

From beliefs to patterns of participation – shifting the research perspective on teachers

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Belief research was introduced to mathematics education in the early 1980s. It challenged the primarily cognitive and mathematical agenda of the time by investigating the character and significance of mental meta-constructs called beliefs. Particular attention has ever since been paid to teachers' beliefs and their role in instruction.

Belief research has been troubled by conceptual and methodological problems since its early beginnings, and most of these are still unresolved. This indicates that it may be time to adopt a different perspective, if we are to understand the role of the teacher for the practices of the mathematics classroom.

Elsewhere we have discussed the problems of belief research at some length and suggested an alternative that we call patterns-of-participation research (e.g. Skott, 2009, 2010). In the present article we briefly recapitulate some of the arguments underlying this suggestion, but our main interest is to use the patterns-of-participation approach for empirical purposes. Consequently the article consists of two main sections. First we summarise some of the problems of belief research and present the contours of our alternative, patterns-of-participation research. Second, we in a much longer section present and analyse data on the case of a teacher, Susanne, whom we follow prior to and after her graduation from college. The overall intention is to suggest a change of research perspective from beliefs to patterns of participation.

Belief research vs. patterns-of-participation research

Belief research has significant conceptual and methodological problems. In this section, we review some of them and present our overall approach when researching patterns in teachers' participation in social practices.

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This brief discussion is merely to function as a basis for our main intention of the article, to illustrate the use of a patterns-of-participation approach for empirical purposes.

The concepts and methods of belief research

It is often pointed out that there is little conceptual clarity in belief research and in particular that there is no consensus about how the key concept of beliefs may be defined (e.g. Philipp, 2007). Looking across a number of standard references, however, it seems that the concept is generally understood as relatively stable, mental constructs that are subjectively true and the result of experiences gained over prolonged periods of time. In some cases definitions include that beliefs have an impact on practice (e.g. Op't Eynde et al., 2002), while in others this is relegated to formally less prominent position (e.g. Schoenfeld, 1998). In general, however, it is an a priori expectation that there is at least a semi-causal relationship between beliefs and practice.

In relation to teachers this means that beliefs are seen as object-like, mental entities that link prior experiences to the practices of the mathematics classroom. The experiences may stem from different contexts, for instance the teacher's personal life, her own schooling, or the theoretical or practical parts of her teacher education. Such experiences are the bases of individual beliefs, which in turn influence the practices of the mathematics classroom (cf. figure 1).

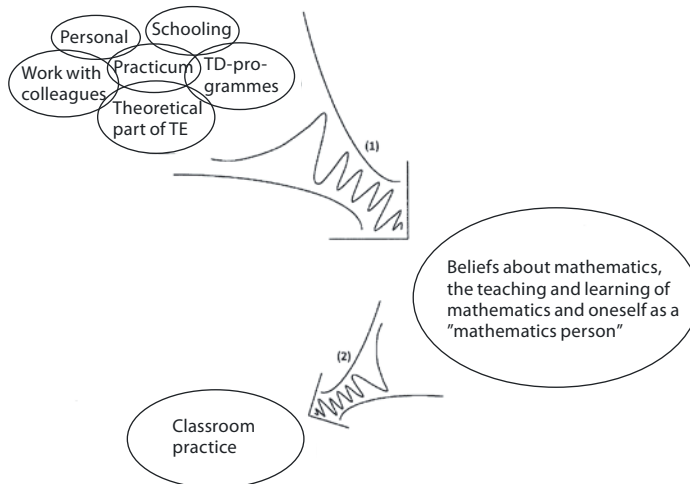


Figure 1.

Elsewhere we have made the point that there are similarities between the individual emphasis in belief research and the acquisitionist metaphor for knowledge in constructivism (Skott, 2010). Building on Sfard (2008), we suggest that in both engagement in social processes is expected to congeal into reified entities in the form of knowledge or beliefs, which become the property of the individual in question. Subsequently these reifications objectify, i.e. they take on a life of their own and lose their connection to the processes that initially gave rise to them.

Constructivist acquisitionism has been challenged by more participatory approaches, not least by Sfard (2008). She claims that we need to maintain the process character of individual meaning making and introduces the notion of commognition in order to do so. As the term suggests, Sfard emphasises that cognition and communication are different manifestations of one and the same social process, and that thinking should be viewed as individualised interpersonal communication. In a somewhat similar sense our point in the present article is that we need to maintain the processual emphasis also in research on what is traditionally referred to as beliefs. Instead of considering the teacher's meaning-making and contributions to the interactions of the mathematics classroom the result of an enactment of objectified beliefs, we view them as the simultaneous engagement in a range of mathematical, meta-mathematical, and broader social practices, the specific character and relative significance of which emerge in the interactions in the locally social.

Even if the conceptual problems of belief research are solved or deemed inconsequential, there are still significant methodological problems to be overcome. Teachers' beliefs are reifications, and in the research process the beliefs that are attributed to or symbolically imposed upon teachers are a result of a similar, but second-order reification process on the part of an observer. The task for the researcher, then, is to infer and/or attribute a set of pre-existing preferences or priorities – beliefs – to the participating teachers based on interpretations of the experiences they appear to have gained from engagement in social practice. The problem of method is how to get access to these elusive mental constructs that reside entirely within the individual.

It is generally agreed that verbal accounts do not necessarily reveal what people "really believe". As Wilson and Cooney (2002) point out, inferences of teacher's beliefs that are based on espoused versions are at best tentative interpretations that may lack explanatory power in relation to classroom practice. The response to this problem is often to use methodological triangulation, and combine surveys and research interviews with observations of classroom interaction. Between them these

different methods are expected to specify teachers' beliefs with greater accuracy and in more detail than if one relies on verbal accounts only.

