentiate between the problem and the problem solver (Björkqvist, 2003). This separation is usually done in order to make generalisations possible regarding mathematical problems or problem solvers.

A common understanding of "problem solving" is even more diverse than the understanding of "problem". When trying to find a definition of problem solving fitting the culture of school mathematics, the literature itself reveals what Schoenfeld (1992, p. 334) indicates as an area ”somewhat ill-defined and poorly grounded”. A major reason for this is that problem solving has contained multiple meanings from ”working rote exercises” to ”doing mathematics as a professional mathematician”. Regardless of Polya's (1945) famous book from 1945, it took almost 40 years before problem solving established roots in school mathematics, exemplified by its shift in status from ICME IV in 1980 to ICME V in 1984 when problem solving became a main theme (Schoenfeld, 1992). (ICME stands for International Congress on Mathematical Education). Polya's (1945) ”short dictionary of heuristics” has been criticized and to some extent been used as an explanation for the late establishment in school mathematics, mainly because of its descriptive rather than prescriptive nature. The latter means that the characterisation of the heuristics makes it possible to recognise the strategies when they are being used, but do not provide the necessary details to enable pupils who are not already familiar with the strategies to implement them. This issue is of special interest for textbook authors and teachers when dealing with heuristic approaches. According to Branca (1980) problem solving can mean different things to different people at the same time and different things to the same person at different times. Many have presented interpretations of ”problem solving” by using slightly different perspectives, for example Hatfield (1978), Branca (1980), Stanic and Kilpatrick (1989), Schroeder and Lester (1989), Ernest (1998), Lester and Lambdin (2004) and Stacey (2005).

Problem solving in the curriculum

Stanic and Kilpatrick (1989) have labelled three themes regarding the role and use of problem solving in school mathematics curricula. These are problem solving as context, as skill and as art. In problem solving as context, problem solving is usually not seen as a goal in itself, but the problems and the solving of these are used in the service of other curriculum goals. The interpretation of problem solving is minimal just meaning working with the problems that have been presented. This theme is divided into five sub-themes; problem solving as justification, as motivation, as recreation, as vehicle and as practice. The first and the last two of these are of special interest for textbooks in schools, because historically problem solving has provided justification for teaching mathematics in the first place, where some of the problems are related to real-world experiences and thereby occupy a convincing role. Textbooks consist of many problems and the solving of these can be interpreted as a vehicle for learning new concepts and skills, and as necessary practice to reinforce skills and concepts that have been taught. Within problem solving as skill solving problems is valuable in its own right, where different heuristics are typically taught as subject matter with practical problems assigned so that the heuristics can be mastered. Schoenfeld (1992) characterised the vast majority of curriculum development and implementation that in the 1980s went on under the name of problem solving as skill. In Norway this was the case in the M87 curriculum (KUF, 1987). Problem solving within this theme is seen as a valuable curriculum goal in itself and leads to certain consequences for the role of problem solving in mathematics textbooks with regard to hierarchy. This can be exemplified by the fact that solving a non-routine problem is considered to be a higher level skill than solving a routine problem. Problem solving as art contains a deeper and more comprehensive view of problem solving, and arises in the work of Polya. He viewed problem solving as a practical art, like playing the piano, and possible to learn by imitation and practice. Polya used the notion of plausible reasoning, interpreted as different ideas about mathematical discovery. The complexity in mathematical problem solving, which Schoenfeld (1985) elucidates, and I have tried to consider in my definition of heuristic approaches, is also found in Stanic and Kilpatrick's (1989, p. 16) discussion of plausible reasoning. They claim on one hand that "if students are to use plausible reasoning, they need to be taught how", but on the other hand they warn against the danger of reducing problem solving as art to problem solving as skill, when attempts are made to implement Polya's ideas and putting them into textbooks. Stanic and Kilpatrick's statement is of interest for textbook authors and teachers dealing with problem solving, and touches the issue of Polya's (1945) descriptive rather than prescriptive nature of heuristics.

The latest TIMSS report for Norway (Grønmo & Ostad, 2009) makes a general call for more discussion and argumentation about problem solving and strategies, and especially states that classes that do so perform far better than classes that do not. The importance of strategies is also exemplified by Alseth's (1995) notion of the process of mathematical thinking, where an individual's cognition is divided into basic knowledge, strategies, meta-cognition and attitudes.

In the current Norwegian curriculum, LK06 (UFD, 2005a), problem solving is stated as a competence: "Problem solving is part of mathematical competence. This means analysing and transforming a problem into mathematical form, solving it and assessing the validity." (UFD, 2005b, p. 1). The content of this quotation seems to indicate an exclusion of problems which already are mathematical. Is this intentional? A closer investigation of the Norwegian curriculum concerning "the five basic skills" indicates that this is probably not intentional: "Numeracy in the mathematics subject is, needless to say, the foundation of the mathematics subject. This involves problem solving and exploration, starting with practical day-to-day situations and mathematical problems." (UFD, 2005b, p. 3). According to LK06 (UFD, 2005b, p. 3) the "basic skills are integrated in the competence aims where they contribute to development of the competence in the subject, while also being part of this competence". These basic skills are common for all subjects in school and known as "being able to express oneself orally, being able to express oneself in writing, being able to read, numeracy and being able to use digital tools".

Out of a total of four times, problem solving is mentioned in the curriculum twice under the basic skills. The third time is: "Being able to use digital tools in the mathematics subject involves [...] learning how to use and assess digital aids for problem solving, simulation and modelling." (UFD, 2005b, p. 4). The quotation can be interpreted as using digital aids as an approach to solve mathematical problems. The fourth time problem solving is mentioned explicitly is as a competence goal within the topic of "numbers and algebra" after the tenth school year: "The aims for the education are that the pupil shall be able to use, with and without digital aids, numbers and variables in exploration, experimentation, practical and theoretical problem solving and technology and design projects." (UFD, 2005b, p. 9). What exactly is meant by practical and theoretical problem solving is not explained. An interpretation from a textbook perspective is that pupils use numbers and variables, for instance in an equation, to solve practical and theoretical problems.

In the LK06 (UFD, 2005a) heuristic approaches are not mentioned but strategies are mentioned twice under the basic skills. The first time in general terms without any explicit connection to problem solving: "Being able to express oneself orally [...] means talking about, communicating ideas and discussing and elaborating on problems and solution strategies with others." (UFD, 2005b, p. 3). The importance of discussion and argumentation about problem solving and strategies is underlined in the latest TIMSS report (Grønmo & Ostad, 2009) and found closely related to the level of performance in mathematics. The second time strategies are mentioned a connection to problem solving is noticeable: "Numeracy in the mathematics subject is, needless to say, the foundation of the mathematics subject. This involves problem solving and exploration [...]. To manage this, pupils must [...] have the ability to use varied strategies, [...]." (UFD, 2005b, p. 3). Even though the exact wording is not "problem solving strategies" or "heuristic approaches" in either of the two quotations above, it is reasonable to interpret them as implicitly dealing with well-known approaches such as "guess and check" and "look for a pattern" to solve problems. The only time specific approaches are mentioned is under the basic skills: "Being able to express oneself in writing in the mathematics subject means solving problems by means of mathematics, describing a process of thinking and explaining discoveries and ideas; one makes drawings, sketches, figures, tables and graphs." (UFD, 2005, p. 3). The quotation is about solving problems, not problem solving, but well-known heuristic approaches such as "make a visualisation" and "make a table" are quite noticeable. The quotation also includes important problem solving components like "describing a process of thinking and explaining discoveries and ideas" (UFD, 2005, p. 3), which Grønmo and Ostad (2009) call for in the latest TIMSS report.

