The theory of conceptual change as a theory for changing conceptions

PETER LILJEDAHL

It has become widely accepted that what and how mathematics teachers teach is linked to what it is they believe. What teachers believe, however, is not always in alignment with contemporary notions of mathematics and the teaching and learning of mathematics. As such, it is important for teacher educators to help facilitate changes in teachers' beliefs in ways that will enable them to become more effective teachers of mathematics. In this article I present the results of a research project designed to examine the feasibility of using the theory *of* conceptual change as a theory *for* changing mathematics teachers' conceptions about key aspects of mathematics and the teaching and learning of mathematics. The results indicate both that the theory of conceptual change is a viable theory for designing interventions for the purpose of changing beliefs, and that the implementation of these aforementioned interventions resulted in the rejection of participants' a priori beliefs.

It has become widely accepted that what and how mathematics teachers teach is linked to what it is they believe about mathematics and the teaching and learning of mathematics (Beswick, 2009; Fosnot, 1989; Skemp, 1978; Skott, 2001). What teachers believe, however, is not always in alignment with contemporary notions of mathematics and the teaching and learning of mathematics (Green, 1971). As such, it is important for teacher educators to help facilitate changes in teachers' beliefs in ways that will enable them to become more effective teachers of mathematics. The research literature, as well as my own experience with such efforts, has informed me that these are both worthy and challenging pursuits.

Peter Liljedahl,
Simon Fraser University, Canada

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Teachers' beliefs can be difficult to change. Too often have I encountered situations where the in-service mathematics teachers I work with have agreed (or acknowledged) that, for example, there is more to mathematics than mastery of the collection of outcomes in the current curriculum, that mathematics is about problem solving and inquiry and reasoning, only to then have these ideas demoted to lower levels of importance for the sake of "preparing students for the exam" or "to save time". Along with this demotion comes a privileging of more traditional teaching methodologies that are seen to be more efficient. What has happened in such situations is that, although the teachers have been willing to assimilate additional views of mathematics and mathematics teaching and learning into their belief structures, the old views have not been eradicated. Schommer-Aikins (2004) points out that beliefs are "like possessions. They are like old clothes; once acquired and worn for awhile, they become comfortable. It does not make any difference if the clothes are out of style or ragged. Letting go is painful and new clothes require adjustment" (p. 22). So, in the end, there is a return to the old, out of style, and ragged beliefs.

But, what if instead of going through a process of assimilation the teachers had gone through a process of replacement – first rejecting their old beliefs and then adopting the new beliefs. In such an instance there could not be a return to the old beliefs – they would have already been discounted as viable. Such a change in beliefs can be seen as a form of accommodation, although the lack of compromise and blending of the old and the new make it a very specific form of accommodation. A more accurate description of such a process is one of conceptual change – a process by which a current conception is first rejected and then a new conception is adopted.

In this article I present a research project designed to examine the feasibility of using the theory of conceptual change as a theory for changing mathematics teachers' beliefs about key aspects of mathematics and the teaching and learning of mathematics. In what follows I first present the theory of conceptual change and argue its applicability in changing conceptions among in-service mathematics teachers. I then present and argue for a theoretical turn in which the theory of conceptual change is not just used as a *theory of* how conceptions may have been changed, but also how it can be used as a *theory for* changing conceptions. This is followed by a description of the methodology used in the aforementioned research project and finally the presentation of some of the results of this project.

Theory of conceptual change

The theory of conceptual change emerges out of Kuhn's (1970) interpretation of changes in scientific understanding through history. Kuhn proposes that progress in scientific understanding is not evolutionary, but rather a "series of peaceful interludes punctuated by intellectually violent revolutions", and in those revolutions "one conceptual world view is replaced by another" (p. 10). That is, progress in scientific understanding is marked more by theory replacement than theory evolution. Kuhn's ideas form the basis of the theory of conceptual change (Posner, Strike, Hewson & Gertzog, 1982) which has been used to hypothesize about the teaching and learning of science. The theory of conceptual change has also been used within the context of mathematics education (Confrey, 1981; Greer, 2004; Lehtinen, Merenluoto & Kasanen, 1997; Tirosh & Tsamir, 2004; Vosniadou, 2006; Vosniadou & Verschaffel, 2004).

Conceptual change starts with an assumption that in some cases students form misconceptions about phenomena based on lived experience, that these misconceptions stand in stark contrast to the accepted scientific theories that explain these phenomena, and that these misconceptions are robust. For example, many children believe that heavier objects fall faster. This is clearly not true. However, a rational explanation as to why this belief is erroneous is unlikely to correct a child's misconceptions. In the theory of conceptual change, however, there is a mechanism by which such theory replacement can be achieved – the mechanism of "cognitive conflict".