Triangulation in belief research, however, also has its problems. As Lester points out, there is a circular argument involved in the attempts to infer beliefs from practice while explaining the very same practice with reference to beliefs (Lester, 2002). It is another problem that triangulation in belief research is based on the assumption that different research methods shed light on the same underlying construct of beliefs. This assumption was questioned by Hoyles (1992) and later by Lerman (2001), who both suggest that beliefs are situated. Consequently there is little reason to expect that beliefs espoused in a research interview resemble those that are enacted in the mathematics classroom, although there may be "a family resemblance" between the two (Lerman, 2001, p. 36).

There are, then, significant conceptual and methodological problems in belief research. We do not intend to come to the rescue of the field by remedying these maladies. Rather than making yet another definition of beliefs or devising other ways of getting access to their "true character", we circumvent the problems by suggesting that we do away with the concept of beliefs altogether. Instead of working with objectified mental constructs we focus on the pre-reified processes that are said to give rise to them. This is the essence of patterns-of-participation research.

The concepts and methods of patterns-of-participation research

Students and teachers of mathematics engage in multiple simultaneous practices in the classroom some of which relate to the teaching and learning of the subject and some of which do not. There are patterns in the ways in which they participate in these practices and contribute to their continuous reconstitution and renegotiation. The patterns specifically related to mathematics implicitly or explicitly address for example what questions to ask, what types of answers to expect and provide, how to seek the answers, and when and how to elaborate on a solution or procedure, if at all. Also, the patterns map out the relative responsibilities of teachers and students as they engage in interaction.

The notions of practice and participation are key constructs in this. As we use the term, *practice* is a social phenomenon. Practice emerges in the locally social and is a result of individual and collective meaning-making and agency. It is embedded in broader social situations, but the emphasis on emergence means that we regard it as an empirical question how and to what extent for instance a school culture, the students' family backgrounds, national or local educational regulations, or recommendations for reform play a role for the practices that evolve. With this understanding of *practice* the phrase of "the teacher's practice" is a misnomer:

a practice does not belong to any one individual. This is the case also for classroom practices in spite of the teacher's unquestionable influence.

The teacher participates in classroom practices. At all times she interprets the students' individual and collective actions symbolically, drawing on a range of other social practices to do so. Some of the practices in which the teacher engages are discursive in an explicit verbal sense, while others are not; and some of them are virtual in the sense that they relate to communities that are not physically present in the classroom or at the school (Skott, 2009). For example a teacher may during an interaction with the students draw on practices related to her colleagues, the school management, the parents, and the participants in her pre-service teacher education programme. In the terminology of Lave (1988), all of these may function as resources that asymmetrically structure the teacher's contribution to the practices that unfold in the situation. It follows that classroom practices are social in the dual sense that they are not determined by any one individual and that each participant interprets, makes sense of, and contributes to the unfolding classroom events by drawing on a range of other prior and present practices that may originate in other contexts.

Our approach in the present article, then, is that the teacher's participation in the practices of the mathematics classroom is a meaning-making activity in which she negotiates and contributes to their continuous (re-)generation. As she manoeuvres in relation to multiple, simultaneous actual and virtual communities of practice, she becomes involved in actions as diverse as repeating procedural explanations, solving disciplinary problems, ensuring a student's position in the classroom, making a mathematical conjecture an object of joint investigation, manifesting her own professional authority, and taking a child's problematic home situation into account. In all of this, patterns from the teacher's prior engagement in social practices are enacted and re-enacted, moulded, fused and sometimes changed beyond recognition as they confront, merge with, transform, substitute, subsume, are absorbed by, exist in parallel with and further develop those that are related to the more immediate situation. Some of the prior patterns are linked to immediate social interaction, for instance in mathematics classrooms. Others are primarily linguistic and discursive in a verbal sense and the result for example of theoretical considerations and exchanges.

From this perspective, teaching is a matter of continuously manoeuvring and (re-)engaging in different and sometimes mutually conflicting forms of practice. The research task is to disentangle the patterns of the teacher's participation in these past and present practices and to understand if and how they influence the learning opportunities that evolve in the classroom.

To a large extent the methods of patterns-of-participation research are the same as those of belief research. We also use for instance interviews, observations, and document analyses, and our previous critical comments about the methods in belief research are only in part a critique of the methods themselves. However, the most important themes of research interviews may differ and the settings that one may wish to observe are extended to for instance staff rooms and team meetings. This reflects that there are major differences between the intentions behind the use of any combination of the methods in belief research and in patterns-of-participation research. In this sense the methodological stance differs between the two fields, even when the methods are the same. This is evident for instance in relation to triangulation. In patterns-of-participation research we do not assume that one might get better access to the true character of contextually and temporally stable constructs like beliefs. Different methods are used exactly because they may shed light on decidedly different forms of practice and decidedly different modes of participating in them. For example, one may use a stimulated recall interview as a supplement to a set of classroom observations because it allows the teacher to become engaged in discussions of classroom practice and indicate if, how, and to what extent she re-engages for instance in the theoretical discourse of her teacher education programme or what she regards as a dominant educational practice at her school when relating to those that unfold in the classroom.

The main purpose of empirical investigation, then, is not to allow a close reading of objectified beliefs and of their (semi-causal) relationship to the practices of the mathematics classroom. Rather, it is to disentangle how participation in other past and present practices come together to form the teacher's interpretation of classroom interaction and influence how she participates in them.

This means that observations of staff room interaction, of communication with the management, of discussions in teams of cooperating teachers and of PTA-meetings may supplement the observations of classroom teaching. Interviews may be used to develop understandings of the teacher's relation to the practices of her college education, invite narratives of the teacher's own schooling, and allow the teacher to explain her views of collaborative opportunities, or the lack thereof, at the school. And stimulated recall interviews may bring to the fore elements of conflict between for instance the current classroom practices and teacher's engagement in more theoretically discursive ones. These methods are not to shed light on objectified mental entities. Rather they are to allow interpretations of the patterns in the teacher's engagement in multiple simultaneous practices.

The case of Susanne at Southern Heights

Susanne is 37 years old when she graduates from a Danish teacher education college in 2008. She initially studied sports at university and after her bachelor's degree she got a part time position for a national sports association. Later she began teaching mathematics, science and English at Southern Heights Primary and Lower Secondary School. She did so without a degree in education, but enjoyed it and decided to enrol in a 2-year college programme for second-career prospective teachers.