Nordic school systems, when comparing themselves internationally, find it especially interesting to investigate the situation in Singapore and Finland. In Singapore important components in Schoenfeld's (1985) chain of thought are quite recognizable. The role of heuristics has been substantial with an explicit list of heuristics together with samples to illustrate their use in the syllabi (Zhu, 2003; Fan & Zhu, 2007b). In the famous "bar modelling" example from Singapore the description of the solution process is also given a substantial role. In Finland problem solving has been one of the overall goals in the curricula for more than twenty years (Pehkonen, 2007). Today the use of problem solving tasks is quite popular in the mathematics lessons and the textbooks are influenced by a desire to especially develop the pupils' thinking skills and problem solving skills.

Methodology

Design and method

I have used a cross-sectional design and a research method known as content analysis. A cross-sectional design involves the collection of data on more than one case at a single point in time in order to collect a body of quantitative or quantifiable data in connection with two or more variables which are examined to detect patterns of association. According to Bryman (2008, p. 274) the two best-known definitions of content analysis are the following: "Content analysis is a research technique for the objective, systematic and quantitative description of the manifest content of communication" and "Content analysis is any technique for making inferences by objectively and systematically identifying specified characteristics of messages". The qualities of "objectivity" and "being systematic" are decisive for any content analysis. Objectivity means that the rules are clearly specified in advance for the assignment of the raw data into categories. Objectivity is therefore linked to transparency in the procedures for assigning the material to categories in a way that minimize personal bias. The ideal in content analysis is simply to apply the rules in question. As the reader will know, the quality of being systematic refers to the consistent manner in which rules are applied so that bias is minimized. Considering the qualities of objectivity and being systematic at the same time has formed the ideal of my content analysis, namely that anyone who employs the same rules to the same data will come up with the same results. In order to secure these qualities in the best way the design of the coding scheme is of great importance. My coding scheme consists of a coding schedule and a coding manual. A coding schedule is a form onto which all the data relating to an item being coded will be entered. The coding schedule that I have developed is presented in figure 4.

Textbook	Example		Main subject area					Heuristic approach								
	total	approaches	numbers and algebra	geometry	measuring	statistics, probability and combinatorics	functions	look for a pattern	make a systematic table	make a visualisation	guess and check	solve part of the problem	work backwards	think of a related problem	simplify the problem	change your point of view

Figure 4. *Coding schedule*

My coding schedule consists of four dimensions with different categories within each dimension. The dimensions of ”textbook” is not coded with anything other than the name of the analysed textbook. The dimension of ”example” is divided into two sub dimensions consisting of the total number of examples in the textbook and the total number of heuristic approaches. The dimension of ”main subject area” is divided into five sub-dimensions consisting of the topics found in the curriculum (UFD, 2005a). I have used ”+” if the topic is present in the textbook and ”–” if it is not. The dimension of ”heuristic approach” is divided into nine sub-dimensions consisting of the above presented heuristic approaches within problem solving. When designing a coding schedule the different dimensions should be discrete, meaning that there is no conceptual or empirical overlap between them. This is unproblematic in my schedule with completely separate dimensions. It is also important that the categories within each dimension are mutually exclusive, meaning that there is no overlap between the categories. This has been a challenge within the last dimension where my initial inspiration was Fan & Zhu's (2007a) international comparative study of how mathematics textbooks in China, Singapore and USA presented 17 problem-solving categories. These categories were based on the different heuristics found in the national syllabuses and standards of the three countries. Several of these heuristics were neither found in the Norwegian curriculum nor the textbooks, some of which called for a great deal of mathematical knowledge and sophistication as well. An example of such an approach is "make suppositions". This

Heuristic approaches	**Descriptions**
1. Look for a pattern	Identifying patterns in the given problem based on observation of common characteristics, variations, or differences in the problem.
2. Make a systematic table	Constructing a systematic list or table containing the possibilities for a given situation.
3. Make a visualisation	Creating a visualisation on the available information to visually represent the problem.
4. Guess and check	Making a reasonable guess of the answer and then checking the result to see if it works. If necessary, repeating the procedure to find the answer, or at least a close approximation.
5. Solve part of the problem	Dividing a problem into sub-problems, then solving them one by one in order to solve the original problem.
6. Work backwards	Approaching a problem from its outcomes or solutions backwards to find what conditions they eventually need to meet.
7. Think of a related problem	Using methods or results of a related problem, or recalling a related problem, or considering a similar problem solved before in order to solve the problem.
8. Simplify the problem	Changing the complex numbers or situations in the problem into simpler ones without altering the problem mathematically.
9. Change your point of view	Approaching a problem from another angle.

Figure 5. *Coding manual*

approach was described as ”making a hypothesis, and then based on the givens and hypothesis, finding out the relationship between the known and unknown, and finally solving the problem” (Fan & Zhu, 2007a, p. 66). I also found difficulties regarding the mutual exclusiveness of some of Fan & Zhu's (ibid.) approaches. This was the case with ”restate the problem” and ”simplify the problem”. One of Fan & Zhu's (ibid.) approaches, ”use an equation”, was excluded because of its major tradition as a more general approach than as a typical heuristic approach in problem solving. Based on the revision of Fan & Zhu's (ibid.) list and additional reviews of approaches in Polya (1945), Lester (1996) and Björkqvist (2003), my study is based on a characterisation of the occurrence of the nine above presented heuristic approaches. Another side of the selection of approaches is, as with much content analysis, the fact that I am just as interested in the omissions of approaches as in what approaches occur. Omissions of specific approaches are in themselves potentially interesting, as they may reveal what is and is not regarded as important to textbook authors.

The coding manual must give clear descriptions of the different categories meaning that the coder should have little or no freedom or discretion in how to allocate codes to the units of analysis. Figure 5 shows my coding manual for the heuristic approaches.