Cognitive conflict works on the principle that before a new theory can be adopted the current theory needs to be rejected. Cognitive conflict is meant to create the impetus to reject the current theory. So, in the aforementioned example a simple experiment to show that objects of different mass actually fall at the same speed will likely be enough to prompt a child to reject their current understanding.

The theory of conceptual change is not a theory of assimilation. It does not account for those instances where new ideas are annexed onto old ones. Nor is it a theory of accommodation, *per se*, in that it does not account for examples of learning through the integration of ideas. The theory of conceptual change is highly situated, applicable only in those instances where misconceptions are formed through lived experiences and in the absence of formal instruction. In such instances, the theory of conceptual change explains the phenomenon of theory rejection followed by theory replacement. The theory of conceptual change, although focusing primarily on cognitive aspects of conceptual change, has been shown to be equally applicable to metaconceptual, motivational, affective, and socio-cultural factors as well (Vosniadou, 2006). And, it has been argued that it is applicable to teachers' beliefs about mathematics and the teaching and learning of mathematics (Liljedahl, Rolka, and Rösken (2007).

This last argument is based on more than the use of synonyms – although given the fact that the terms beliefs and conceptions are often used interchangeably in the beliefs literature, such an argument could be a valid one. The theory of conceptual change, as the explanatory framework described above, is applicable only in those instances where misconceptions are formed through lived experiences and in the absence of formal instruction, there is a phenomenon of concept rejection, and there is a phenomenon of concept replacement. In Liljedahl, Rolka, and Rösken (2007) we showed that each of these criteria was equally relevant to some instances of rejection and replacement of pre-service teachers' beliefs about mathematics or the teaching and learning of mathematics. In such instances relevant lived experience can be drawn from their time as students. As learners of mathematics they have both experienced the learning of mathematics and the teaching of mathematics, and these experiences have impacted on their beliefs about the teaching and learning of mathematics (c.f. Chapman, 2002; Skott, 2001). These experiences can be viewed as having happened outside of a context of formal instruction because, although they are situated within the formal instructional setting of a classroom, the object of focus of that instruction is on mathematics content rather on theories of learning, methodologies of teaching, and philosophical ideas about the nature of mathematics are not. For more experienced teachers, the relevant lived experiences can come from their daily practice where routinization of teaching may have long since replaced the formal instruction and teacher education that at one time underpinned their practice. In either case, the relevant lived experience serves to construct conceptions, or beliefs, about mathematics and the teaching and learning of mathematics that is impacting on their practice. When these conceptions are rejected and then new conceptions are adopted then a conceptual change has occurred.

Theory of ⇒ theory for

In the aforementioned example of a child developing naive views of the effects of gravity on objects of different masses and then changing that view on the heels of a demonstration the theory of conceptual change gives a viable explanation of the learning that that particular child has experienced. When a teacher is aware that there may be a number of children in her class that have a similar misconception decides to run such an experiment for the purpose of changing her students' conceptions she is using this theory for the purposes of promoting learning. That is, she is using the *theory of* conceptual change as a *theory for* changing concepts. More generally, she is using a *theory of* learning as a *theory for* teaching.

In this aforementioned example such a move on the teacher's part is both intuitive and natural. She needs not have any special knowledge of the theory of conceptual change to know that challenging the students naive views would be a pedagogically sound move. However, in proposing this shift as a change from a *theory of* learning into a *theory for* teaching I suggest that it is equally applicable in situations where sophisticated knowledge of a learning theory is needed. Simon (1995), for example, uses deep knowledge of the theory of constructivism to guide his planned and implemented teaching actions. This is not without challenge, however:

> Although constructivism provides a useful framework for thinking about mathematics learning in classrooms and therefore can contribute in important ways to the effort to reform classroom mathematics teaching, it does not tell us how to teach mathematics; that is, it does not stipulate a particular model. (p. 114)

To overcome this lack of a model Simon must "hypothesize what the [learner] might learn and find ways of fostering this learning" (Steffe, 1991, p. 177 as cited in Simon, 1995, p. 122). It is within this hypothesizing that the sophisticated knowledge of constructivism comes to bear as Simon designs mathematics pedagogy on the constructivist view of learning (paraphrased from Simon, 1995, p. 114). I propose that Simon's actions in this regard can be described as turning the *theory of* students' constructing knowledge into a *theory for* promoting knowledge construction.

There is a need in mathematics education to have a way to discuss the distinctions between theories with respect to learning and with respect to teaching. Whereas there is an abundance of theories to discuss learning – from constructivism (Piaget, 1970) to commognition (Sfard, 2008) – these same theories don't explain teaching.

> While theory provides us with lenses for analysing learning (Lerman, 2001), the big theories do not seem to offer clear insights to teaching and ways in which teaching addresses the promotion of mathematics learning. (Jaworski, 2006, p. 188)

In fact, it can be argued that there can never exist such a thing as a theory of teaching.