At college Susanne specialises in mathematics and is granted an exemption for a specialisation in physical education because of her university background. Formally, the mathematics programme at the college emphasises not only the subject itself, but also educational issues. The local regulations state that "mathematics and mathematics education are viewed as two sides of the same coin, as the identity of the subject [at the college] connects mathematics, the theory of mathematics education, and teaching practice". In mathematics the students are expected to work in ways that model the teaching-learning processes they are to initiate upon graduation, for instance doing investigations.

Susanne continues to teach full time at Southern Heights while at college. Upon graduation, she accepts the offer of a permanent position at the school. She now teaches mathematics in grades 5 and 6 as well as a number of other subjects in middle school.

Notwithstanding the connotations of the name, Southern Heights is located fairly closely to the city centre. It is a big school with 85 teachers, and according to Susanne it enrolls students from mixed social backgrounds. Some are children of career-minded professionals and academics, while others are from families with weak educational backgrounds, some of which are hard hit by social problems. Approximately 25% of the students have a different mother tongue than Danish. Susanne sees the mixed backgrounds of the students at Southern Heights as one of the advantages of the school, as everybody realises that some people are very different from themselves, but "nice and sensible people anyway" (the first interview with Susanne). However, this mixture is also one of main challenges of working at the school.

Methods

The study of Susanne is part of a larger study involving two other practising teachers and three prospective teachers. The main interest of the study is the relationships and possible tensions between the teacher education programmes and the teachers' instructional approaches in mathematics, both during their practicum and after their graduation. Susanne

was deemed particularly relevant for the study, because she was working as a teacher while at college.

The study of Susanne spans almost two years. We first contacted her, when she was in the final term of her teacher education programme and teaching at Southern Heights. We audio-recorded a semi-structured, qualitative interview with her about her pre-service education and about classroom teaching. Also, we visited her at Southern Heights and video-recorded altogether six mathematics lessons from her grade 3 classroom. We contacted Susanne again 18 months later, i.e. slightly more than a year after her graduation. We video-recorded six more lessons and conducted two more semi-structured interviews, one prior to the observations and one after having observed four lessons. All audio- and video-recordings were transcribed in full and analysed using methods inspired by grounded theory, but without the objectivist connotations sometimes associated with grounded theory.

Initially the project built on a relatively traditional approach to belief research. The data, then, were not collected with the intention of using a patterns-of-participation perspective. Consequently the dataset has its limitations when viewed from such a perspective, and the study would benefit from additional observations of for instance the practices of mathematics teaching and learning in Susanne's teacher education programme as well as of staff room conversations and team meetings at Southern Heights. With these limitations in mind, the data available may be interpreted in terms of patterns of participation.

Tales of change and tradition

In the interviews Susanne uses her educational and professional experiences to position herself in relation to three somewhat disjoint sets of social practices. One of them consists of a public meta-Discourse¹ about school mathematics and is represented by dominant rhetorical accounts of the subject at the college and in curricular documents. It seems inspired by what is often referred to as *the reform* (cf. Skott, 2004). The other set of practices is Susanne's dominant experiences with mathematics teaching and learning as a student. This is less meta-Discursive in an explicit, verbal sense and does not to the same extent as the reform produce rhetorical accounts of what school mathematics is or should be. However, it introduces Susanne to norms for social interaction and mathematical activity in mathematics classrooms. These two sets of experiences are concerned with (mathematics) education in fairly general terms. A third set of practices also plays prominently in the interviews with Susanne. This is more local in the dual sense that it is particularly related to her

experiences at Southern Heights, and that it concerns the specific issue of how to deal with students who face problems in or out of school.

Before discussing episodes from Susanne's classrooms, we look at how she relates to each these three sets of social practices. It is important to mention that when we refer to "traditional practice", "the reform" and "the practices at Southern Heights", we are talking neither about observer independent entities, nor about results of empirical investigation of these practices per se. We are referring to the meaning Susanne seems to make of them as she participates in the research interviews. It should be mentioned also that while Susanne uses the notion of "traditional teaching" repeatedly, she does not use the word "the reform". This is our shorthand for her reference to aspects of the meta-Discourse mentioned above.

Susanne's educational experiences

Susanne claims that the teaching-learning practices in her mathematics classroom are somewhat traditional. She promotes this tradition, because she enjoyed classes organised in a similar manner, when she was in school herself. For instance she says in an interview a year after her graduation from college²: "I had a fantastic time at school. I really liked all that, you know, when you sat there listening, 'It is like this and like this', and then you did like twenty exercises yourself afterwards [...]" (II). Susanne is explicit that she builds on these early educational experiences in her own teaching.

Second, Susanne's dominant modes of instruction at Southern Heights are implicitly supported by parts her pre-service teacher education. In contrast to the intentions of the course, Susanne claims that a substantial part of it is similar to what she did in secondary school and dominated by the teacher's exposition of proofs for the students to remember and copy and of related exercises for them to do. While still at college she describes the programme in a terminology that strongly resembles her wording about school mathematics 18 months later: "It is [...]'Then you need to know that proof and that proof and that proof', and then we do 30 exercises on it afterwards. It is exactly the same as when I was in upper secondary school" (I).

Susanne is not particularly fond of this approach to teacher education. Her criticism, however, is not directed against this way of working in mathematics. Rather, she suggests that prospective teachers should not spend their time studying the subject itself, but need to be "pumped full of great ideas for how to teach. [...] I would like the college to be more practical" (I).

Susanne, then, claims to be drawing on her own educational experiences when teaching. To a lesser extent she is supported in her approach by discussions about mathematics teaching and learning with her colleagues, or rather by the limitations of these discussions. She regrets that they to a very small extent discuss issues related to school mathematics. Officially they work in teams of teachers teaching a particular year group as well as of teachers teaching the same subject. However, the mathematics team meets rarely and irregularly, and when it does, they discuss practicalities and organisational issues rather than questions related to mathematics teaching and learning. Elaborating on the lack of genuine collaboration, Susanne says that the school consists of "a lot of one-man armies, with each teacher running his own race" (I). In a later interview she mentions one exception to this, namely that she sometimes shares a few ideas with a colleague who also teaches mathematics in grade 6. Susanne emphasises that this colleague also likes "teaching-from-the-board and exercises" (II).