Suitability of the method

There are several advantages gained by using content analysis in textbook research. First of all it is a very transparent method. The coding scheme together with a detailed description of possible sampling procedures makes both replication and follow-up studies feasible. This transparency explains why content analysis is often referred to as an objective method. Content analysis is also suitable for longitudinal studies such as Jakobsson-Åhl's (2008) study of mathematics textbooks, and it thus creates possibilities for further research. Content analysis is also known as an unobtrusive method (Cohen, Manion & Morrison, 2011), meaning that one can observe without being observed, thus making it a possible non-reactive method. The main use of content analysis is traditionally in the examination of printed texts, documents and of mass media items (Bryman, 2008).

As with every research method, content analysis also has some disadvantages. A content analysis can only be as good as the documents on which the researcher works. This means that the documents should be assessed in terms of authenticity, credibility and representativeness. Authenticity is whether the textbooks are what they purport to be and credibility concerns whether the books may be distorted in some way.

This is unproblematic because I have informed all the publishing houses about my study and received the textbooks directly from them. Representativeness is whether the documents are representative of all possible relevant documents. A lack of representativeness could be noticed if there were to exist textbooks for lower secondary school based on the curriculum which are not included. In order to minimize this have I asked publishers, lower secondary school teachers and people working with textbooks within the mathematics community which textbooks they know exist on the market. Based on this informal investigation I have identified seven series. Another disadvantage is connected to the coding manual which is almost impossible to devise without entailing some interpretation on the part of the coder(s). In order to reduce this disadvantage I have used an experienced teacher and researcher within mathematics education to code separately and independently all the 214

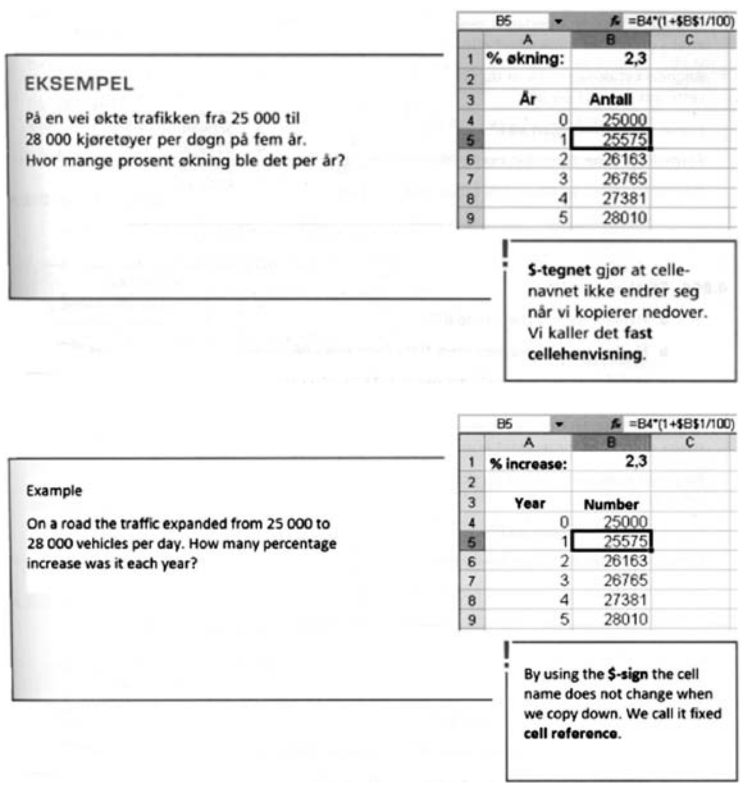


Figure 6. *Example of an inconsistency (Torkildsen & Maugesten, 2007b, p.51, author's translation)*

examples in Sirkel (Torkildsen & Maugesten, 2007a, 2007b). Our coding results were to a large extent consistent. Whenever there was an inconsistency we scrutinised the example together once more and agreed upon a coding. An example of a characteristic inconsistency is presented in figure 6. In the example one coder had coded this as ”make a systematic table” whereas the other had coded it as ”make a systematic table” and ”guess and check”. There was no problem agreeing about the coding and we decided that the example should be coded as dealing with both approaches.

Selection of textbooks

As mentioned earlier, the Norwegian requirement for the national approval of textbooks was removed in 2000, and on the open market today there are at least seven different mathematics textbook series being used in lower secondary school. Because the publishers of several series had not published books for grade ten when I started my research, I have analysed textbooks from the highest available grade at that time, ninth grade. I wanted to analyse textbooks from the highest available grade based on the assumption that these were more suitable for treating heuristic approaches. I have analysed the textbook series which have the classical structure of one textbook for each school year. These six textbook series are Grunntall (Bakke & Bakke, 2007), Kode X (Christensen, 2007a, 2007b), Mega (Guldbrandsen, Melhus & Løchsen, 2007a, 2007b), Faktor (Hjardar & Pedersen, 2006), Sirkel (Torkildsen & Maugesten, 2007a, 2007b) and Tetra (Hagen, Carlsson, Hake & Öberg, 2006). All six series are written by Norwegian authors except the latter which is a translation from Swedish.

Data collection

All problems presented in the main text part in the textbooks, usually labelled and intended as examples, have been analysed. No problems in the sets of exercises have been included, because no heuristic approaches are illustrated in these, only a single answer is given. Many examples were coded using more than one heuristic approach.

Findings and discussion

Looking at the six textbook series and the different dimensions in the coding schedule together, is generating a matrix of data (table 1). The matrix demonstrates for example that Faktor (Hjardar & Pedersen, 2006) has 89 examples which include 107 heuristic approaches. Since the total number of heuristic approaches is larger than the total number of examples it is obvious that several examples include more than one approach. The matrix also demonstrates that Faktor (ibid.) has chapters covering all topics treated in the curriculum. The codes in the dimension of ”heuristic approach” demonstrate that Faktor (ibid.) has one example coded as ”look for a pattern”, eight ”make a systematic table”, 31 ”make a visualisation”, 52 ”solve part of the problem” and 15 ”change your point of view” approaches. The four ”0” demonstrate that ”guess and check”, ”work backwards”, ”think of a related problem” and ”change your point of view” have not been found in Faktor (ibid.).

The matrix and the bar diagram (figure 7) shows that ”make a visualisation”, ”solve part of the problem” and ”change your point of view” are the most common approaches, and that especially ”guess and check”, ”work backwards”, ”think of a related problem” and ”simplify the problem” are rarely presented in the textbooks.