> Theories help us to analyse, or explain, but they do not provide recipes for action; rarely do they provide direct guidance for

> practice. We can analyse or explain mathematics learning from theoretical perspectives, but it is naive to assume or postulate theoretically derivative models or methods through which learning is supposed to happen. Research shows that the sociocultural settings in which learning and teaching take place are too complex for such behavioural association. (Jaworski, 2006, p. 188)

Yet at the same time, teaching is perpetually informed by theories of learning.

> It seems reasonable that the practice of teaching mathematics can and should draw on our depth of knowledge of mathematical learning, and learning theory, but to theorise teaching is a problem with which most educators are struggling. (Jaworski, 2006, p. 188)

I propose that the source of this tension between theories of learning and theories of teaching is the assumption that theories should play the same role in teaching as they do in learning. This does not need to be the case. Teaching and learning are inherently different activities. And to theorize about them requires, not (necessarily) the use of different theories, but the use of theories differently. More specifically, theories of learning are models that can be used to understand, explain, and make predictions about learners exposed to specific *experiences*. On the other hand, theories for teaching (as used in the paper) are the use of theories of learning to construct these *experiences*. In both cases, observation, conjecture, empirical testing, and recursive refinement can improve these theories.

This is not a new idea. For example, it is very similar to the work on Realistic Mathematics Education (RME) coming out of the Freudenthal Institute in the Netherlands, in particular, the work on *model of* and *model for* as a way of describing a schematization.

> About fifteen years ago, Streefland (1985) elucidated in a Dutch article how models can fulfill the bridging function between the informal and the formal level: by shifting from a "model of" to a "model for". In brief, this means that in the beginning of a particular learning process a model is constituted in very close connection to the problem situation at hand, and that later on the contextspecific model is generalized over situations and becomes then a model that can be used to organize related and new problem situations and to reason mathematically. (Van den Heuvel-Panhuizen, 2003, p. 14)

Streefland (1993) later used the idea of *pre-image* and *post-image* to describe the distinction between a design theory that emerges out of the analysis of a cycle of instructional design (post-image) and the subsequent use of that design theory for future instructional design cycles (pre-image).

In so doing he draws a distinction between how a model or image is formed and how they are subsequently used. Similarly, there is a distinction between how, and for what purpose, a particular theory of learning is formed and how it may be used as a theory for teaching.

Taken together, the *theory of/theory for* distinction allows, not only for the differentiation between the development and use of a theory in general, but also for the use of the same theory to discuss aspect of both learning and teaching in particular.

Methodology

As mentioned, in this article I explore the feasibility of using a particular learning theory – the theory of conceptual change – as a theory for changing teachers' conceptions about mathematics and the teaching and learning of mathematics. The research for this is situated within a course for in-service secondary mathematics teachers wherein the participants are subjected to six interventions designed to change their beliefs about six core aspects of mathematics education: (1) the nature of mathematics, (2) the nature of mathematics teaching, (3) the nature of assessment, (4) the nature of student knowledge, (5) the nature of student learning, and (6) the nature of student motivation. In what follows I first describe the general setting of this research and then detail the particular aspects of the methodology used to answer the research questions.

Setting and participants

Participants for this study are in-service secondary mathematics teachers who were enrolled in a master's program at Simon Fraser University in Vancouver, Canada. The program was specifically designed to help teachers develop insights into the nature of mathematics and its place in the school curriculum, to become familiar with research on the teaching and learning of mathematics, and to examine their practice through these insights. There are six core courses in the program, each one looking at a different aspect of secondary mathematics education, from technology to pedagogy, from geometry to history. The courses, although discrete in their topics, were designed with the other courses in mind. The result is an extremely cohesive and comprehensive program aimed at providing teachers with the very best of mathematics education research and practice.

The program is designed for practicing teachers. As such, the enrollees[1] take one course each semester for six consecutive semesters (three semesters per year). Each course consists of 52–65 contact hours, and convenes one evening a week for 13 weeks. To accommodate this structure the program is designed on a cohort model with the students staying together for the entirety of the program. The particular course that this study took place in is called *Teaching and learning* and was the second course in the sequence. This particular course was designed to examine closely the teaching of mathematics from the perspective of, and with the goal of, students learning mathematics. This may seem like an overly obvious (perhaps even trivial) association, but as was mentioned in the introduction and will be seen in the data, this is not at all the case. The course was constructed on four pillars of activity:

1. Exposure to literature.
2. Submersion in a constructivist learning environment.
3. Experimentation with teaching and observation of learning within their own classrooms.
4. Discussion of critical questions pertaining to the above three components as well as past teaching experiences.

These activities will be elaborated on as part of the presentation of results.