While Susanne vehemently argues for the advantages of her teaching approach, she is not blind to some of the possible drawbacks. In particular there is a dilemma between wanting to provide thorough explanations and keeping the explanations short. On the one hand you want "[...] that everybody understands, and on the other hand it should last only three minutes" (I). Short explanations are important, because there are "quite a few [students] who do not learn anything, who opt out. They focus on infinity³, when you stand up there at the board, and all they hear is her saying bla, bla, bla, bla. [laughs]. I think." (III).

It is difficult for Susanne however, to find other ways of organising instruction than those of the tradition, because "this is the way I was brought up" (II). She knows about the reform Discourse from her teacher education, from national curricular documents and from textbooks. However, she does not consider it a suitable alternative.

Susanne and the reform discourse

As mentioned before, Susanne describes the bulk of her pre-service teacher education in mathematics in terms of its resemblance to what she did in secondary school. This is in contrast to the formal college regulations, as well as to the dominant rhetoric about school mathematics at the college. Susanne refers to the latter as "college talk".

Susanne says that "college talk" focuses on students' investigations and on the use of manipulatives in mathematics. Students need to work independently, using informal methods before they are introduced to formal mathematics. Although she is not unsympathetic to all aspects of

this, Susanne is generally critical of the reform, and primarily associates "college talk" with what she in different interviews describes as a pedagogy of "cut and paste", of "fiddle and touch", and of "cubes and gadgets". She finds it hard to see the mathematical potential of these approaches.

As Susanne sees it, "college talk" is generally in line with the wordings in the national curricular documents. She acknowledges that her scepticism towards the former also brings her somewhat in opposition to the latter. She does not worry though that her teaching is incompatible with the formal requirements, because she and her students follow a textbook scheme closely, in which "you can even smell the college talk" (I).

Susanne is explicit that she uses little of what she sees as the recommendations of her teacher education programme. In this sense her everyday teaching at Southern Heights differs from what she and her fellow students did in their teaching practicum. At that time they

used all that stuff that we learned at the college [..] we made some really funny, alternative teaching-learning sequences. And now, here I am out in the real world, and I don't use very much of it [...] I think the college teachers would like to see some more "landscapes of investigation" – to use a real "college word". [...] There is much more teaching from the board in the real world than what the college teachers would like. (I)

Susanne has two somewhat practical objections to the practices promoted by "college talk". One is that it takes too much time to prepare. The other is that it creates a very noisy classroom atmosphere that she finds it hard to cope with. This last worry is a recurrent theme in the interviews. Susanne focuses on ensuring a reasonable amount of order in the classroom. Explaining her reluctance to engage in practices promoted by "college talk" she says:

I don't like teaching, if there is such a lot of noise and din and fooling around. I get really tired and that is probably why there isn't so much of it. It's okay sometimes as an alternative. "Now we are to do the cut-n-paste-stuff and you can wander around and talk to each other". But it is not going to happen every day in my classes. (I)

There are also other reasons for Susanne to question the college rhetoric. Based on her teaching experiences, she is especially worried that the weaker students are vulnerable if you choose such an approach. As a teacher you may want to help these students by using "fiddle-and-touch", but actually they get more confused. [...] All these alternative ways benefit the strong students. That is what I think." (I).

Susanne returns repeatedly to a theme that is linked to the reform rhetoric: the emphasis on the students' understandings, and the relationship between their understanding and their procedural competence. During our first visit to Southern Heights this is the one part of "college talk" that she considers important: "You know that doctrine that they need to understand and not just follow the rules" (I). Later in the same interview she says that the best part of "college talk" is about "giving the understanding of mathematics. I think it is such a good mixture that first you get the understanding ... 'Ahh, now I know what this is!' And then you learn it by heart. [...]" (I)

As implied by the last comment, Susanne's emphasis on students' understanding does not indicate that she discredits procedural knowledge. On the contrary she is explicit that for instance the students need to know how to multiply and "bloody hell, they do need to know the multiplication tables. [...] that is the basic tenet for all teachers" (I). When she was still at college, however, Susanne emphasizes that procedural knowledge is to be built on conceptual understanding.

A year after her graduation Susanne maintains that "college talk" and the curricular documents focus on the need to build procedures on understanding, "instead of just knowing the algorithm the way I learnt it: that one goes at the bottom and that one on top, and that's it, over and done with" (II). She still keeps this aspect of "college talk" in mind, but she now questions the general recommendation that students need to understand first and practice later. In line with her comment a year earlier, she finds this especially problematic for the weaker students:

Sometimes it is just the opposite. You know [...] for the ones who find it difficult it becomes even more difficult if you first have to develop all these different understandings and "so this is why, errr, this is a result of that and connected to that", and then afterwards I also have to remember that there is this rule that you must remember and that is tied up to all that tedious twaddle. In fact there are some of them who may think "may not understand this, but I know I am going to use it". [...] if you are mathematically weak it doesn't make it any better [to have to understand first]. (II)

In the final interview, she returns to this and says that for some of the ones who find mathematics difficult "it is just great that they know how to do something by heart. And then later, when it comes to understanding, they can focus on infinity" (III).

In summary, then, Susanne is at all times critical of "college talk", and she does not find what she sees as the practical recommendations helpful for addressing her problems of instruction. On the contrary she

claims that the recommendations aggravate existing problems, especially that they contribute to creating a noisy classroom atmosphere. Further, Susanne becomes increasingly critical of the college rhetoric as she gets more experience with classroom teaching. She gradually comes to question the one element of the reform that she initially favours, i.e. the one of building procedural competence on conceptual understanding. Such an approach does not help the weak students for whom it is primarily intended, but leaves them in a situation in which they neither understand, nor get a grasp of the basic procedures of mathematics.

