

Cooperation and collaboration as zones of proximal development within the mathematics classroom

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Beyond understanding the Vygotskian construct of zone of proximal development or ZPD with reference to an individual student, this paper explores the formation of ZPD within the pedagogical constructs of cooperation, wherein students cooperate with each other within their groups; as well as collaboration, wherein students from different groups that constitute the classroom collaborate with each other. Identified on the basis of functions that are in the process of maturing, the formation of either ZPD is exemplified from a socio-cultural-historical study at an upper secondary mathematics classroom in Norway. An emphasis on what distinguishes events in instruction that are educational from those that are not is also explored, before illustrating what nature of ZPD is established. The role of guidance received, imitation and cultural resources in the development of higher mental functions is understood as these ZPD are formed, enabling students to act independently within the classroom teaching-learning of mathematics.

Empirical studies within mathematics education that explore the Vygotskian construct of *Zone of proximal development* or ZPD seem to fall under two broad categories – those that focus on the interaction of individual students participating in a ZPD (e.g. Tudge, 1990; Forman & McPhail, 1993; Goos, Galbraith & Renshaw, 1999, 2002) and those that examine the role of cultural resources or artefacts, tools, signs, language and gestures embedded within micro-social contexts (e.g. Abreu, 2000; Albert, 2000; Lerman, 2001). While studies in the first category have attempted to understand the ZPD on the basis of individual ability, communication and metacognitive functioning those in the second category have

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attempted to grasp the role of cultural resources in mediating mathematical meaning. While either category has aimed to understand how to make classroom practices more mathematical, a need for more research explicating the interactions that actually take place in a ZPD is called for by both. In this paper I draw on my classroom study (Gade, 2006) and examine the role of guidance, imitation and cultural resources in the formation of a ZPD. Established within the pedagogical constructs of cooperation, wherein students cooperate with each other within their groups; as well as collaboration, wherein students from different groups that constitute the classroom collaborate with each other, I ask how upon their formation a student is able to act independently in the classroom teaching-learning of mathematics. Towards such understanding I first discuss ZPD as theoretical and analytical construct, then locate functions in the process of maturing in instruction, before describing the ZPD formed in group cooperation and classroom collaboration. I conclude by reflecting upon the many aspects of a ZPD which enable students to act independently in their learning of mathematics.

ZPD as theoretical and analytical construct

Postulating that the psyche of human beings is social in nature Vygotsky (1978, 1997b) viewed cognitive development in terms of their utilisation and mastery of cultural tools and resources. Covering a range of abilities like the ubiquitous use of language on the one hand and being able to exhibit and exercise voluntary attention on the other, these abilities Vygotsky argued were acquired initially under guidance from others in social practices and became available for independent use and control of one's own actions and behaviour only later. Resting upon biological functions that are innate and which he termed as lower mental functions, Vygotsky identified those functions that were mediated by cultural resources and involved a history of learning by an individual as higher mental functions. It was at the juncture of learning to master the utilisation of cultural resources under guidance, that Vygotsky formulated the construct of zone of proximal development or ZPD. Each of the terms zone, proximal and development need a closer examination. In explicating a Vygotskian approach in education, Bodrova and Leong (2007) point out that the term development refers to the formation of higher mental functions upon an individual's utilisation and successive mastery of cultural tools and resources. The term zone they say alludes to the fact that human development is not fixed and once and for all but a continuum of behaviours in various degrees of maturation. They finally argue that the term proximal in describing this zone, alludes to the fact that the zone

of development is itself limited by those behaviours that are closest to emergence in any individual and have the possibility of developing in the near future at any given point of time.

Interest in the zone of proximal development, or ZPD hereafter, lies also in its potential use in investigating the relationship between teaching, learning and development. It is here that the role of assistance that a student obtains from another individual becomes significant. While some form of learning happens at all times and from birth for any individual, Vygotsky's construct of ZPD focuses specifically on those abilities that a student or child can perform with the assistance of others as against those that she or he can perform independently. While independent performance is indicative of mental functions that have matured, the ability of a child to perform with assistance was for Vygotsky indicative of the existence of mental functions that had not already matured – but were in the process of maturing. Vygotsky (1997b) argued that a child's performance of various actions under guidance depended on the child's ability to imitate actions of those adults or more capable peers who were offering assistance. That this ability to imitate is dependent on the presence of corresponding mental functions that are in the process of maturing in a ZPD, is drawn attention to by Chaiklin:

The presence of maturing functions is the reason the zone of proximal development exists. Alternatively, one can say that the zone of proximal development is defined as referring to those intellectual actions and mental functions that a child is able to use in interaction, when independent performance is inadequate. [...] We see here that Vygotsky used the term *imitation* to refer to situations in which a child is able to engage in interaction with more competent others around specific tasks that the child would not be able to perform alone, because of the presence of maturing psychological functions. [...] In an interaction situation (collaboration), the child can only imitate that for which the maturing functions are present. If the child has no capability to imitate, then this would be taken as an indication that the relevant maturing functions were not present.