There were 14 teachers enrolled in the course, all of whom agreed to be part of the study. Their experience as classroom teachers ranged from 0 years to 23 years with an average of 8.7 years. There were 8 males and 6 females. Of the 14 participants, only 6 had undergraduate degrees in mathematics, 7 had a degree in one of the sciences, and 1 had a degree in history. Having said this, the majority of the group had been teaching mathematics either exclusively or in conjunction with some other subject areas for the majority of their careers and *all* of the participants considered themselves to be mathematics teachers. Table 1 gives a breakdown of the participants. Not only is this diversity in educational experience indicative of the population teaching secondary mathematics in the local geographical context where there is often a migration away from the area of formal training but it is also indicative of the past four cohorts of master's students I have worked with.

Data sources

This study started long before the actual course did. It began with an examination of the feasibility in constructing interventions designed to create cognitive conflict and to promote the rejection of beliefs that I anticipated some participants may have[2]. Careful records of this

Table 1: Breakdown of participants' teaching experience

Pseudonym	Gender	No. of years teaching	Undergraduate major	No. of years teaching mathematics
Alicia	female	0	mathematics	0
Betty	female	1	mathematics	1
Chad	male	3	mathematics	3
David	male	3	physics	3
Eric	male	4	engineering	4
Frieda	female	6	biology	6
Grant	male	7	history	3
Henry	male	7	mathematics	7
Ingrid	female	10	biology	7
Jenny	female	12	physics	12
Kris	male	13	mathematics	13
Lori	female	15	chemistry	15
Marcus	male	18	chemistry	18
Nicholas	male	23	mathematics	23

planning process were kept. A journal was started in which initial ideas, tentative plans, apprehensions, and unresolved questions were recorded. Once the course began, this journal became the place where field notes were recorded. Records were kept of the observed effects of each planned intervention with specific focus on things that were predicted and things that were surprising. Also recorded in the field notes were pieces of discussions with and among participants.

With respect to the changes in beliefs of the teachers enrolled in the course the data is constituted of four sources. The first has already been mentioned – the field notes produced from observations within the classroom setting. The second and major source of data came from the journals that each of the participants kept. As part of the requirements for the course the teachers were to keep a reflective journal within which they were to respond to specific prompts. Sometimes these prompts were of a quick-write nature. That is, they were to respond immediately and quickly to the prompt. Most often, however, the responses were done in their own time as homework. The third source of data was informal interviews that were conducted with each participant at varying times during the course. These were requested by me and were held either immediately after the class or during breaks and lasted from between

5 and 15 minutes. The structure of the interview was casual – almost conversational – and were used as a mechanism for getting the participants to elaborate or clarify things heard from them during class time. In some cases, these interviews were used to get insights into the thoughts of the more reserved teachers. The final source of data was from one of the course assignments in which the teachers had to write an essay. The details of this essay will be discussed in a subsequent section.

Data analysis

These data were sorted in two different ways. First they were sorted according to the specific planned interventions wherein all the data pertaining to a specific planned activity were grouped together and analysed. This analysis focused on changes in beliefs as related to each of the specific intervention. The second sorting consisted of bringing together all of the data for each participant so a more longitudinal analysis of changes in beliefs over the entire course could be done.

In both cases the data were recursively coded for emergent themes. This began with a search for anticipated outcomes. Evidence of cognitive conflict and belief rejection was sought out as was evidence pertaining to more assimilatory behaviour. But the process was open and as this cursory analysis progressed themes beyond the anticipated effects began to emerge. These were assigned new codes and the process began all over again. This was continued until no more themes were forthcoming. The emergent themes were then examined more closely and similar themes were collapsed together and overly diverse themes were disaggregated. Ten themes remained after this process.

Results and discussion

As can be expected, the results from the aforementioned analysis are many. As such, only three of the interventions have been selected for presentation in this article. In what follows, results from the interventions pertaining to (1) the nature of mathematics, (2) the nature of mathematics teaching, and (3) the nature of assessment are presented in their own subsection. Each of these begins with a detailed description of the intervention and how it is designed to promote cognitive conflict and the rejection of beliefs. This is then followed by exemplification from relevant data regarding the effect of this intervention on the participants' beliefs.

Nature of mathematics

Experience, prior research (Liljedahl, in press), and literature (Beswick, 2009) show that teachers' beliefs about mathematics are often anchored firmly in the context of the school mathematics curriculum learning outcomes. Such a view is often punctuated with a belief that mathematics is about facts and procedures, where facts are to be memorized and procedures are to be mastered. This is a rather narrow view of mathematics and one I felt was important to change. Adhering to the theory for conceptual change being implemented here, any such change needs to be preceded by a rejection of any such *a priori* beliefs.

Lockart's Lament (2008) was chosen for this purpose. The first few pages of this essay tell the tale of first a musician and then a painter who wake from their own respective nightmares in which their craft has been reduced to a compulsory curriculum of skills to be practiced and mastered. Lockhart's purpose for doing this is clear. Using reductionism he effectively showcases the absurdity that results when complex activities such as music and painting, and by analogy – mathematics, are reduced to the mastery of the tools of the trade. These first few pages were selected to be read by the participants during class in week 2 of the course. This was then followed by a 15 minute quick-write where the teachers responded to the prompt – "... meanwhile on the other side of town a mathematician wakes form a similar nightmare. What nightmare did he wake from?" This was succeeded by a whole class discussion. The full text of Lockhart's Lament was then assigned as homework as was the journal prompt – "what is the relationship between the curriculum learning outcomes and mathematics?"