It is apparent when the textbook series are seen individually that Sirkel (Torkildsen & Maugesten, 2007a, 2007b) with 359 approaches distributed among 214 examples stands out in respect to both the total number of

Table 1. *Matrix of data*

Textbook	**Example**		**Main subject area**					**Heuristic approach**								
	total	approaches	numbers and algebra	geometry	measuring	statistics, probability and combinatorics	functions	look for a pattern	make a systematic table	make a visualisation	guess and check	solve part of the problem	work backwards	think of a related problem	simplify the problem	change your point of view
Mega	97	137	+	+	-	+	+	0	6	32	0	72	0	0	0	27
Grunntall	105	162	+	+	-	+	+	3	7	39	1	76	0	0	2	34
Tetra	126	235	+	+	-	+	+	7	10	64	2	104	0	1	10	37
Kode X	109	175	+	+	+	-	-	2	6	51	0	106	0	0	0	10
Sirkel	214	359	+	+	+	+	-	10	28	103	5	146	1	0	3	63
Faktor	89	107	+	+	+	+	+	1	8	31	0	52	0	0	0	15

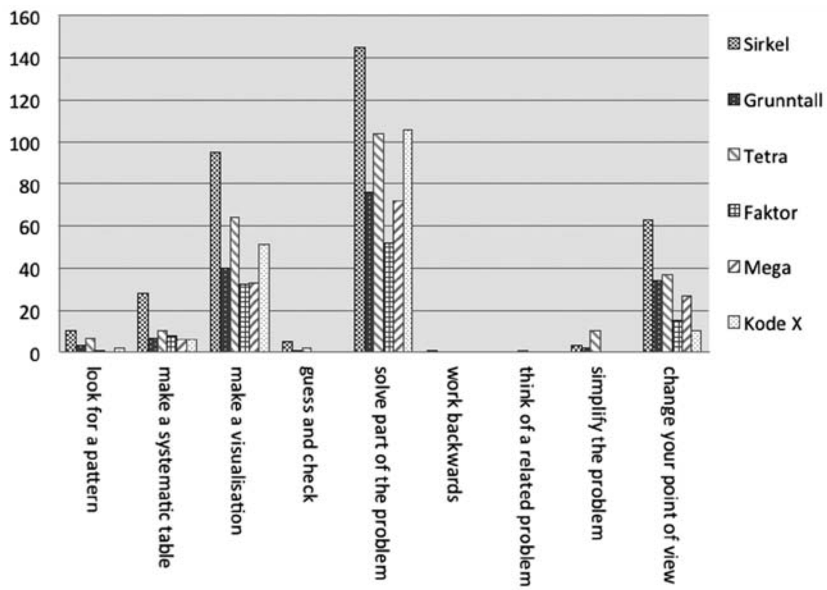


Figure 7. *Bar diagram of heuristic approaches*

approaches and examples. In addition Sirkel (ibid.) together with Tetra (Hagen et al., 2006) cover most approaches with eight out of nine. These two series also stand out with the only examples of, respectively, "work backwards" and "think of a related problem". Faktor (Hjardar & Pedersen, 2006) has the lowest number of approaches and examples. Despite being the only textbook covering all topics in the curriculum, Faktor (ibid.) only covers five approaches, and of these "look for a pattern" is exemplified once. Mega (Gulbrandsen et al., 2007a, 2007b) is covering the fewest approaches by exemplifying the four most common approaches.

Examining the approaches individually and observing their distribution among the textbooks, reveal some interesting features (table 2). All analysed textbooks include the topics of "numbers and algebra" and "geometry" which one might expect to include several "look for a pattern" and "guess and check" approaches. But "look for a pattern" is exemplified only 23 times, of which 17 are found in Sirkel (Torkildsen & Maugesten, 2007a, 2007b) and Tetra (Hagen et al., 2006). "Guess and check" with its general applicability and the curriculum's (UFD, 2005b, p. 3) requirement for pupils to have "the ability to use varied strategies", is only exemplified eight times, of which seven are found in Sirkel (Torkildsen & Maugesten, 2007a, 2007b) and Tetra (Hagen et al., 2006). However, even though the total number of examples and approaches vary a lot between the

Table 2. *Relative distribution in percentage*

Textbook	Heuristic approach									Total
	look for a pattern	make a systematic table	make a visualisation	guess and check	solve part of the problem	work backwards	think of a related problem	simplify the problem	change your point of view	
Mega	0	4	23	0	53	0	0	0	20	100
Grunntall	2	4	24	1	47	0	0	1	21	100
Tetra	3	5	27	1	44	0	0	4	16	100
Kode X	1	3	29	0	61	0	0	0	6	100
Sirkel	3	8	29	1	41	0	0	1	17	100
Faktor	1	7	29	0	49	0	0	0	14	100

series, the relative distribution is to some extent consistent. For instance the relative distribution of "make a visualisation" varies from 23 % in Mega (Gulbrandsen et al., 2007a, 2007b) to 29 % in Kode X (Christensen, 2007, 2007b), Faktor (Hjardar & Pedersen, 2006) and Sirkel (Torkildsen & Maugesten, 2007a, 2007b).

Despite the fact that a few approaches in some textbooks stand out, for instance "change your point of view" in Kode X (Christensen, 2007a, 2007b), the overall impression of the relative distribution in each textbook is similar. "Solve part of the problem" varies most but is also the approach used most often. An example of this approach is shown in figure 8. In the example the original problem is divided into sub-problems, which are solved one by one, before the original problem is solved. By checking the answer in the end a part of Polya's last stage in his four-stage model is also visible. This stage is usually labelled as "looking back" and only occasionally found in the textbooks, always in connection with specific examples and hardly ever connected to problem solving in general or to Polya's model in particular. In fact Grunntall (Bakke & Bakke, 2007) is the only textbook with a reference to a list of explicitly named heuristic approaches. A three page sub-chapter, called "Hvis vi står fast" ("If we get stuck", author's translation) (ibid., p. 142), gives two

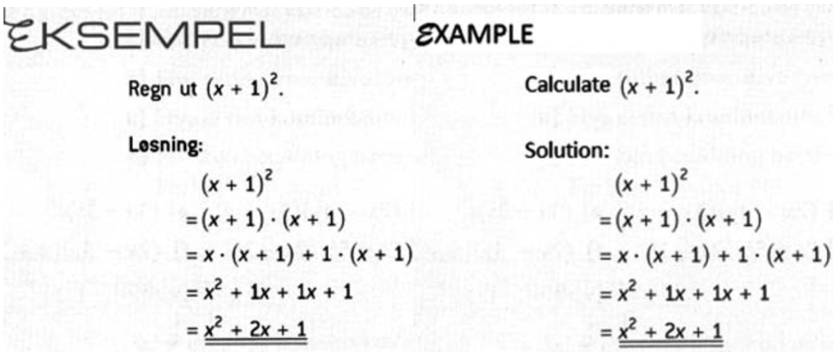
Eksempel	Example
Herman arbeider i 3 timer, og Sara arbeider i 4 timer. De får 560 kr til sammen for dette arbeidet. Hvor mye får hver av dem?	Herman works for 3 hours, and Sara works for 4 hours. Together they get 560 kr for this work. How much does each of them get?
Løsning	*Solution*
Herman arbeider: 3 timer	Herman works: 3 hours
Sara arbeider: 4 timer	Sara works: 4 hours
Til sammen: 7 timer	Together: 7 hours
Lønnen for én time blir: 560 kr : 7 = 80 kr	The salary for one hour: 560 kr : 7 = 80 kr
Herman får: 3 · 80 kr = 240 kr	Herman gets: 3 · 80 kr = 240 kr
Sara får: 4 · 80 kr = 320 kr	Sara gets: 4 · 80 kr = 320 kr
Vi kontrollerer svaret: 240 kr + 320 kr = 560 kr	We check the answer: 240 kr + 320 kr = 560 kr