Handling "students with problems" at Southern Heights

As mentioned earlier, Susanne claims that the teachers at Southern Heights are "one-man armies". However, at least to some extent they fight the same battle. In particular everybody shares a concern for how to handle students who are in some sort of trouble.

"Trouble" may mean different things. Susanne is explicit that some of the students at Southern Heights are from families with severe social problems, while others are not. Some are from low-income families; other students are beaten by their parents; and still others have fathers or mothers who are alcoholics or drug addicts. Many of them "go to school every morning without breakfast and without a kiss and a hug and without all the other things that the rest of us consider matters of course" (I).

Susanne is proud that the school "takes incredibly well care of" the students' individual problems by use of different organisational measures (I). For instance there is a special needs department for students with learning problems in particular subjects, and two teachers are particularly responsible taking care of other students in problematic situations. These "BCW-teachers" are to address particular students' problems as they relate to Behaviour, Contact, and Well-being⁴.

Other initiatives to assist students with problems include an "observation class" for students who are violating school norms. The teacher can send unruly students off to "the obs", in order to ensure that they do not disrupt ordinary teaching-learning processes. In Susanne's own words, "the school has decided that we need somewhere, if you [as a student] go over the top, you can go there and calm down" (I). Susanne emphasizes that this is a high priority at the school and that "the obs" is financed entirely by the school's ordinary resources. A few students go there for a single lesson, others go for up to a month, while some stay even longer. This is up to the teachers of the observation class to decide, as they "know these kinds of students, and sometimes they say: 'You are not going back',

and then it becomes a sort of stopover until you get another offer" (I). A considerable number of students are referred to other institutions, because they are not "school-minded" (I).

The social problems at Southern Heights manifest themselves also in Susanne's classrooms. In general her grade 5 is very noisy. Also, there are three students who have different psychological diagnoses and two more, who are in the process of getting one⁵. Other students are repeaters and really weak in mathematics. However, there are still others who perform well in the subject and are difficult to handle together with the other students because of that. And then there is a group of four or five "nice and quiet girls, model students, but it is all just killing them sometimes" (I).

Susanne explains that she sometimes sends students off to the observation class, and that she refers some of the weaker ones to the special needs department. She also separates the students who are not sent off to other classes or teachers into more manageable groups. For instance she asks students who are good in mathematics and who behave well to work together, if possible in a separate room. Other students doing well in mathematics, but who do not behave if in a group or if they are not under continuous, direct supervision are encouraged to work alone within the classroom. Besides, Susanne has agreed with the two colleagues who teach the same grade level that once a week they divide all the students into three groups according to ability levels so that each of them teaches a more homogeneous group.

The rest of the students remain as one organisational unit for most of the time and are rarely divided into groups. This is so although Susanne thinks it would preferable to use group work sometimes and is explicit that she should try to do it more often. Her scepticism, however, is due to her experience that it is bound to result in "sheer hell" and "I'll end up not with one, but with two or three failure groups [...] it is pretty awful to call them that, but you know those [groups] that just do not function in any way" (III).

There are shifting emphases in Susanne's descriptions of the students with problems. When she talks of students in fairly general terms, she focuses on problems that arise due to the students' diverse social backgrounds; when she talks of issues that are closer to the practices of classroom instruction, she emphasises handling problematic students. Using the latter terminology she points to her experience that these students are difficult to manage within the context of instruction, as their presence conflicts with the mathematics education practices in which she seeks to engage herself as well as the class. She ensures that they are taught separately or work on their own by using organisational measures

that appear to be much in line with the general segregational approach to students with problems at Southern Heights.

Observing Susanne's classroom: teaching multiplication in grade 5

Susanne describes the teaching-learning processes in her own classroom as similar to the traditions of school mathematics, i.e. much in line with her own educational experiences with the subject. For instance she says:

I begin by drawing and telling and explaining. "Now listen, this is what you are to do; you are to do these pages". Then they sit there and work for half an hour, and whatever they do not manage to do in class, they have to do at home. My teaching is fairly traditional. Clearly. But then again, they [the students] are to work individually some of the time. (I)

The last comment in this quotation indicates that Susanne sees "the tradition" in school mathematics as dominated by whole class instruction. She views her own teaching as inspired by this tradition, although it involves a greater element of individual work for the student.

In general the observations made in Susanne's classroom are in line with her own description of the dominant teaching-learning processes. This is so both in the first and the second set of observations. The three transcripts presented below are all from a teaching-learning sequence on multiplication in grade 5 during our second set of visits to Southern Heights, i.e. from just over a year after Susanne's graduation. Two are from whole class sessions. The first one is exemplary for the way Susanne presents concepts and procedures in the observed classes. In the second episode a boy, René, suggests an alternative to Susanne's way of solving a particular task. The third transcript is from Susanne's interaction with René during the students' individual work.

Episode 1: Presentation of a multiplication algorithm

Susanne begins the first lesson on multiplication by asking the students to suggest a number between 2 and 9 and another number between 11 and 99. The numbers proposed are 5 and 55, and Susanne writes " 5×55 " on the board. She asks, if anybody can explain what this means. A girl, Mira, says that you have "Fifty-five five times" or "the reverse". Susanne continues:

Susanne: Or the reverse, yes, or five fifty-five times. Exactly. Okay, but that means that I can say that now I take those five [points to 5 on the board] five times first, and then afterwards I take them fifty times.

That should be the same, right? Then I get fifty-five times altogether. It does not matter if I take fifty-five times at once, or whether I first take one pile and then the other pile and add them up, does it? So, let us do that. We begin by taking five five times [points to 5 and the last 5 in 55]. Five times five.

Dagmar: Twenty-five.

Susanne: That is 25. And then this one, this is all the ones, so I write all the ones down here [writes 5 underneath the 5×55].

Dagmar: And the twos go down there? [next to 5 in the result].