(Chaiklin, 2003, pp. 52–53)

In correlating the ability of a child to imitate actions received under guidance, to the presence in her of mental functions that are in the process of maturing, Chaiklin offers a yardstick with which to identify whether it is possible to establish a ZPD for any child. Following Chaiklin, a child's inability to imitate is a crucial and visible indicator of the fact that a child lacks the presence in her or him of corresponding mental functions that are in the process of maturing. The inability of a child to imitate and

establish a ZPD, in turn emphasises the importance this aspect has for a child's utilisation and successive mastery of cultural resources and the development of higher mental functions.

Following Vygotsky, Chaiklin's (2003) attention to the ability or inability of a child to imitate others provides a powerful means for both teaching and research to identify the basis upon which higher mental functions mediated by cultural resources can be formed. We may have here a key with which to resolve the frustration faced by many teachers – that of being unable to assist children and guide their utilisation and successive mastery of cultural resources. This key, which is the ability or inability of a student to imitate others, is very much observable in instructional situations. With respect to the nature of guidance that students can receive from others, I find Vygotsky (1997a) to argue in addition that not all events that are instructional in nature can be termed as educational. Taking an everyday example Vygotsky explained:

If upon leaving home, I arrange with my child where I'm going to leave the key for him, there can be no doubt that I am thereby forming a new relation with my child. But if this reaction does not have any purpose other than help him find the key, from the psychological point of view it cannot be termed educational. Consequently, not everything we do with children is education, in the scientific understanding of the term. (Vygotsky, 1997a, p. 58)

Vygotsky's example draws attention to the nature of relationships that need to be formed with a child or student, if the guidance offered to them is to be considered as educational. Examples of events in my study that would *not* be considered educational in line with Vygotsky, could include one of my teachers Olaf drawing the attention of his students to a certain page in their textbook or his urging students to consult other students in their respective groups when in doubt. An event that could be considered educational would include Olaf discussing with all his students how to obtain the value of the expression $(-2)^2$ both with and without a calculator. The manner with which to obtain a positive value in both cases, either by even order of multiplication or by using the appropriate button on the calculator, would be illustrative of what Vygotsky identified as educational relations. With Olaf forming new psychological relationships *for* his students, his students could either execute an even order of multiplication or utilise the calculator appropriately. They thereby had the potential to imitate Olaf's guidance both in the present and at any point in the near future, paving way to their utilisation and subsequent mastery of either operation on their own. In so doing Olaf's students would be able to utilise mental functions corresponding to these

operations, which had not already matured but were in the process of maturing. Van der Veer (2007) argues that it is the receiving of such nature of guidance by students that enables them to progress beyond their former zone of proximal development or ZPD and realise a new zone of actual mental development.

Much research teases out and extends the theoretical construct of ZPD beyond the basic Vygotskian formulation that I have outlined so far. Drawing upon anthropology and with an objective of not wanting to take the cultural context in which cognitive development takes place as a given, Cole (1985) argues ZPD to be a zone in which both culture and an individual's psychological processes create each other. The ZPD as a unit of analysis Cole argues, provides insight into how individuals acquire specific kinds of behaviour under guidance when engaged in specific goal-directed activity, for example the goal of obtaining the value of $(-2)^2$. Alluding to the Marxist origins of Vygotskian perspectives, Bruner (1984) considers ZPD to be an instrument that enables the sharing in any collective of not only knowledge but also of consciousness. It is under guidance and with the cultural resource of language says Bruner, that society provides for a child to be able to enter into a relationship with somebody more able. This enables the child to both develop and become conscious of one's own individual capabilities. On deploying ZPD as an analytical construct Wertsch (1984) draws attention as well to the notion of intersubjectivity amongst individuals who are engaged with each other. For Wertsch the existence of a ZPD underlines the fact that cognitive development cannot be actualised in individual terms alone and can be so only upon instruction – accompanied by various degrees of agreement or disagreement. It is in the formation of a ZPD Wertsch says that Vygotsky instantiates how interpsychological or socially mediated origins of higher mental functions become intrapsychological and available for individual utilisation. The role of cultural resources in enabling individuals to utilise and eventually master cultural resources and through them one's own mental functions is highlighted in particular by Stetsenko (1999). The use of the calculator cited above, she points out, would at the same time be an instance of students learning how the calculator-can-be-used-for-a-certain-purpose. Drawing attention to the crucial role of both cultural resources and guidance within a ZPD and human development more generally she says:

When one looks at the links that bridge social interaction, cultural tools, and ZPD then – and only then – does it become possible to understand why learning plays a leading role in development, why it constitutes the very essence of development rather than merely

following or supporting development. Learning leads development because it is through learning that the child comes to be able to master – through and within interactions with an adult – the new cultural tools; this mastery constitutes the very cornerstone of human development. (Stetsenko, 1999, p.248)

In furthering appreciation of the nexus between guidance, imitation and cultural resources Stetsenko draws attention to two important aspects. Firstly that any claim to mastery and human development by any child is a process of learning, and secondly that this process is simultaneously crucial to how a child has the possibility to master his or her own behaviour, attaining the freedom that is needed to take independent actions (Stetsenko, 2004). Having outlined how a child's independent actions are a consequence of his or her learning in a ZPD, I now turn to locating mental functions in the process of maturing.

Locating functions in the process of maturing

In being able to discuss how I located functions in the process of maturing in the students in my study, I first turn to relevant aspects of methodology. At the first year of an upper secondary school, the students in my classroom study learnt mathematics bilingually – Norwegian and English. The two teachers, Olaf and Knut, who taught this class lay emphasis on cooperative learning by students in their groups, which they combined with teaching at the whiteboard. As participant observer in this class I conducted a naturalistic and qualitative study so as to capture some aspects of this complexity (Strauss, 1995). My chief instruments of data collection were field notes, students' responses to group-tasks set by their teachers as worksheets and my audio-recording of problem solving sessions that I conducted with different groups of students I interacted with. In a longitudinal manner lasting most of the academic year I viewed both teachers and students as participants in the historic process of instruction, within which Olaf and Knut made special efforts to facilitate student participation both in group cooperation and classroom collaboration. Towards analysis I sectioned observations made with each of my instruments in a cyclical manner coinciding with the seven chapters of the textbook that was followed. Drawing on Vygotskian perspectives, I chose units of analysis that were not just descriptive but also explained development (Vygotsky, 1997b). In the extracts that I offer in this paper from the many in my study, I deploy mediated action as unit of analysis (Wertsch, 1991). By this I attempt to comprehend the mental functioning of individuals in relation to available cultural resources within the context of the teaching-learning of mathematics in my study. I offer two

extracts each to locate functions in the process of maturing and two each for the formation of ZPD in group cooperation as well as classroom collaboration. In these I identify myself as Res, the teachers as Olaf and Knut and students by their pseudonyms. I also distinguish two audible levels of talk within instruction, using the *italic* font for talk audible within a group and normal font for talk audible across the whole classroom.

The first of two extracts with which I locate mental functions in the process of maturing relates to my interaction with three students who were attempting to express the product of $(2 \cdot 10^4)(5 \cdot 10^3)$ as a single exponent in their notebooks.

- Res *What is the confusion?*
 Jan *Does 2 multiply 4?* [Pointing to the number 2 and the exponent 4]
 Res *Remove the brackets.*
 Jan $2 \cdot 10^4 \cdot 5 \cdot 10^3$ [Writing]
 Res *Do you get a 10?*
 Jan $10 \cdot 10^4 \cdot 10^3 = 10^8$ [Writing]
 Res [Now guiding Erik to remove brackets and obtain a 10]
 Erik $= 2 \cdot 10^4 \cdot 5 \cdot 10^3 = 10^8$ [Writing]
 Res [Now guiding Tove to remove brackets and obtain a 10]
 Tove $= 2 \cdot 10^4 \cdot 5 \cdot 10^3 = 10 \cdot 10^{4+3} = 10^8$ [Writing]
 Res *What was the power of the first 10?*
 Tove *One.*
 Res [Surmise that $a = a^1$ has not yet been explicitly discussed as a rule in instruction]
 Olaf Do we need the brackets? [Speaking at the whiteboard]
 Olaf No in multiplication, yes in addition. [Writing on the whiteboard]
 $= 2 \cdot 10^4 \cdot 5 \cdot 10^3 = 2 \cdot 5 \cdot 10^4 \cdot 10^3 = 10 \cdot 10^4 \cdot 10^3 = 10 \cdot 10^7 = 10^8$
 Olaf That's a hundred million.
 Thor A lot of zeros!