... Results

The quick write produced not only interesting, but also creative, results. Common among the 14 nightmares were intonations about students being shown, and being required to practice, algorithms without any explanation of what its purpose was or why it worked. This is succinctly demonstrated in an excerpt from Jenny's passage wherein a high school mathematics teacher is telling her student:

> Don't worry about it dear, you'll learn what it is for next year.

This was a recurring phrase in her nightmare appearing in four separate places. It was also a recurring theme in Alicia's and Marcus' nightmares. During the follow-up whole class discussion it was mentioned, and agreed to by many that not only has mathematics been reduced to a collection of curricular skills but it has been reduced to a collection of pre-requisite skills that need to be mastered now.

Analysis of the journal entries revealed that this reduction of mathematics to curricular learning outcomes was layered, situated, and in many cases surprising. In his journal Chad wrote:

> I never really thought about it before, but for me math is all just about what I'm teaching tomorrow, or the next day, or last week. Even when I'm talking to the kids it's all about "you have to learn this ... it's important ... you'll need it next year!" After last class I started looking at my top students and thinking about why they like math and I realized that they don't even know what math is. They think math is this stuff I'm teaching them and they like the fact that they can be completely right at it and know that they are completely right. Then I started to think about why it is that I like math and I realized that I had forgotten that I used to love to figure out logic puzzles and solve difficult problems. I don't do that anymore, but I should.

Chad's entry shows how his own view of mathematics is bound up with the view of his students. They are inseparable, situated within the context of the mathematics that they are experiencing on a daily basis. The difference, however, is that Chad has a memory of mathematics as something else and for this reason he is surprised that he has allowed his views to stray from what it is he used to enjoy about mathematics.

Lori also talks about memories of a distant experience, albeit quite different.

> The thing about the music really struck a chord for me (ha-ha). I used to play the piano when I was a kid. I hated all the practicing and I wanted to quit. My mom would tell me that I just needed to get through it and then I would start enjoying it. But I didn't have the patience and so I quit. The ironic thing is that I was the one who wanted to start taking piano lessons. I wanted to play the piano. I wanted to be able to play the music. But I gave up. Isn't this what we are doing to our students? How many of my students have given up? We have reduced math to a bunch of drills. No wonder kids hate it. Math needs to be more than this. Kids need to have an opportunity to play maths as well, to forget about the drills and just enjoy it.

Although Lori does not make clear what she means by play math or what mathematics needs to be she is very clear about what mathematics shouldn't be.

All of the teachers had journal entries similar to these – entries that lamented the systemic reduction of mathematics to curriculum and with it, the reduction of their own views about mathematics. They also, uniformly rejected this view even if they did not have an alternate view to replace it with. This is nicely captured in an excerpt from an interview with Grant.

Interviewer: A few weeks ago I had you read Lockhart's Lament in class. Tell me what you thought about that.

Grant: It made things pretty clear, didn't it?[laughs] I mean, the way that he described music and art sure made you think about what we are doing to kids in math.

Interviewer: And?

Grant: And we sure as hell shouldn't be doing it?

Interviewer: So, what should we be doing?

Grant: I'm not sure. Problem solving for sure. And probably more group work. But I haven't figured it all out yet.

Interviewer: So, what made you come away from your initial ideas about mathematics?

Grant: To be honest, I had never really thought about it before ... I hadn't really looked at what I was teaching in such a stark way. Lockhart paints a picture that is hard to ignore.

[...]

Interviewer: Let me ask you something – just off to the side a bit. Can you tell me about something from mathematics that you enjoy, but that you don't teach ... something not in the curriculum.

Grant: [long pause] Can it be something I have taught in the past?

From this transcript it is evident that Grant's belief about the nature of mathematics may be what Green (1971) would call an unconscious and non-evidentiary belief. That is, a belief that just is. For Grant, like Chad, this belief has likely been formed from his daily experience of marching through curriculum content. For Grant though, this experience is all that he has to draw on his understanding of what mathematics is. Consequently, he is unable to name anything from mathematics that isn't curricular. This does not prevent him from rejecting this view of mathematics, however. The process of making conscious – of making stark – his belief is enough to make it clear to him that rejection is warranted.