Susanne: Well, these are the tens, aren't they? I add those to the next pile, because now I am to multiply the tens. Right. So in reality this is twenty, even though I have written 2 up here, it is really ...?

$$\begin{array}{r} 2 \\ 5 \times 55 \\ \hline 5 \end{array}$$

Figure 2. *On the board*

Dagmar: Twenty.

Susanne: It is really twenty, because it was *twenty-five*, wasn't it [says twenty-five slowly, emphasizing both parts of the word]? But we just write 2. Okay? Then I say, well really I say five times fifty, don't I? I really say five times fifty, but we just do five times five.

Molly: Well, it is 25, but//

Susanne: Yes.

Molly: But isn't it 125? [This may be Molly's suggestion for the result of the whole task].

Susanne: No, because you need to add those two [points to the number carried]. Twenty-five and two?

Molly: Twenty-seven.

Susanne: Then it is twenty-seven. In reality it is two hundred and seventy, because it is five times fifty, this is what I says isn't it? But we did already put the ones down there, so we just write 27 [writes 27 in front of the 5 in the results line].

[...]

Michael: I don't understand this.

Susanne: No, but then I try to explain it once more. [Repeats the explanation in 18 seconds].

Peter: I thought it was five hundred and twenty-five.

Susanne: What did you say?

Peter: I thought it was five hundred and twenty-five.

[Other students suggest other results]

Susanne: Five hundred and twenty-five? ... Okay, then we try to do it differently [goes over the same explanation again].

Between them the introduction and the subsequent whole class examples to multiplication last more than half an hour. In the transcript above Susanne initially asks the students to provide the notion of multiplication with some meaning. Mira explains how the task may be interpreted, and Susanne extends the explanation. Next, she goes over the multiplication of the ones. She explains where to write the partial result, that the 2 is "really twenty", and that she needs to write 2 over the five tens in 55. She then moves on to multiplying five with fifty, and says that you only need to multiply five by five and add the 2 that was carried. In what follows she reacts twice to students' comments that they do not understand her explanations. Both times she does so by going over the procedure again, but without the conceptual clarifications about the meanings of the different digits. In her last explanations, however, her written algorithm is changed somewhat as she writes more elaborate partial results to the task.

Episode 2: René suggesting an alternative procedure

Later in the same whole class session Susanne goes over yet another example of how to multiply one-digit by two-digit numbers. This time the task that Susanne has written on the board is " 7×49 ". When the task is solved a boy, René, raises his hand to make a suggestion:

René: Well, can't you just add two more zeros to the 7, and then take half and minus 7?

Susanne: [Reluctantly tries to make sense of his idea and writes two zeros after 7, see figure 3] So, that I write 700 ...?

$$\underline{700 \times 49}$$

Figure 3. *On the board*

René: No, you know, if the result, well, if that for example was a hundred, then you add two extra zeros. And then you just take half of seven hundred, and then it is three hundred and fifty, and then you minus seven, because it is not fifty, but fifty-nine [René says fifty-nine, but clearly means forty-nine].

Susanne: Ohh, yes! [excited] [deletes the 00 that she wrote next to the 7. It again says 7×49] Yes, yes, now I understand it. You say seven times fifty//

René: Then I just write two extra zeros on the result.

Susanne: Ohh, and that is just one seven too much. And then I subtract seven from seven times fifty.

[Several boys interrupt and talk loudly and all at once. Among the things said: "That is taking half" and "This is cheating"].

René: So you know, first I say like this, you write two extra zeros, when it is one hundred, right?

Susanne: Yes.

René: Then I have that. And then I take half of seven hundred, 'cause it is fifty, sort of.

Susanne: Yes, yes, yes.

René: And then it is three hundred and fifty, but then because it is one less than fifty, I minus 7.

Susanne: That was very advanced, but you are completely right. It was so complicated that I did not understand it at all, but you are completely right. And that is the way it always is when you multiply, if you can get it in some other exciting way, you just do that.

René suggests is that you can multiply by 49 by multiplying by 100, halving the result, and subtracting the multiplicand. At first, Susanne does not understand his suggestion. When she does understand, she becomes excited and she and René takes turns in providing partial explanations, René focusing primarily on how to use his procedure, Susanne convincing herself, rather than René or the other students, why it works. Susanne's last comment is directed to the whole class as well as to René. Here she makes a general statement to the effect that there are other ways of finding a result than the one she has suggested, at least when working with multiplication.

Episode 3: Susanne in one-to-one interaction

For the subsequent individual work, Susanne has written eight multiplication tasks on the board: 2×567 , 3×613 , 4×86 , 5×167 , 6×73 , 7×703 , 8×2136 , and 9×755 . While the students work, Susanne walks around to the students who indicate that they are in need of assistance. René is one of them. When she joins him he has already solved the two first tasks correctly, using procedures that resemble the one he has suggested during the whole class session. When Susanne joins him they first discuss the task 5×167 . René suggests first multiplying by 10 to get 1670 and halving this partial result to obtain the correct answer. Susanne says that she wants to see whether he can also do it her way. René rubs out his correct

results, but Susanne says he should not have done that and asks him to rewrite them, as they are still just visible in René's notebook. They then begin doing 4×86 :

Susanne: Let us begin here: 4 times 6. That is? [She points at 4 and 6 in 4×86 in René's notebook].

Rene: 24.

Susanne: Yes, then you start by writing 4 over here on the result line [points a few centimetres to the right of the equals sign]. You need to make room for some numbers in front of it, and then you carry 2 [Rene begins to write 2 as in figure 4]

$$\underline{4 \times 86} = \quad \begin{array}{c} 2 \\ 4 \end{array}$$

Figure 4. Rene's notebook

Susanne: No, over there. [Rene corrects it].

Susanne: Okay and then 4 times 8? [She points at the numbers]

Rene: 32 [inaudible]

Susanne: Plus 2. You have to write it here on the other side [points; René writes]. Did you get the same result using your own method? There is nothing wrong with the way you did it. You can do it the way you want.

Rene: I just make it shorter.

