# An investigation of Norwegian students' affective domain in mathematics

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After decades of research in the affective domain in mathematics education, and search for ways to enhance students' positive attitudes towards the discipline, the perception that to be able to do mathematics is innate remains a widespread belief. Already twenty years ago the Fourth NAEP study concluded that students believe mathematics to be important, difficult and based on rules, and theses attributions also characterise the view of mathematics even two decades later. As a relationship exists between the claims "mathematics is difficult" and "mathematics is boring" one could assume that students lack interest towards mathematics. The conclusions about the present situation are based on a study carried out in Norway in 2005. This paper documents and analyses the data from the study. Six factors are identified and analysed in relation to students´ affective domain in mathematics. The six factors are: interest, hard-working, self-confidence, usefulness, insecurity, and MAD (Mathematics as an Absolute Discipline).

Students' affective domain in mathematics education is the focus of the study presented in this paper, based on fieldwork carried out in Norway in spring 2005. This study is part of the LCM-project (Jaworski et al., 2007) within the Norwegian Research Council's KUL[1] programme and it addressed the question of the nature of Norwegian students' beliefs, attitudes, and emotions in mathematics (and its teaching and learning). Many studies have provided evidence that students' learning outcomes in mathematics are strongly related to their beliefs about mathematics (Furinghetti & Pehkonen, 2000; Pehkonen, 2003; Schoenfeld, 1992; Thompson, 1992). Pehkonen (1995) notes that beliefs do influence not only how students learn mathematics but they "may also form an obstacle

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Kislenko, K. (2009). An investigation of Norwegian students' affective domain in mathematics. *Nordic Studies in Mathematics Education*, 14 (4), 33–64.

for effective learning in mathematics" (p. 21). Students, holding negative beliefs about mathematics teaching and learning, more often become passive learners, who place more emphasis on remembering than understanding (Pehkonen, 1995). Thus, to improve the teaching and learning of mathematics (one of the aims of the LCM-project), teachers and teacher educators need to be aware of the students' beliefs and the influence these beliefs may have on their learning.

The data collection of the study fulfilled two aims – first, to have a kind of post data, and secondly baseline data. The post data collection is in relation to the study carried out in 1998–1999 in Norway, called the KIM [2] project, which collected national data on students' understandings of key concepts in the national mathematics curriculum in Norway (Brekke, Streitlien & Wiik, 2004; Brekke & Streitlien, 2005). This survey used a Likert scale questionnaire that contained 125 items designed to expose students' beliefs about mathematics and mathematics teaching and learning. In 1997 the Norwegian school system was reformed and a new curriculum (referred to as L97) was developed and implemented, which "affected the whole of the compulsory education system" (Norwegian directorate for education and training, 1997). Therefore it evoked an interest to detect possible differences between students' answers in the very beginning of the curriculum implementation, and seven years later (this comparison is not presented in this paper, see the comparison in Kislenko, Grevholm & Lepik, 2007). Moreover, as the instrument used in my study is virtually the same as the one used in the KIM study (with respect to the comparative interest), then there was a curiosity to see if the structure of the affective variables measured with the same tool is stable.

From another perspective, one of the goals of the project was a baseline data collection before any intervention planned to be undertaken. For improving teaching and learning of mathematics one needs to have a rich picture of the current situation in mathematics teaching and learning. Therefore, students' beliefs, attitudes and emotions towards mathematics should be taken into account as these play an important role in mathematics education. Before any possible changes are implemented it is necessary to explore the present situation. From that, the implications could be, for example, splitting students based on their beliefs or achievement or socio-economic status, and inquiring into the development of the prerequisites in these groups. Or when detecting the relationships between different variables of which one is not directly changeable, for example if there exists a relationship between interest and self-confidence in mathematics, then it might be possible to evoke a positive change in a so called "hardly approachable" variable through improving the other one. Or when explicit differences in some areas of affect between different groups of students (e.g. gender, age groups) could be exposed it might give a clear sign that in some of these groups a conscious change is needed (e.g. through different teaching methods) for improving the situation in favour of the ones suffering. As pointed out in the present curriculum implemented in 2006 "Both girls and boys must have the opportunity to gain rich experiences that create positive attitudes to and solid competence in the subject" (Norwegian directorate for education and training, 2009).

The specific research question for this paper is: what characterises the affective variables – beliefs, attitudes, and emotions – towards mathematics that students from one urban area in Norway hold? It includes the following sub-questions inspired by the outcomes of the KIM-study:

1. In what way are the affective variables structured?
2. What are the general tendencies in pupils' answers?
3. Are there differences from the perspective of gender?
4. Are there differences from the perspective of particular grades?