In portraying the attempts of Jan, Erik and Tove in arriving at the product of exponents in the two brackets, I argue for the presence of corresponding mental functions in each of them – where these mental functions had not yet matured but were in the process of maturing. My conjecture is based on the fact that my guiding each of them to remove the brackets and obtain a 10 enabled each of them to reach the solution of 10^8 . I point out as well that the guidance I offered was in addition to that of their teacher Olaf at the whiteboard, allowing opportunity for students to imitate actions demonstrated by either of us. I take this extract to be

illustrative of an instance of classroom instruction, wherein students who possessed mental functions that were in the process of maturing (corresponding to the multiplication of exponents) had opportunity to utilise cultural tools (like algebraic terms and brackets) under specific kind of guidance.

It is possible to consider the mental functions that I identify in the process of maturing in my second extract, as not being entirely desirable with respect to the classroom teaching-learning of mathematics. In my interactions with Per, I portray his attempts at another question from the topic of exponents – by which time Olaf is drawing the attention of his students to use formulae compiled in their formula book *Formelsamling i matematikk*.

Olaf It is important to use formulae.

Olaf $= 3^5 \left(\frac{x}{3}\right)^4 = 3^5 \cdot \frac{x^4}{3^4}$ [Writing on the whiteboard]

Olaf What is the rule?

Olaf $= 3^{5-4} \cdot x^4 = 3x^4$ [Writing]

Res [Observe that many students are still not making use of the rules in the formula book. Work with Per who seems to look for assistance.]

Per $\frac{2^5}{5^2} \cdot \left(\frac{5}{2}\right)^3 = \frac{2^5}{5^2} \cdot \frac{5^3}{2^3}$ [Working in his notebook]

Res $\frac{6^4}{6^3} = 6^{4-3}$ [Showing an example in Per's notebook]

Per $= 2^{5-3} \cdot 5^{3-2} = 2^2 \cdot 5 = 20$ [Continuing in his notebook]

Per *I am not very good at mathematics. I don't have confidence.*

Res *Just go ahead and try!*

My second extract portrays how Per is able to simultaneously utilise two aspects of guidance – Olaf's example of opening the bracket on the whiteboard and my example in his notebook of the division of exponents. I contend these actions of Per to constitute his ability to imitate, revealing also the presence in him of corresponding mental functions that are in the process of maturing. As with Jan, Erik and Tove the ability of Per to utilise cultural tools like exponents and brackets was a process of learning under guidance and something he may not have been able to achieve by himself alone. Per's admitting that he was not good at mathematics and that he lacked confidence leads me to argue in addition, that along with those maturing functions that enabled Per to work with exponents, Per's lack of psychological comfort with mathematics could also be in the process of maturing. As part of my ethical concerns as participant observer, during fieldwork Per and I spent time together working at his mathematics between teaching-hours.

My locating mental functions in the process of maturing in the two extracts just offered follow Chaiklin (2003) and drew on my ability to gauge whether students were able to imitate the guidance they were receiving. Towards this students in either case evidenced their ability to utilise cultural tools and resources, like mathematical symbols and algebraic notations towards mastering their own actions. In so doing they were able to act independently, giving opportunity and basis for me to infer the development in them of corresponding higher mental functions. Following Vygotsky(197b) the ability of students to act at first under guidance and later alone can be indicative also of the formation of a ZPD. Following Bruner (1984) and in such formation it can be argued that these students acquired symbolic tools that society had to offer, allowing each of them to be conscious as well of their own abilities. Following Stetsenko (1999) there is evidence in addition of students learning how to utilise cultural resources in specific ways like say bracket-for-multiplication and subtraction-of-powers-when-you-divide-exponents. Finally following Wertsch (1984) and in Thor expressing that he found a lot of zeros in a million and in Per expressing doubts in his capabilities at mathematics, the two extracts illustrate the opportunities any ZPD has potential for – for students to experience and express various nuances of intersubjectivity. I now turn to the nature of opportunities made possible in a ZPD formed when students cooperated with each other within their groups.