Figure 8. *Solve part of the problem (Hjardar & Pedersen, 2006, p. 25, author's translation)*

explicit examples of using equations to solve problems, together with a reference to five approaches presented in the textbook for grade eight (Bakke & Bakke, 2006, p. 163-180). These five approaches are called ”uvanlige metoder” (”unusual methods”, author's translation) and put under the heading of ”kreativitet og fantasi” (”creativity and imagination”, author's translation) (Bakke & Bakke, 2006, p. 164). They are not explicitly related to problem solving but given explicit names and would have been covered by the approaches known as ”solve part of the problem”, ”work backwards”, ”make a visualisation”, ”guess and check” and ”make a systematic table” in my study. The presentation of the approaches is in line with Lester's (1996) programme for how to teach problem solving by following both phases. The three page sub-chapter in the analysed ninth-grade textbook (Bakke & Bakke, 2007) presents ”use an equation” as the sixth approach. The presentation of the approach does not fit the second phase in Lester's (1996) programme because all the problems afterwards all can be solved using only this approach. The presentation of these six approaches in Grunntall (Bakke & Bakke, 2006, 2007) for grades eight and nine is nevertheless a contribution to the treatment of heuristic approaches in textbooks.

”Solve part of the problem” is by far the most exemplified approach in all six textbook series. This can be a result of its usefulness and a shared understanding of its importance among textbook authors. However, it can also be that by grade nine textbooks include several multi-steps problems. As with every description, a description of an approach will result in examples being coded that might not fit the typically wanted example.

In "solve part of the problem" this has been the case with examples which include one or more intermediate steps in the solving process (figure 9).


EKSEMPEL

Regn ut $(x + 1)^2$.

Løsning:

$$(x + 1)^2$$
$$= (x + 1) \cdot (x + 1)$$
$$= x \cdot (x + 1) + 1 \cdot (x + 1)$$
$$= x^2 + 1x + 1x + 1$$
$$= \underline{\underline{x^2 + 2x + 1}}$$

EXAMPLE

Calculate $(x + 1)^2$.

Solution:

$$(x + 1)^2$$
$$= (x + 1) \cdot (x + 1)$$
$$= x \cdot (x + 1) + 1 \cdot (x + 1)$$
$$= x^2 + 1x + 1x + 1$$
$$= \underline{\underline{x^2 + 2x + 1}}$$

Figure 9. *Example with intermediate steps (Christensen, 2007a, p.60, author's translation)*

The intermediate steps between $(x+1)^2$ and x^2+2x+1 in the solution process can be interpreted as dividing the problem into sub-problems which are solved one by one. Examples including intermediate steps like this can be seen as a sub-category of "solve part of the problem" and to a large extent explain the high number of "solve part of the problem" approaches. According to Schoenfeld (1985) many heuristic approaches subsume half a dozen sub-categories or more. This means that the typical description of a heuristic approach is more like a label for a category of closely related approaches. Examples that display intermediate steps are beside their role as a sub-category also occupying an important role regarding how one can explain a solution process by using mathematical symbols.

"Make a visualisation" is one of two specific approaches found in the curriculum (UFD, 2005a) and the second most occurring approach (figure 10). The first solution starts by making a drawing to visually describe, and in this case practically also solve, the problem. The second solution uses simple calculation to solve the problem. Since the example presents these two different approaches to solve the problem it is coded as "make a visualisation" and "change your point of view". In the example it is quite clear that the visualisation in the first solution is part of the solution process. This has however not been the case for all examples of this type. In order to get a more balanced impression of the representativeness of examples like this, I have coded all the 320 "make a visualisation" approaches into three sub-categories (table 3). The first category deals with visualisations

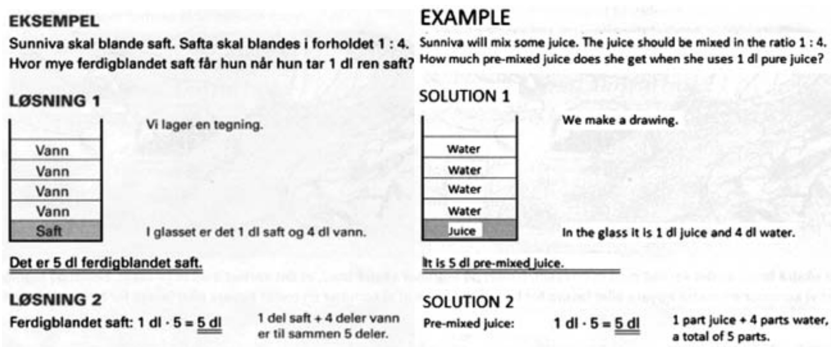


Figure 10. *Make a visualisation (Bakke & Bakke, 2007, p. 46, author's translation)*

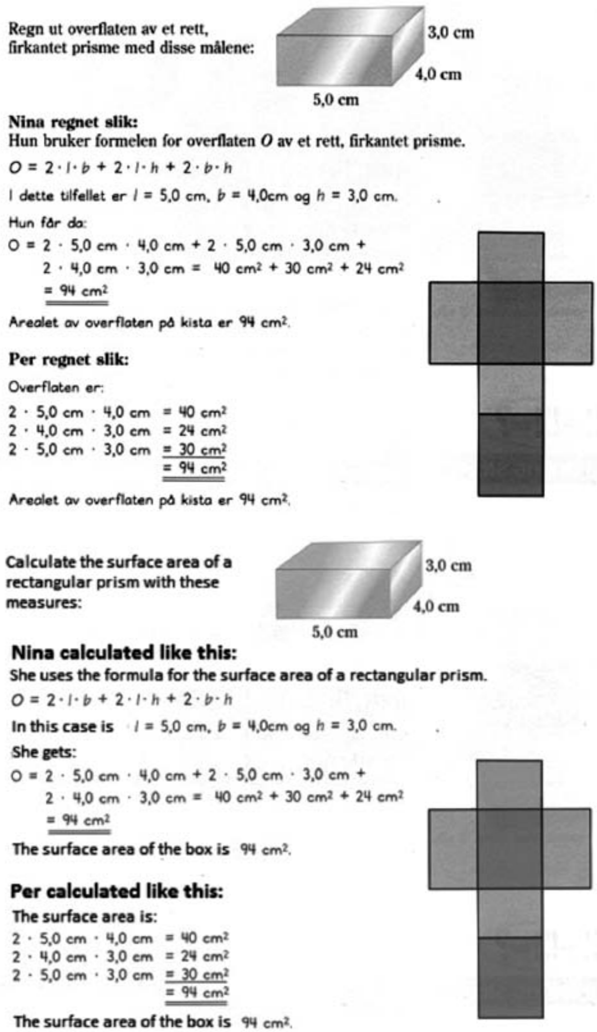
that are a part of the problem statement. The second deals with visualisations that are part of the solution process. The third deals with cases where the wording of the problem directly asks for a visualisation. The first category is further divided into an informative visualisation and a decorative visualisation. The informative visualisation concerns mathematical information, which is necessary in order to state the problem, whereas the decorative visualisation only serves as a decoration without any informative purpose.