Susanne: Yes. It just requires a little more. You have to remember some numbers [inaudible]. But as long as you get it right, it doesn't really matter how you did it. I just wanted to see if you could do it my way as well, and you could. [She leaves him].

In this brief interaction Susanne takes René through the multiplication algorithm on the task of 4×86 . She tells him to begin finding the solution by multiplying 4 and 6. Although he knows the result, he writes the digits in the wrong places in spite of Susanne's assistance, but she corrects him. Next she asks him to multiply 4 and 8 and add 2. René writes the correct answer in his note book, and Susanne finishes off by letting him know that she just wanted to make sure that could do it her way as well and that he can do as he pleases as long as he gets the correct result.

In the next sections we elaborate on how Susanne's contributions to the practices that emerge her classroom relate to the verbal accounts of her engagement in the three sets of social practices mentioned previously, i.e. those of the reform discourse, of her previous experiences with mathematics education, and the approach to students with problems at

Southern Heights. We do so by identifying two key emphases in her tales of mathematics teaching and learning.

Interpretation and discussion of classroom practices

Interpreting the above episodes, we consider Susanne's contributions to the classroom practices a result of the meaning she makes of the interactions that unfold. Beyond the numerical discourse on multiplicative procedures and place value, this meaning-making activity relates to her engagement in the three types of practices already described, i.e. the ones of her own educational experiences, of the reform Discourse, and of the way she conceives of the approach to students with problems at Southern Heights.

Susanne is in line with the latter approach when she segregates students from the immediate classroom situation either physically by sending them off to other locations or mentally by making them work on their own within the classroom. This does not necessarily involve an element of marginalisation of the students in the sense that it threatens their position within the classroom community. For students who are sent to "the obs" or the special needs department this may be the case; other students who are asked to leave the classroom and to continue on their own accord may be positioned even more centrally in the community, despite their physical absence. In either case, Susanne's removal of the students serves to solve what she sees as problems between the students' actions and the classroom situation. She does not, however, describe the problems as the result of classroom interactions. Rather they are seen as manifestations of social problems that exist prior to and somewhat independently of the classroom. The removal of the students is to allow her to keep in line with the traditions of mathematics teaching and learning that dominate her own educational experiences.

Susanne's contributions to the interactions with the rest of the students are informed by, but also mould her engagement in the traditional classroom practices as well as in the reform Discourse. In the first whole class episode, Susanne presents a multiplication algorithm that she wants the students to use. She apparently tries to promote the students' understanding of the functioning of the algorithm by mentioning the place value of the individual digits. Several students suggest different results and others complain that they do not understand. None of this is elaborated on, and for instance there is no evidence to suggest whether the complaints concern the meaning of the steps of the algorithm or procedural knowledge of how to perform it. Susanne reacts by going over the calculations again, using a written format that is somewhat more

transparent in terms of the meaning, but she does so without explicating the values of the individual digits. All in all this may be read as her attempt to provide the students with an opportunity to develop their understanding of the algorithm within a whole class format, while avoiding that the presentation becomes too long. In this case elements of her peripheral participation in the reform Discourse on understanding are inserted into a practice dominated by procedural competence.

In the next episode a student, René, suggests an alternative procedure. It works well with the particular task, although there are strong limitations to its general applicability. The suggestion indicates that René has developed a thorough understanding of numbers and of multiplication, but possibly also that he cannot make sense of Susanne's procedure. Once Susanne understands the suggestion she becomes excited and praises him for it. In spite of her enthusiasm and of her comment that "if you can get it in some other exciting way, you just do that", she does not invite a discussion of the limitations and potentials of his method. Instead, René's suggestion is left as an example that there may be ad hoc methods that work when multiplying and that you are allowed to invent and use such methods. The implied message, however, is that the methods are indeed ad hoc and of little real significance as they are distinct from more general mathematical competence. Susanne repeats this dual message in a subsequent interview, saying that René's suggestion was "super" (III), but that he needs to know other methods as well, as it would be too difficult for him to multiply for instance 36 and 53, if he does not. It appears that she sees his learning of these other methods as distinct from his initial suggestion.

This interpretation is substantiated by the third episode. Susanne works with René at his table. He has solved the first two tasks correctly using similar methods to the one he suggested in the whole class session. When Susanne joins him he rubs out his work. Moving on to the next task, he responds to Susanne's questions about the multiplication of the individual digits and follows her instructions about where to write the partial results. This sets a new agenda, and René does not attempt to link his previous work to the general algorithm. Further, Susanne does not capitalise on his work and alternative procedures to further develop his general understanding of multiplication or his procedural competence. Also, there is no indication that Susanne considers the development and refinement of René's procedure a valuable mathematical activity in its own right.

Susanne and her students continuously make sense of each others' moves. René is struggling to interpret and balance the conflicting requirements of finding the correct answer to the tasks and coming to grips with

a procedure that seems to make little conceptual sense to him. His difficulty with doing so is apparent from his interaction with Susanne during the students' individual work. He erases his correct results as Susanne arrives because of his expectation that she accepts only the standard procedure. On her part Susanne attempts to make sense of René's suggestion and trying to substantiate it she becomes involved in a decidedly mathematical discourse (cf. Sfard, 2008). Simultaneously she needs to address the more general question of whether to allow the students to develop their own ways of multiplying and sends the dual signal that on the one hand this is acceptable if you get the right answer, but on the other you need to know also a standard algorithm.

Sending this signal Susanne is drawing on and re-engaging in the meta-Discursive practice of the reform as well as the one of "traditional teaching". Students' work with their own algorithms is a recurrent theme in the reform literature, also at Susanne's college. It relates closely to the discussion of the relationship between conceptual understanding and procedural competence that Susanne refers to repeatedly. In contrast, "the tradition" focuses on competence in handling standard procedures. Susanne's response to René may then be read as her shifting engagement in these two distinct practices within the very short interaction with René.