Group cooperation as ZPD

Towards my objective of illustrating the formation of a ZPD in group cooperation, I offer two episodes – the first relating to Thor working at a single group-task and the second relating to another student Egil and his group working at two group-tasks. Thor's attempts at his group-task refers to the topic of similarity in triangles, of which I reproduce in figure 1 the three questions that made up *task 1*, along with his attempts.

After allowing students time to make sense of what was expected of them in *task 1*, as well as discuss the three questions within their group, I record Olaf and Knut to call for whole class discussion which transpired as follows:

Olaf The angles of triangle ABC are similar to ...

Levi Those of XYZ.

Thor *The angles of the two triangles are equal?*

Thor *They are similar.*

Kim *The reason is they add up to 180 degrees.*

Similar Triangles

TASK 1

Explain why the two triangles are similar.

Because of same vinkles / angles

Calculate the length of YZ.

$$\frac{32}{8} = 4 \quad \frac{40}{4} = 10$$

Find the length of a side (y) in $\triangle ABC$ in terms of the corresponding side (x) in $\triangle XYZ$.

$$\frac{y}{x} = 4 \quad \underline{y = 4x}$$

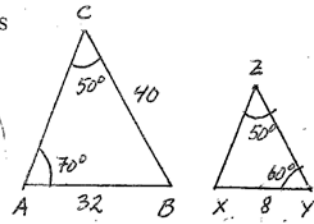


Figure 1. Task 1

Thor *Values?* [Addressing the researcher]

Res *Values.* [Confirming Thors usage in English]

Levi [Questions Knut who is visiting the group tables about the third task]

[...] [After a while]

Thor *Are angles and values the same as vinkles?*

Res [Discussion switches over to some dialogue from a fantasy or musical movie]

In the above extract I draw attention to the attempts of Thor who is Norwegian speaking at his learning of both Mathematics and English. In confronting the word angles in Levi his group-mate's response to Olaf, and in observing that the angles of the two triangles in the task were equal, Thor sought my guidance in ascertaining if he could use the word "values" to refer to the magnitude or measure of the angles. That this may be the case is demonstrated also by Thor's explanation as to why the two triangles are similar (task 1) in which the word "vinkles" may be a mispronunciation for the Norwegian word vinkler for angles. I argue Thor's actions to be indicative of two aspects – a search by him of using words in two languages that refer to the same idea and the presence in him of functions that were in the process of maturing. Thor's search for words corresponding to the same meaning and his responses in turn evidence two other aspects – his ability to imitate the guidance he sought and obtained, as well as the development in him of higher mental functions that corresponded to these abilities. This conjecture is based on Thor's ability to utilise and master to some extent the cultural resource

of the English language. It was the possibility of such an opportunity that Cole (1985) directed attention to, when any ZPD is formed. In Levi, Kim, myself and Thor's teachers all participating in his attempts at the task there is possibility following Wertsch (1984) to trace and follow the interpsychological and social origins of Thor's individual and intrapsychological thoughts. Following Van der Veer (2007) there is basis as well to argue that it was via participation in his group-task that Thor made progress beyond his former ZPD and established a new zone of actual development.

In my second episode relating to group cooperation, I illustrate yet again the efforts of Norwegian speaking students to express themselves in English. I offer two extracts in which I relate the attempts of Egil and his group at their utilisation of the textbook as a cultural resource in responding to two successive group-tasks – *proportionality* and *inverse proportionality*. The first extract relates to a set of proportional values of the extension of a spring with increasing weights. In this group-task students were asked to calculate the ratios of given values and plot these on a graph paper. Students were then asked to answer the question: What can you say about the graph? Towards responding to this question I record Egil pulling out his textbook from his bag and locate the topic of proportionality within. On finding the appropriate page Egil then drew the attention of his group-mates to the Norwegian text beneath an illustration of a straight line graph which read "Vi ser at grafen er ei rett linje gjennom origo" which translates as "We see that the graph is a straight line passing through the origin". Upon reading this text, the responses of Egil and his group mates in their respective worksheets was as below:

- Anja This graph is a proportional graph. When we double the force, the extension is doubled as well. The graph is a straight line.
- Egil When we double the force, the extension is doubled as well. It increases proportionally. The graph goes through the origin.
- Lea It's a straight line going through the origin. It's linear.
- Stine It's a steep straight line, and its goes through origin.