From a problem solving perspective "part of the solution process" is of most interest. Out of the 320 approaches there are 128 (40 %) which have the visualisation as part of the solution process. 117 have visualisations as part of the problem statement, where 90 of these are informative- and 27 are decorative visualisations. 75 approaches deal with cases

Table 3. *Sub-categories of "make a visualisation" approaches*

	Part of the problem statement		Part of the solution process	Directly asking	Total
	Informative	Decorative			
Sirkel	28	16	43	16	103
Grunntall	8	0	13	18	39
Tetra	7	5	36	16	64
Faktor	17	3	5	6	31
Mega	12	0	14	6	32
Kode X	18	3	17	13	51
Total	90	27	128	75	320

where the problem directly asks for a visualisation. Between the textbook series there is great deal of variation, where Sirkel (Torkildsen & Maugesten, 2007a, 2007b) has 43 "part of the solution process" approaches whereas Faktor (Hjardar & Pedersen, 2006) has five. Grunntall (Bakke & Bakke, 2007) stands out with 18 out of 39 (46 %) visualisation approaches where the problem directly asks for a visualisation, and Faktor (Hjardar &


Regn ut overflaten av et rett, firkantet prisme med disse målene:

3,0 cm
4,0 cm
5,0 cm

Nina regnet slik:
Hun bruker formelen for overflaten *O* av et rett, firkantet prisme.

$O = 2 \cdot l \cdot b + 2 \cdot l \cdot h + 2 \cdot b \cdot h$

I dette tilfellet er l = 5,0 cm, b = 4,0cm og h = 3,0 cm.

Hun får da:

O = 2 · 5,0 cm · 4,0 cm + 2 · 5,0 cm · 3,0 cm + 2 · 4,0 cm · 3,0 cm = 40 cm² + 30 cm² + 24 cm² = 94 cm²

Arealet av overflaten på kista er 94 cm².

Per regnet slik:
Overflaten er:

2 · 5,0 cm · 4,0 cm = 40 cm²
2 · 4,0 cm · 3,0 cm = 24 cm²
2 · 5,0 cm · 3,0 cm = 30 cm²
= 94 cm²

Arealet av overflaten på kista er 94 cm².

Calculate the surface area of a rectangular prism with these measures:

3,0 cm
4,0 cm
5,0 cm

Nina calculated like this:
She uses the formula for the surface area of a rectangular prism.

$O = 2 \cdot l \cdot b + 2 \cdot l \cdot h + 2 \cdot b \cdot h$

In this case is l = 5,0 cm, b = 4,0cm og h = 3,0 cm.

She gets:

O = 2 · 5,0 cm · 4,0 cm + 2 · 5,0 cm · 3,0 cm + 2 · 4,0 cm · 3,0 cm = 40 cm² + 30 cm² + 24 cm² = 94 cm²

The surface area of the box is 94 cm².

Per calculated like this:
The surface area is:

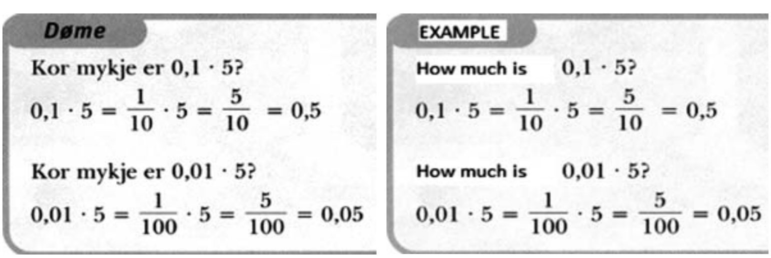
2 · 5,0 cm · 4,0 cm = 40 cm²
2 · 4,0 cm · 3,0 cm = 24 cm²
2 · 5,0 cm · 3,0 cm = 30 cm²
= 94 cm²

The surface area of the box is 94 cm².

Figure 11. *Change your point of view (Gulbrandsen et al., 2007, p. 38, author's translation)*

Pedersen, 2006) stands out with 20 out of 32 (63%) visualisation approaches where the visualisation is part of the problem statement. Out of the total 1175 approaches found in the textbooks 128 of these have visualisation as part of the solution process, thus making it almost 11%. The corresponding approach in Fan & Zhu's (2007a) international comparative study showed that "draw a diagram" was the most common approach in China, Singapore and USA.

The third most common approach in my study is "change your point of view" (see figure 11). In the first approach Nina uses the formula for the surface area of a rectangular prism to solve the problem, whereas Per uses the visualisation of the net of the prism in the second approach. In this example the distinction between the different approaches is very clear, but this is not the case for all examples. One such case includes what I have called "change of form of representation" (figure 12). The original problem of multiplying the decimal number with the whole number, $0{,}1 \cdot 5$, is approached from another angle by transforming the decimal number into a fraction. The same change of form of representation is done when trying to calculate $0{,}01 \cdot 5$. When comparing this example with the previous "change your point of view" example, it is noticeable that whereas the calculation of the surface area is done by presenting two different approaches that solve the problem, the calculation of the decimal number and the whole number is solved by only one approach. The latter is, although more concealed, closer to the common description of the heuristic, meaning that it usually takes effect when a previous approach is no longer effective. This heuristic is in its nature problematic for textbooks due to their written form and impracticality of first presenting an unworkable approach before a suitable approach. Within the context of mathematics textbooks in school both examples, Gulbrandsen et al., (2007, p. 38) and Hage et al. (2006, p. 28), are possible for teacher and pupils to recognize, appreciate and mimic, but the former is probably



Døme

Kor mykje er $0{,}1 \cdot 5$?

$$0{,}1 \cdot 5 = \frac{1}{10} \cdot 5 = \frac{5}{10} = 0{,}5$$

Kor mykje er $0{,}01 \cdot 5$?

$$0{,}01 \cdot 5 = \frac{1}{100} \cdot 5 = \frac{5}{100} = 0{,}05$$

EXAMPLE

How much is $0{,}1 \cdot 5$?

$$0{,}1 \cdot 5 = \frac{1}{10} \cdot 5 = \frac{5}{10} = 0{,}5$$

How much is $0{,}01 \cdot 5$?

$$0{,}01 \cdot 5 = \frac{1}{100} \cdot 5 = \frac{5}{100} = 0{,}05$$

Figure 12. *Change of form of representation (Hage et al., 2006, p. 28, author's translation)*

easier to recognize as an actual change of view, although different from the common problem solving view of the heuristic.