By all accounts, the first few pages of Lockhart's Lament (2008) created a conflict for many of these participants. The reduction of a craft to a collection of discrete and closed skills in such a stark fashion created enough of a disturbance for them that they rejected their self professed understanding that mathematics is learning outcomes. For some, like Chad, this was a moment of remembering mathematics as something else. But for most, like Grant, it wasn't. It was just a rejection of his current viewpoint. Grant, at a later date, commented in class after this intervention that he felt like he had "been set adrift", that "he had lost sight of land and he wasn't sure where he was going". This was a sentiment acknowledged to be shared by many, and is indicative of the process of rejection of a belief without replacement with a new belief (... yet).

Nature of mathematics teaching

One of the implicit goals of the masters program within which this course is set is to provide the enrolees with the knowledge, the will, and the ability to teach mathematics using more contemporary and progressive teaching methodologies. The achievement of this goal is made difficult by the fact that many teachers that come to the program are often very traditional teachers, are deemed to be very qualified with high teaching abilities, are seen as leaders within their schools and districts, and are teachers who like the way they teach and feel that they do it well. For many, these more traditional practices will have originated in their own successful experiences as learners of mathematics (Ball, 1988). Taken together, most of the teachers believe that they are good at teaching the way that they do and they believe that what they do is effective for student learning. It is this second belief that was targeted for intervention.

Jo Boaler's book *Experiencing school mathematics* (2002) was chosen for this purpose. In this book, Boaler presents the results of her doctoral research in which she compares two very different teaching methods employed at two otherwise very similar schools. At Amber Hill teachers use a more traditional method of teaching while at Phoenix Park they construct their teaching around problem posing (Brown & Walters, 1983). What is powerful about this book is that Boaler's descriptions of the traditional teaching practices and classroom norms of Amber Hill are so rich that they reflect back at a reader their own teaching practices while at the same time exposing the consequences of this style of teaching in terms of students' attention, retention, performance, and enjoyment. At the same time Boaler counters this stark depiction of traditional teaching with an equally descriptive account of an antithetical teaching style, associated classroom norms, and subsequent student learning.

In week 1 of the course the participants were introduced to the book and assigned a 2500 word essay on the following:

> It can be said that when we read a book we read ourselves into the text. In what ways do you read yourself into Boaler's book? Speak about your own teaching practice (past, present, and future) in relation to the book.

This essay was due in week 6 of the course at which time we engaged in structured debate on the merits and demerits of teaching and learning in the dichotomous settings of Amber Hill and Phoenix Park. This was followed up by a journal prompt asking them to reflect on the most powerful aspect of the book.

... Results

The most common theme that emerged from the data regarding this intervention was the unanimous declarations that the participants saw themselves at least partially reflected in the teaching of Amber Hill. Lori's, Kris', and Alicia's journal reflections are succinct examples of this.

> It was as though I was looking at my own teaching.

> I couldn't help but think that Boaler was describing my classroom.

> [...] it was a really good description of the classroom I did my practicum in.

Nicholas, in an interview, was not as committal, choosing to temper his acknowledged traditional teaching styles with some of the more progressive aspects of his practice.

Nicholas: I certainly would fit in well with the teachers at Amber Hill, especially with the focus on testing. But I'm not exactly the same. I tend to make more use of group work especially during project work.

Also common among participants was how incongruous their reading of the book was. In an interview immediately after the essay was due, Chad commented on his experience in reading the book.

Interviewer: So, what did you think of the book?

Chad: It was good ... it was eye-opening. As I was reading it I kept trying to identify myself with Jim at Phoenix Park but I kept coming back to Amber Hill. It was really troubling when I finally realized that I was an Amber Hill teacher.

Ten of the participants commented, either in their essay, their journals, or in the interviews on a similar experience of trying to will themselves into the more progressive classroom but not being able to avoid the reality that their teaching is more traditional than they may have been willing to admit ... even to themselves. This realization was troubling to some – Frieda in particular.

> I think the most powerful part of the book for me was the fact that it made traditional teaching look *so* traditional. When I realized that that was me it really bothered me. I used to think of myself as a really caring teacher. I guess I still am but it is really troubling to know that my students are probably just as off task and uninterested as the kids at Amber Hill.

Frieda is straddling two aspects of this realization at the same time – the realization that she is a traditional teacher *and* the realization that this form of teaching, her teaching, is not as beneficial to the students as she thought. In this she is not alone.

Each of the participants wrote at length in their essays and their journals about the negative effects of traditional teaching portrayed in Boaler's book. In particular, they commented on the issues of retention:

> If the students are not going remember the stuff we teach them then have they really learned? And if not, then what was the point in the first place? (Ingrid)

attention:

> If the students are not engaging with the lesson then there is no way that they can learn. (Eric).

and issues pertaining to student affect:

> Math needs to be fun. Sitting in rows and listening to the teacher is not fun. (Alicia).

In so doing they were rejecting the traditional teaching paradigm, and by extension, many of their beliefs about what mathematics teaching should look like.