The responses of Anja, Egil, Lea and Stine's allow me to evidence the manner in which they as Norwegian speaking students utilised the Norwegian language in their textbook to express themselves in English. In they being able to do so I conjecture the presence in each of them of mental functions (corresponding to their utilisation of the English language) as in the process of maturing. It is possible to argue as well, that it was through a process of imitation of the text from the textbook and the guidance that Egil was able to offer, that these functions would develop in each of them as higher mental functions.

I now turn attention to the actions of Egil and his group at the group-task of inverse proportionality in which numerical values relating to pressure and volume of a mass of gas were under consideration. After plotting a graph as was required of them in the first group-task, Egil and his group mates had to once again respond to what they could say about the graph they had obtained. I record Egil's group not referring to their textbook this time and offer the following written responses in their individual worksheets:

Anja This is a inverse proportional graph. The graph is curved.

Lea It's an inverse proportional graph. It decreases.

Stine It's not straight. It goes like a bow.

Though I was unable to collect Egil's response, the responses of Anja, Lea and Stine enable me to evidence far more than the presence in each of them of mental functions that were in the process of maturing. Presented with goals similar to those in their first group-task, the actions of these students allow me to argue that their mental functions developed beyond their former ZPD and realised a new zone of actual development as pointed out by Van der Veer (2007). My conjecture is based on the fact that the actions of Egil and his group mates which were guided by the textbook initially were independent of such assistance subsequently. Following Stetsenko (2004) it can be argued as well that the presence of social interaction within Egil's group and their utilisation of the textbook as cultural resource enabled them to master their behaviour and act independently, within the ZPD that was established between Egil and his group-mates.

Classroom collaboration as ZPD

I discuss the formation of a ZPD in classroom collaboration with two episodes. In the first I portray how Olaf and Knut privilege in their instruction the use of the mnemonic shown in figure 2 – as a cultural resource which enabled students the opportunity to consciously reflect on its use (Wertsch, 1991). The episode relates to Rolf's use of the triangular mnemonic in attempting the following problem translated from his textbook:

If we travel t hours with a speed of v km/hr, then the distance s in kilometers is given by $s = vt$. Find the speed if we travel 259 km in 3.5 hours. Find the formula for speed v . Find the speed if we travel 364 km in 4 hours 40 minutes.

(Oldervoll, Orskaug & Vaaje, 2001, p. 73)

Rolf had utilised the mnemonic to remember the formula involving speed, distance and time and solve the numerical question at hand. As

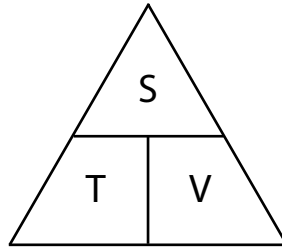


Figure 2. *Mnemonic*

part of their regular visits to the tables of student groups, as participant observer I record Olaf and Knut to notice Rolf show his group mates his use of the mnemonic. Soon upon this Olaf and Knut called the attention of all other students to a diagram of the triangular mnemonic they drew on the whiteboard. Olaf and Knut went on to discuss Rolf's use of the mnemonic where Rolf offered that he remembered the mnemonic as "Siv has a TV in the basement". It is the use of the mnemonic introduced by Rolf within instruction that I focus upon in this episode of classroom collaboration. I trace the use of the mnemonic introduced by Rolf and privileged by his teachers (Wertsch, 1991) to Tia from a group other than Rolf's in the classroom. I do this by discussing Tia's response to a question in the same group-task of inverse proportionality that Egil and his group had attempted. As I offer Tia's response in figure 3, I mention that within the historical progression of instruction in my study the events relating to Rolf occurred prior to the conduct of the group-tasks on *proportionality* and *inverse-proportionality* in the classroom.

Task 5: Use your result from task 4 to find a formula for y in terms of x ($y = \dots$)

$$y = \frac{a}{x} \qquad y = \frac{40}{x} \qquad \begin{matrix} x \cdot y = a \\ y = \frac{a}{x} \end{matrix}$$

In this case the quantities x and y are said to be inversely proportional.