There is a remarkable similarity between Susanne's interaction with René and the way she distances herself from a pedagogy of "cut and paste" based on her negative experiences with it. She considers such methods a possible way of varying classroom activity for the good students. However, they create insurmountable disciplinary problems if used in general, and she considers them neither valuable for the development of the students' conceptual understanding nor a possible foundation for their procedural competence. On the contrary, they challenge her own engagement as well as that of the students in the core practices of mathematics teaching and learning.

Susanne also has negative experiences with aspects of "traditional practice". That, however, does not make her reconsider her attempts to engage the students in it. She segregates the "problematic students" from the rest in an organisational measure inspired by those at Southern Heights in general. The rest of the class is a somewhat homogeneous group with which "traditional teaching" functions to the extent that the students accept the predominantly repetitive character of their task and that the disciplinary problems become manageable.

Apart from the insertion of elements of conceptual understanding into Susanne's presentations of mathematical procedures there is a sharp

discontinuity between her re-engagement in the traditions of school mathematics and the reform Discourse as represented for instance by "college talk". She does not, then, tap into the reform rhetoric to shift the practices that are based on her own educational experiences, or use those experiences to make different sense of the reform. The two sets of practices co-exist in their distinct forms. It seems that none of her previous experiences have initiated her sufficiently into ways of using manipulatives, investigations or other student questions and suggestions to support student learning of standard results and procedures. And none of them has sufficiently engaged her in ways of working that indicate that mathematical investigations may be of value in their own right.

We suggested previously that the multiple activities in which a teacher engages may function as structuring resources for her contributions to emerging classroom practices. For Susanne, these activities include presenting procedures, making sense of the students' mathematical suggestions, emphasising elements of understanding, solving disciplinary problems, and taking the students' social problems into consideration. Becoming engaged in these activities, Susanne relates to a range of social practices beyond the mathematics classroom. Apart from mathematical discourse itself, the most important ones are "the tradition", the reform Discourse, and the organisational segregation of students at Southern Heights. In Susanne's case the practices of "the tradition" constitute a set of value laden and highly resilient *modus operandi* involving norms for action on her own part as well as on the part of the students. In spite of the resilience both "the tradition" and the reform's emphasis on students' understanding are remodelled, but not integrated as Susanne attempts to incorporate elements of the latter into the former. Also, the attempts to take students' social problems into account changes when aligned with the ways in which Susanne positions herself in relation to "traditional practice". It shifts and takes on a flavour of handling problematic students instead.

One of the special characteristics of the case of Susanne at Southern Heights is that the different practices in which she re-engages do not merge to any great extent. "The tradition" is an almost monolithic structure and other practices function primarily by suggesting ways of handling issues at the outskirts of the main practice. They allow her to engage in "traditional practice" by separating out students who constitute problems or by inserting elements of the reform Discourse on understanding in isolated pockets of instruction. At all times, however, Susanne is manoeuvring in relation to multiple prior and present practices as she contributes to the interactions that unfold in her classroom.

Conclusions – what to gain from the notion of beliefs?

Susanne's explanations about school mathematics and their relation to the practices that unfold in her classroom may be interpreted in the terms of mainstream belief research. Using espoused and enacted or attributed beliefs one can emphasise that Susanne believes in "the tradition" and that her classroom practices comply with her beliefs. One could elaborate and suggest that her college education made her add another element that is somewhat alien to "the tradition", namely the need to focus on understanding. This belief, however, is in a separate cluster and less centrally held than the ones related to the tradition (cf. Green, 1971). Also, her shifting emphases on understanding over time may be seen as expressions of belief change in view of the experiences she gains from her own teaching practice. This could be read as yet another indication that beliefs that are the result of verbal or theoretical learning are less likely to be enacted and more susceptible to change than those that are the result of prolonged, real-life experience.

Our approach in the present article, however, is different. We suggest doing away with the assumption of reified entities called beliefs that take on a life of their own and impact mathematics teaching and learning. Instead, we suggest interpreting the teacher's contributions to classroom interaction from the perspective that they consist of her shifting engagement in a range of simultaneous practices, the most immediate of which is obviously the one of the mathematics classroom. As that evolves the teacher relates to a range of other ones. Some of these are based on immediate physical interaction, while others are primarily theoretical and meta-Discursive; some of them are currently active, while others consist of the re-engagement in prior practices; and some are positively laden while others are used primarily negatively to make other ones stand out in relief. They mutually structure each other as resources for the practices that emerge in the mathematics classroom (cf. Lave, 1988). Beyond mathematical discourse itself, the most important ones to Susanne in her mathematics classroom are "the tradition", the reform Discourse, and the organisational segregation of students at Southern Heights.

This approach addresses the conceptual and methodological difficulties of belief research. The most significant advantage though is that it shifts the research endeavour away from depicting second order objectifications and postulating their impact on practice and towards focusing on the processes that allegedly give rise to beliefs. This is in line with the growing interest in participationism as a metaphor for learning and thinking in mathematics education. When further developed it may allow us to analyse engagement in mathematical, meta-mathematical, and broader social processes within the same participationist framework.

It may then allow us to overcome the limitations of conducting research in the three distinct and mutually isolated fields of knowledge, beliefs and identity.

The point is, then, that there is little to gain from assuming the existence and significance of objectified mental constructs called beliefs; and that there may be a lot to gain from doing away with the notion of beliefs altogether.

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Notes

- 1 Gee (2000-2001) uses Discourse, with a capital D, to describe how one can become socially recognised as a certain kind of person via a combination of ways of talking, interacting, moving, dressing, etc. We follow him in this and use "the reform" as a description of the discursive formation of notions of quality teaching in mathematics.
- 2 In the rest of the article we refer to each of the three interviews by (I) – prior to Susanne's graduation, (II) – the first interview after her graduation and (III) – the final interview.
- 3 This is our attempt to translate a non-standard expression that Susanne uses repeatedly into English. The meaning seems to be that the students opt out of all current activities, look beyond anything within their immediate reach, and "focus on infinity".
- 4 The Danish abbreviation is AKT-teachers.
- 5 One of them has problems with her short term memory; we do not know what the other diagnoses are.

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