Like the intervention designed around Lockhart's Lament (2008), the impactful aspect of Boaler's book (2002) seems to be the starkness of the picture it paints. This can be seen in both Chad's and Frieda's comments (above) and was evident in most of the data collected. It seems as this starkness makes explicit that which has previously been implicit, and brings to light exactly what the participants beliefs about teaching are. The evidence against the effectiveness of such a teaching style makes it almost impossible to sustain their traditional views of teaching and rejection seemed inevitable.

Surprisingly, as quick as the participants were to reject the teaching practices of Amber Hill, and thus many of their *a priori* beliefs about teaching, they were not as quick to simply adopt the paradigm extolled in the descriptions of Phoenix Park. For them, the classroom norms presented as part of the problem posing environment were far too chaotic and unstructured to easily embrace. This makes even more prominent the fact that they were rejecting without replacement many of their traditional beliefs about teaching.

Nature of assessment

Teachers' plans often begin and end with considerations of assessment. Although in traditional setting the culmination of a unit of instruction is usually a test, preparation for that test most often begins on day one. It is unavoidable; teachers tend to teach towards whatever assessment instruments they intend to use. As such, if changes in teaching practice are to be realized then changes in assessment practices need to follow (or lead). Although the research presented here is not about teachers' practices, per se, it is about the beliefs that guide those practices. Thus, changing teachers' beliefs about the nature of assessment in mathematics is of equal importance to that of mathematics and mathematics teaching.

Like with the aforementioned interventions, the plan for changing teachers' beliefs about assessment is organized around exposure to provocative literature – in this case two articles. The first article by Lew Romagnano (2001), called the *Myth of objectivity in mathematics assessment*, is a look at three instances wherein seemingly straight forward and objective marking criteria are anything but. Through these cases the article highlights the inherent subjectivity of teachers' marking habits and thus calls into question the idea of objectivity in not only marking but also in students' grades. The second piece of literature is a pre-publication version of a working group report on assessment (Liljedahl, 2010). In this report assessment in mathematics is examined from four perspectives: as communication, as a form of valuing that which is valuable, as a way of reporting out progress in meaningful and helpful ways, and as *not* ranking. Although not offering exacting models of how to implementing any of these models within a teaching practice, the report effectively calls forth the main purposes of assessment and what it needs to accomplish. The last section on *not ranking* is particularly provocative in that it challenges the paradigm of ranking that much of the traditional assessment and evaluation practices are constructed on.

These articles were given to the participants at the end of 7th week of classes along with the assignment to read them before the next week and respond to the following prompt in their journals:

> Comment on the articles with respect to your own assessment practices.

At the beginning of class in the 8th week the teachers were put into groups of 3 or 4 and challenged with the following scenario, for which they had a full hour to discuss:

> You have just posted the letter grades for the second term and one of your students, who is receiving a B (with 84 %)[3], comes to you and asks what they could have been better at in order to have received a higher mark. You open your grade book and look at the marks you have recorded for that student. Based on what is in your grade book, what can you tell them?

This was followed up with a presentation and a series of activities designed to expose the participants to a number of alternate assessment strategies.

... *Results*

It was clear from the journals that the articles that the teachers were asked to read had a powerful effect on all of them. Many of them wrote at length about the realization of the infallibility of marks. Ingrid's comments, in particular, are quite profound:

> In math we collect marks, lots of marks. Then we weight them and combine them to produce a percentage for the term. These percentages are then turned into letter grades. The marking program I use can record marks to the nearest hundredth of a percentage. Somehow, a long list of marks combined with the preciseness of the marks program has given the illusion that assessment is an exact science. After reading the Myth of Objectivity I started to wonder just how exact it is. I have a student who has 72.12 % and I'm trying to decide if I should bump him up to a B. As I look at his marks I realize that each one of these could be off. Even if they are off just a little bit by the time I weight each score and add them all together I could be WAY off. I have no idea what he really has, but I know for sure that it doesn't end in a .12.

Ingrid has identified one of the key problems with the formulaic aggregation of marks. Six other participants commented on a similar idea with four of them building their thoughts around the decision as to what letter grade to assign a student who *is so close*. Chad takes this a little further. In his anecdote he identifies one of the key issues associated with the conversion of marks between continuous (percentages) and discrete (letter grades) scales.

> Even if I do decide that John with his 85 % gets an A how does that make him so much better than Jason who is sitting at 84 % and getting a B. So, then I decide that they are not that different so Jason also gets an A. But what about Michelle who sits with Jason and is getting 83 %? And so on. Where does it end? More importantly, where does it start? Who decided that an A is 86 % in the first place?

But there is more to issues of assessment than the gathering of marks. At first, the 60 minutes allotted for the aforementioned discussion seemed to be too long. Once the conversations got going, however, it wasn't enough. Whereas the journal entries were dominated by references to the *Myth of objectivity* (2001) the classroom discussions were dominated by references to the working group report and the four purposes of assessment. Here the dominant theme that eventually emerged pertained to how ineffective their current record keeping practices were when it came to understanding exactly what a student was able to do.