Figure 3. *Tia's response*

Tia's response as offered in figure 3 shows her using a mnemonic very similar to the one introduced by Rolf in the classroom. I find Tia's response to be indicative of two aspects – her recognising the inverse relationship between the values of pressure and volume of gas given in this group-task, as well as her ability to utilise the mnemonic that was privileged by Olaf and Knut. Beyond Tia's actions demonstrating the presence in her

of functions that were in the process of maturing, I conjecture that Tia's actions also demonstrate that she was able to master the utilisation of a cultural resource. The mnemonic that Tia used had the triangular structure that was introduced in the classroom, yet dealt with variables A , x and y , that were applicable in her question at hand. Following Stetsenko (2004) I argue that the utilisation and mastery of the mnemonic by Tia is indicative of her being able to achieve a certain degree of freedom and even act independently. These actions of Tia can be seen as a result of her being able to imitate the guidance she received from her teachers and the development in her of higher mental functions within the formation of a ZPD. Following Cole (1985) Tia's utilisation of her version of the mnemonic can also be taken as evidence of the manner in which cultural resources and individual psychological processes create each other when a ZPD is formed – within classroom collaboration.

The second episode of classroom collaboration and the last of extracts that I offer in this article relates to whole class discussion steered by Olaf at the whiteboard, on the topic of scale factor or ratio of corresponding lengths in similar geometrical figures.

- Olaf [Draws two squares with sides 2 and 6 units on the whiteboard]
 Olaf If we remember the square.
 Olaf What is the scale factor of the side?
 Jan Three.
 Tove Three.
 Levi Or one by three.
 Olaf What is the scale factor of the area?
 Olaf $f = 3$ [Writing on the whiteboard]
 $A_2 = A_1 f^2 = A_1 3^2$ [A referring to areas of the squares]
 Olaf The ratio of the areas of the square is the square of the scale factor.
 Olaf What is the scale factor of volume?
 Ulrik Cube.
 Olaf Cube ... sure?
 Ulrik No.
 Olaf Don't you trust yourself?
 Olaf [Now draws diagrams of two cubes with sides 2 and 6 units]
 Olaf What is the scale factor between the volumes 8 and 216?
 Res [Olaf recounts ratio between the corresponding sides and faces of the two cubes]
 Olaf $\frac{216}{8}$ [Writing on the whiteboard]
 Olaf How much is this?

Dan 27.

Olaf 27! [Showing surprise at Dan's response]

Olaf $\frac{216}{8} = 3^3$

Olaf So you were right Ulrik.

Olaf So if you have the volume of one of them we can calculate the value of the other.

Levi What if we have to do it the other way?

Olaf If we know volume of the larger we find the volume of smaller.

Olaf $\frac{2}{6} = \frac{1}{3}$ [Writing on the whiteboard]

$$\left(\frac{1}{3}\right)^3 = \frac{1}{27}$$

Olaf Good question.

I quote this last extract at length to evidence the presence of functions in the process of maturing in many students – Jan, Tove, Ulrik, Dan and Levi, none of whom belonged to the same student group within the classroom. I draw attention in particular to Levi's responses since these seem to run counter to the grain of discussion of scale factor that Olaf has been guiding and steering within the classroom. While most students were working with a ratio of the larger dimension to the smaller, Levi is seen working in terms of the ratio of the smaller to the larger. In Levi initially suggesting to Olaf that the scale factor being discussed could be one by three and persisting by asking if their scale factor could not have been calculated in another way, Levi evidences that his conception of scale factor is his personal version of the ratio. Way beyond evidencing the presence in him of functions that are in the process of maturing, Levi's actions demonstrate his ability to act independently and master utilisation of the cultural concept of scale factor. Levi's actions allow me to postulate not just the presence of a ZPD in classroom collaboration but also a trajectory of features that constituted such a creation – the ability to imitate by Levi, the role of educational guidance by Olaf, opportunity to utilise and subsequently master cultural resources for Levi, the possibility for the development of higher mental functions in Levi and his classmates and finally Levi's ability to attain a certain degree of freedom and act independently towards achieving the goals he set had for himself.

Acting independently in a ZPD: synthesis and conclusion

In bringing arguments related to the analysis made possible by the formation of a ZPD to conclusion, the aspect that seems to be of greatest significance to the teaching-learning of mathematics is the ability of

students like Thor and Egil in group cooperation and Rolf, Tia and Levi in classroom collaboration to act independently. The independent abilities displayed by them is worthy of note since their very occurrence evidences the gaining by each of them of a certain degree of freedom made possible, as Vygotsky (1997b) would argue, by the development in each of them of higher mental functions. Mediated by cultural tools or resources like the textbook, algebraic notations, mnemonics or simple arithmetic ratios these functions, as Vygotsky postulated, rested upon innate, biological and lower mental functions. In the ZPD formed in either case, the guidance provided by Olaf, Knut and myself or even Egil to his group mates must thus be educational in the Vygotskian (1997a) sense, since it would only be when new psychological relationships were created with students, that they would be able to eventually work independently. The possibility of a ZPD to be formed when students received educational guidance in turn depended on their ability to imitate when guidance was offered and of the presence in each of them, as Vygotsky argued (Chaiklin, 2003), of mental functions in the process of maturing. Synthesising my arguments so far, I represent diagrammatically in figure 4 this trajectory of a student possessing mental functions in the process of maturing up until his or her ability to be able to act independently, one which is made possible upon the establishment and formation of a ZPD.