Many examples in the textbooks have been coded with more than one approach. Based on the distribution of single approaches, is it not surprising to find that ”solve part of the problem” combined with ”make a visualisation” and ”solve part of the problem” combined with ”change your point of you” are the two most common combinations of two approaches. The diagram in figure 13 motivates a few comments. In the same way as it was necessary to dig into concrete examples of single approaches to get a more informative and balanced impression, this is also the case for the combinations of two approaches. The concrete examples found in the textbooks of the most occurring combinations are to a large extent consistent with the findings for single approaches. This means that several combinations of approaches such as ”solve part of the problem” and ”make a visualisation” just include intermediate steps and visualisations that are part of the problem statement. Several combinations of ”solve part of the problem” and ”change your point of view” just include intermediate steps and a change of form of representation. The actual content in both combinations are quite unorthodox in relation to traditional mathematical problem solving. However, my findings show that such combinations occur several times in the textbooks. In figure 14 is an example representing the obvious possibilities textbook authors have to present a more traditional content of heuristic approaches. In the first approach, coded as dealing with ”make a systematic table”, the problem

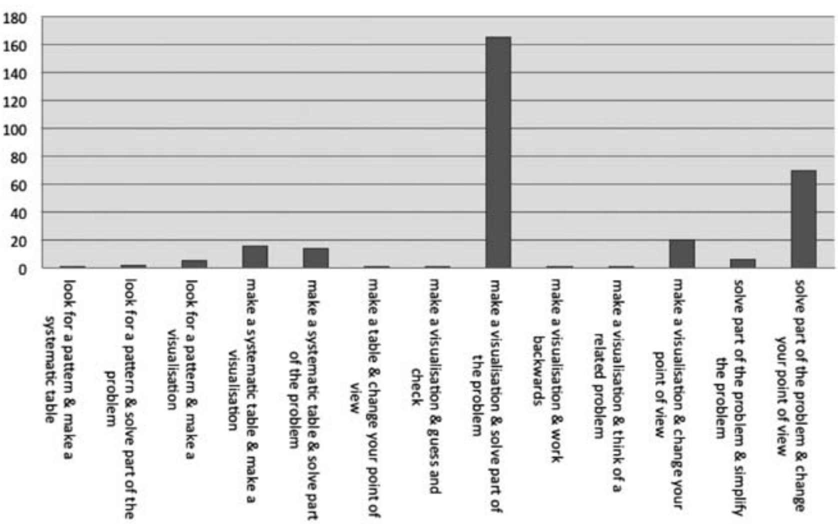
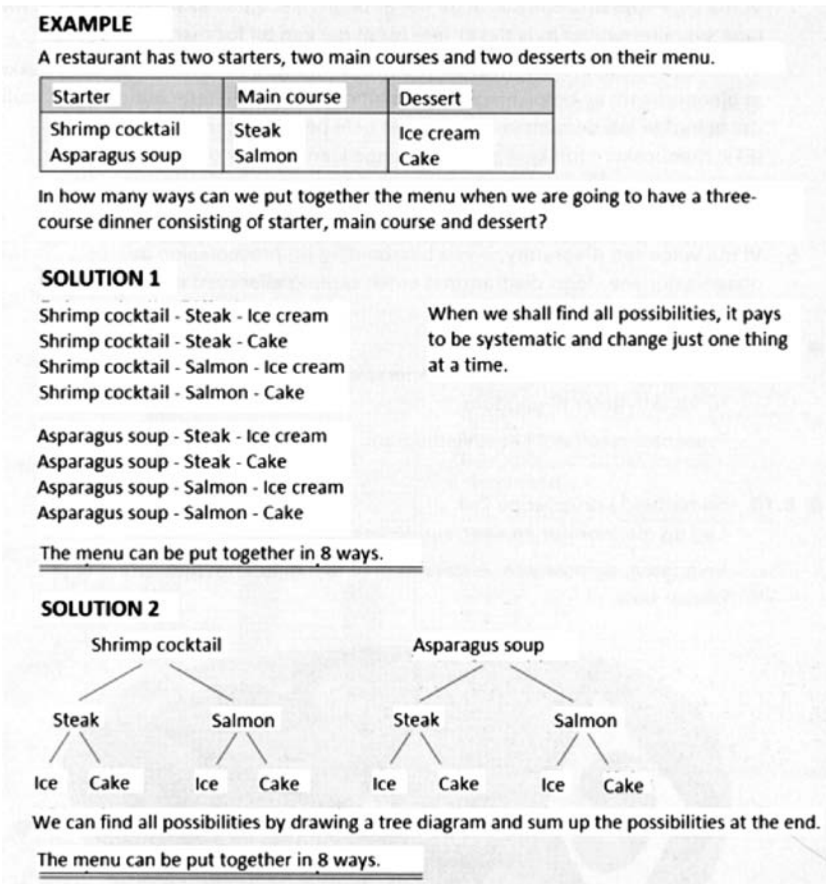


Figure 13. *Bar diagram of combinations of two approaches*

is solved by constructing an organised table containing all the possibilities for the given situation. In the second approach, coded as dealing with ”make a visualisation”, the creation of the tree diagram is the visualisation that helps to solve the problem. This well-presented example is distinguishing between the only two specific approaches found in the currculum (UFD, 2005a) by calling them respectively solution 1 and 2. The example also includes a description of the two approaches thus making relatively clear both the content of the approaches and the differences between them. In order to make the clarification even more visible the approaches could have been labelled with more informative names such as ”make a systematic list” and ”visualisation by tree diagram”, and



EXAMPLE

A restaurant has two starters, two main courses and two desserts on their menu.

Starter	Main course	Dessert
Shrimp cocktail Asparagus soup	Steak Salmon	Ice cream Cake

In how many ways can we put together the menu when we are going to have a three-course dinner consisting of starter, main course and dessert?

SOLUTION 1

Shrimp cocktail - Steak - Ice cream
Shrimp cocktail - Steak - Cake
Shrimp cocktail - Salmon - Ice cream
Shrimp cocktail - Salmon - Cake

Asparagus soup - Steak - Ice cream
Asparagus soup - Steak - Cake
Asparagus soup - Salmon - Ice cream
Asparagus soup - Salmon - Cake

When we shall find all possibilities, it pays to be systematic and change just one thing at a time.

The menu can be put together in 8 ways.

SOLUTION 2

We can find all possibilities by drawing a tree diagram and sum up the possibilities at the end.

The menu can be put together in 8 ways.

Figure 14. *Combination of approaches (Bakke & Bakke, 2007, p. 270, author’s translation)*

thus more in line with the "bar modelling" example presented from the Singaporean textbook (Fan & Zhu, 2007b, p. 496).