Early on in their discussions one group mused about the different scenarios that could lead to a student having an 84 % from poor performance on a single test to neglecting to do their homework, to handing in some assignments late. Each of these scenarios led to a fictional response that they would provide the student. It was David, who re-read the prompt and then turned to his group to state that perhaps what the student was looking for is a better understanding of what mathematical concepts he could have performed better at. This led to further discussion. When they were asked to summarize Lori made the comment that:

> If we base this on the way I keep records now the only thing we can tell this student is that he should have scored better on this or that test, or that he should have handed something in on time. In essence, the only thing we can say to him when he asks what he could have done to receive a better mark is that he should have been better.

The intention of this prompt was to push the participants past the issues of marks to start to really think about the consequences of not thinking about assessment as a means of effective communication. Although most of the groups began these discussions talking about marks and the different ways in which students loose marks they all finished on the issue of how their current assessment practices are not well linked to student attainment of specific learning outcomes *and* that this was an important deficiency. This was nicely summarized by Jenny in her interview immediately after the group discussion.

Interviewer: So, what did you think of the discussion today?

Jenny: I'm not sure yet. It was good ... but I just don't know what to do about it. Clearly what I am doing needs to be improved. Not only are my marks not accurate, they aren't helpful in guiding students, or myself, into knowing what areas need to be worked on.

[...]

Interviewer: So, what about giving zero's. I heard you talking about this.

Jenny: Yeah, I've always been a big fan of using zero's as a way to force my students to do their work. But when I use it as mark I'm really saying that a student knows nothing. This doesn't say much about my teaching ability, does it?

Although the follow up activities addressed many of the deficiencies identified in the aforementioned excerpts, of relevance here is that the teachers were rejecting their beliefs about assessment before they were exposed to these alternate ideas. Abandonment of ideas such as the objectivity and precision of marks, the value in sorting students into letter grade groupings, the ability for a mark to speak for itself, and the use of zeros as an extrinsic motivator made room for new ideas to come in.

Conclusions

The results of the analysis of data pertaining to the three aforementioned interventions, as well as the three not presented here, indicate that: (1) the *theory of* conceptual change is a viable *theory for* designing interventions for the purpose of changing conceptions, and (2) implementation of these interventions resulted in cognitive conflict and eventually rejection of the participants' *a priori* beliefs. The cognitive conflict that precipitates this belief rejection seems to be greatly affected by the starkness of the images present in some of these interventions – especially when those images are both troubling and undeniably reflective of the participant's practice. Further, the data is replete with evidence that the participants not only rejected beliefs pertaining to their current practices, but that often they did so without an immediate replacement at hand. As such, this study not only reaffirms the much of what is known about cognitive conflict but also expands the scope of this body of literature to the domain of mathematical conceptions. More work is needed in this regard in order to ascertain whether or not expansion of the theory of conceptual change into this domain reflexively informs the theory in general.

In the introductory sections of this article I argued that learning and teaching were inherently different constructs, and as such, needed to be explained, not necessarily with different theories, but with theories in different ways. In this article, the *theory of* conceptual change was used as a *theory for* designing teaching. The empirical data and analysis of results show that within the aforementioned context this process was effective in producing the desired changes in teachers' beliefs. As such, this study adds to the small but important body of literature that attempts to bridge the gulf between theory and practice in practical (as opposed to theoretical) ways. More such work is needed in order to fully capitalize on the rich and abundant theories of learning that we have accessible to us.

Absent from this article, but present in the analysis are detailed accounts of how the theory for changing conceptions in general, and belief rejection in particular, has on the subsequent belief replacement. The results of this analysis will be presented in a forthcoming publication, as will the effects of the entire process on these participants teaching practice.

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Notes

1 In the context of this paper the term *students* can be ambiguous. It can refer either to the teachers participating in the study (as graduate students) or the K–12 students of these teachers. To avoid this confusion the term *students* will be reserved for the K–12 students while the terms *participants* or *teachers* will be used to describe the graduate students enrolled in the aforementioned program.

2 As the course designer and researcher I am heavily implicated in this research. In particular, with regards to what beliefs were chosen to be targeted and the relative worth of these beliefs vis-à-vis the ones I wished to promote. Although my decisions can be supported in the literature, they are still my decisions.

3 In the local context the range for a B is 73 %–85 %. 86 % is awarded an A.

Peter Liljedahl
Dr. Peter Liljedahl is an Associate Professor of Mathematics Education in the Faculty of Education and an associate member in the Department of Mathematics at Simon Fraser University in Vancouver, Canada. He is a co-director of the David Wheeler Institute for Research in Mathematics Education. His research interests are creativity, insight, and discovery in mathematics teaching and learning; the role of the affective domain on the teaching and learning of mathematics; the professional growth of mathematics teachers; mathematical problem solving; and numeracy.

liljedahl@sfu.ca