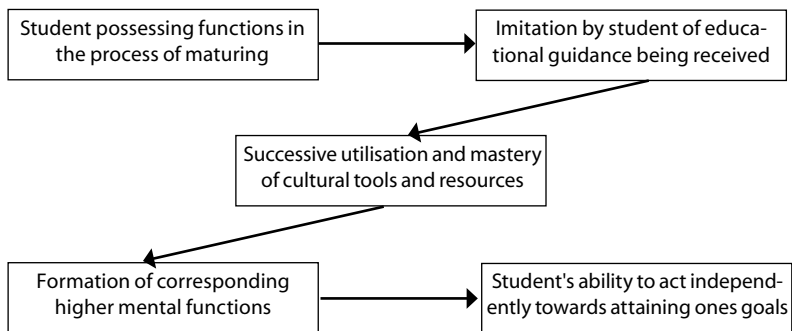


Figure 4. *Trajectory of ability to act independently made possible by a ZPD*

Having outlined the trajectory of events by the end of which students are able to act independently by drawing upon various ZPD formed in my study, I draw attention to issues relevant to the teaching-learning of mathematics in classrooms. Towards this aim there seems adequate evidence of the nature of complexity that needs to be understood while conceptualising how students learn in varying pedagogical contexts. For

one the role of both teacher and student in the educational process seems to be a highly active and simultaneous one, making the process of teaching and learning an inseparable one of teaching-learning. There is evidence also of the nature of opportunities within classroom instruction, which can either be deliberately brought about or made use of, if these are to foster the kind of relationships that Vygotsky (1997a) would argue as educational. The utilising of Rolf's mnemonic by Olaf and Knut is an example of the latter, while the allowing of students to utilise their textbooks while attempting tasks in classroom practice can be an example of the former. The examples of group cooperation and classroom collaboration that I offer are evidence in addition of how these constructs could offer a discursive basis for learning – in line with what Olaf and Knut had intended and desired as teachers. As is understandable and depending on immediate and day-to-day considerations, it was also the case that Olaf and Knut designated certain tasks to be attempted individually, even while students were seated in their groups.

The issue of imitation that Chaiklin (2003) draws attention to seems critical in being able to identify whether students possess functions in the process of maturing – necessary for the formation of a ZPD. It was only upon locating these that it became possible for me as researcher, to correlate the guidance offered by teachers to those actions of students that could be considered as independent. This aspect holds specific relevance to the subject of mathematics since both rudimentary and highly specialised cultural tools and resources of the body of mathematics are those that need to be learnt from others in society. Based on my study, it is possible to argue as well that it was the utilisation and mastery of the cultural tools within mathematics, that gave rise in students of behaviours that differed from each other not so much in quantitative but in qualitative terms. These behaviours were neither acultural nor ahistorical, in that they depended heavily on the cultural tools that were made available within learning as well as the progression of such learning that transpired. As Bordova and Leong (2007) point out, there was nothing static and once and for all about the formation of these behaviours as these were always maturing in various ZPD formed. As argued by Van Der Veer (2007), in some cases these behaviours exhibited the formation of new zones of actual mental development. In relation to the many extracts in which I deployed ZPD as both analytical and theoretical construct it is possible to say that as a researcher I stood informed in terms of whose mental functions, with which kind of cultural resources and what nature of guidance that was received, enabled mental functions that were in the process of maturing to develop within teaching-learning. Adopting such an analytical approach was notable in that there was

opportunity for me and research more generally, to understand students' abilities not merely in terms of what they came to the classroom with, but of how the abilities they come to participate with to the classroom could be nurtured into their ability to act independently. I would like to propose as well that unlike the term internalisation – referring to the process of how a shared mental function becomes an individual mental function (Bordova & Leong, 2007) or the term appropriation – referring to the making one's own of any cultural tool or resource (Wertsch, 1991); an emphasis on processes that enable students to become independent seem easier to implement within classroom teaching-learning, besides being at the heart of a collective of processes we may call education.

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