Examples in textbooks are to a large extent designed for the teachers' instructional purposes (Love & Pimm, 1996), and by explicitly labelling the approaches teachers and pupils would have greater opportunities to create a common professional language for use in the mathematics classroom (Grevholm, 2004). Being well aware of the danger of reducing problem solving as art to problem solving as skill (Stanic & Kilpatrick, 1989, p. 16) the example from Grunntall (Bakke & Bakke, 2007, p. 270) reveals the potential textbooks have to deal with heuristic approaches explicitly, thus creating better possibilities for teachers to teach and pupils to learn heuristic approaches by imitation and practice in the spirit of Polya. By increasing the numbers of examples like this it would probably be easier for teachers to fulfil Grønmo and Ostad's (2009) call for more discussion and argumentation about problem solving and strategies.

During the analysis of the textbooks an approach which was not listed among the nine emerged. This approach is natural to call "use of ICT" and is exemplified in total 20 times. A typical example of this approach uses a spreadsheet on the computer to solve the problem. In the curriculum the content of such an approach is found in the description of the basic skills concerning the ability to "use and assess digital aids for problem solving". (UFD, 2005b, p. 4).

Summary of findings and reflections

The three most common heuristic approaches in all the textbooks analysed are "solve part of the problem", "make a visualisation" and "change your point of view". Sirkel (Torkildsen & Maugesten, 2007a, 2007b) with its 354 approaches distributed among 214 examples stands out in respect to both the total number of approaches and examples. In addition Sirkel (ibid.) together with Tetra (Hagen et al., 2006) cover eight out of the nine approaches recognised in the study. Especially well-known approaches such as "look for a pattern" and "guess and check" are surprisingly rarely exemplified in all textbooks analysed. When analysing the heuristic approaches in more detail different sub-categories emerged which to a large degree could explain the number of displayed approaches. In order to get more detailed information about the variants within the nine analysed approaches, each of them could be further investigated in relation to possible sub-categories. But even though the total numbers of examples and approaches vary a lot between the textbooks the overall impression of their relative distribution are highly alike. Many examples displayed two approaches, and the most common combinations of two heuristic approaches were not surprisingly a result of the most often occurring individual approaches. Grunntall (Bakke & Bakke, 2007) is the only textbook with a reference to a list of explicitly labelled heuristic approaches. The obvious possibilities for presenting different heuristic approaches in a single problem are exemplified by the above exposed combinatorics problem about putting together menus from Grunntall (ibid., p. 270).

My overall impression of the role of heuristic approaches is that the majority of approaches seems not to be consciously presented but rather as a consequence of an unconscious cultural practice. Zhu (2003) found that that there is a substantial degree of homogeneity among approaches presented by textbook authors. Fan and Zhu's (2007a) international comparative study showed that "draw a diagram", "use an equation", "restate the problem" and "make a table" were the most common approaches in the textbooks from China, Singapore and USA.

By using Wyndhamn and Säljö's (1997, p. 363) interactionist perspective – "What people do and what they learn reflect as much contextual premises and constraints, such as those represented by formal schooling as an environment for communication, as characteristics of individuals" – as a backdrop it is possible to argue that the characteristics of the examples and the attention to the approaches, or rather the lack of attention to the approaches, found in the textbooks constrain the activities that pupils and teachers take part in, and thus influence the process of defining mathematics, problem solving and heuristic approaches. The interactionist perspective includes attempts to account for pupils and teachers taking part in certain activities and not in others. Thus contextual constraints such as limited treatment by textbook authors of what the heuristic approaches are about and how to use them, the possibility of pupils and teachers taking part in discussion and argumentation regarding such approaches decreases significantly. This means that if the textbooks do not treat heuristic approaches in an explicit and systematic manner one cannot expect teachers to teach and pupils to learn it either, thus exacerbating Schoenfeld's (1985) issue of heuristic approaches becoming part of a strategy for solving problems over time. Lester (1996) states that for the pupils who are struggling to become better problem solvers the complexity of problem solving itself is reinforced by the fact that most of them do not get the proper teaching in respect to either quality or quantity. In Lester's (ibid.) literature review of problem solving four significant and general principles are found. The first principle is that pupils have to solve a lot of problems. The second is that problem solving ability develops slowly over time. In consequence textbook authors should consider this when designing a textbook and choosing how to present problem solving ingredients like heuristic approaches. The third, and according to Lester (ibid.) the most important, is that in order to get pupils to respond to the teaching they have to believe that their teacher thinks problem solving is important. This principle together with my findings indicates potentially consequences for teacher educators, curriculum developers, authors of textbooks and teachers. The fourth is that most pupils benefit by systematic teaching in problem solving, which also provides guidelines for textbook authors and teachers of mathematics. Lester's principles (ibid.) and the textbook practice in Norway (Schmidt et al., 1996; Alseth et al., 2003, Utdanningsdirektoratet, 2005) together with my findings suggest a rather problematic picture of heuristic approaches in particular and problem solving in general in lower secondary school in Norway. By taking into account Schoenfeld's (1985) chain of thought, the criticism of Polya's (1945) descriptive heuristics, Grønmo and Ostad's (2009) call for more discussion and argumentation and Lester's (1996) programme for how to teach problem solving, this highlights the importance of an explicit, prescriptive, systematic presentation of heuristic approaches in textbooks over time.

Conclusion

Despite the large number of examples being solved by one or more heuristic approaches, the characteristics of the examples and the textbooks' lack of attention to the approaches themselves make it challenging to teach and learn these within the Norwegian culture of school mathematics. The heuristic approaches seem to be used rather incidentally. Typically the textbook does not label the heuristic approaches and does not present any guidance of how and when to use them. The fact that none of the textbooks explicitly treat or mention problem solving to a large extent shows the invisibility of mathematical problem solving in the traditional sense.

The study has added new knowledge about what characterises the heuristic approaches in mathematics textbooks used in lower secondary school in Norway. It has also demonstrated how textbooks actually could contribute to the teaching and learning of heuristic approaches by offering carefully designed examples in the textbook, such as the combinatorics problem in grunntall (Bakke & Bakke, 2007, p. 270). Niss (2003) claims that if we really want pupils to learn something this has to be taught explicitly to them. Here we find that it depends on the teachers if a variety of heuristic approaches will be taught, because the textbooks do not care to be explicit.

Many well-known approaches are almost totally absent. Approaches such as ”look for a pattern”, ”guess and check”, ”work backwards”, ”thinking of a related problem” and ”simplify the problem” seem to be something every pupil has the right to know something about and to have experienced in mathematics class. It is doubtful if the authors of the investigated textbooks share the view that pupils have the need to explicitly and systematically learn about heuristic approaches. There is a lot of potential for improving mathematics textbooks in lower secondary school in Norway concerning heuristics approaches.

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Appendix

Textbooks

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Torkildsen, S. H. & Maugesten. M. (2007b). Sirkel 9B. *Grunnbok. Matematikk for ungdomstrinnet.* Oslo: H. Aschehoug & Co.

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