Thus, this paper addresses the following issues. Firstly, to detect the structure and character of affective variables, and to compare it with the structure identified in the KIM study; secondly, to find relationships between the variables; thirdly, to determine the general tendencies in students' answers about their beliefs, attitudes and emotions in mathematics; and, fourthly, to compare the answers from the perspective of gender and grade.

## Theoretical considerations

Researchers have noted that there is a lack of well-developed well-defined theoretical frameworks in the study of beliefs and attitudes in mathematics education (e.g. Goldin, 2004; Hannula, 2004). Di Martino and Zan (2003) have explored several studies and conclude that one of the reasons for this lack of clarity is that most studies focus on the creation of instruments rather than on the improvement of the theoretical underpinnings and "a large portion of studies show a lack of a clear definition of attitude" (p. 451). This problem might be hidden in the meanings of the conceptions *belief* and *attitude*. These two notions are crucial in the study of individuals' affective domain and they are hardly definable as they are not directly observable and their meanings overlap (Leder & Forgasz, 2002). Not only do the definitions themselves differ but the type of definition differs as well (McLeod & McLeod, 2002). Some researchers see an attitude as a collection of beliefs (e.g. Rokeach, 1972; Sloman, 1987), others classify belief as one component of attitude (e.g. Aiken, 1980; Statt, 1990) or distinguish these two notions from each other (McLeod, 1989). Kulm (1980) suggests that there is no possibility "to offer a definition of an attitude toward mathematics that would be suitable for all situations, and even if one were agreed on, it would probably be too general to be useful" (p. 358). Nevertheless, Leder and Forgasz (2002) claim that despite not having a precise fully agreed definition for these notions, much useful work can be done in the affective domain of mathematics education.

In this study my tool was the questionnaire already developed by the project KIM (see discussion above). Therefore, theoretical underpinnings of the study are highly similar to the ones used by the KIM developers, who took Douglas McLeod's work (1989, 1992) as a theoretical base of the study when developing the questionnaire. The term *affective* here refers to "a wide range of feelings and moods that are generally regarded as something different from pure cognition" (McLeod, 1989, p. 245), and is structured as presented in table 1.

Table 1. *The affective Domain in Mathematics Education (McLeod, 1992, p. 578)*

| Category | Examples |
|---|---|
| Beliefs | |
| About mathematics | Mathematics is based on rules |
| About self | I am able to solve problems |
| About mathematics teaching | Teaching is telling |
| About the social context | Learning is competitive |
| Attitudes | Dislike of geometric proof |
| | Enjoyment of problem solving |
| Emotions | Joy (or frustration) in solving non-routine problems |

The main terms in the study are *beliefs*, *attitudes* and *emotions*. Taken in that order, they represent an increasing degree of affective involvement, and intensity of responses; and a decreasing level of cognitive involvement, and response stability. Beliefs are pointed out to be mainly cognitive in nature and are developed over a long period of time (Goldin, 2002; McLeod, 1992). This is coherent with the idea of beliefs being an individual's subjective knowledge (Lester, Garofalo & Lambdin Kroll, 1989; Pehkonen, 2003), and being situated somewhere between humans' affective and cognitive domain, what Pehkonen (2003) calls the "twilight zone". Keith R. Leatham (2006) explains that "beliefs and knowledge can profitably be viewed as complementary subsets of the set of things we believe" (p. 92) because

> Of all the things we believe, there are some things that we "just believe" and other things that we "more than believe – we know". Those things we "more than believe" we refer to as knowledge and those things we "just believe" we refer to as beliefs. (p. 92)

Attitude is an affective response that includes negative or positive feelings of moderate intensity and stability (e.g. dislike of problem solving, enjoyment of algebraic proof). Emotions can be considered as states of feelings that rapidly change (Goldin, 2002), and they are highly affective in nature (McLeod, 1992).

## Research participants

Six schools (that were not randomly chosen as they were all partners in the KUL-LCM project) from the same urban area in Norway, took part in this study. The study included a total of 245 students – 85 students from grade 9, 97 students from the first year in upper secondary school who had chosen the advanced mathematics course X (later called "X-course students"), and 63 students from the first year in upper secondary school who had chosen the mathematics course Y, which focuses more on practical applications of mathematics (later called "Y-course students")[3] – responded to a questionnaire. For clarification, lower secondary school in Norway comprises 10 grades i.e. students in the first year in upper secondary school are 16–17 years of age.

## The instrument of the study

As pointed out several times before the instrument used in my research was adopted from the KIM study. Some of the items were removed based on my research question, and the final questionnaire contained 98 statements divided into 11 groups based on the specific content of the statements[4]. For example, the statement "Mathematics is difficult" belonged to the group "your thoughts about mathematics" whereas the statement "Most of my classmates find it important to work hard during mathematics lessons" to the group "environment in class".

The first four groups dealt with the issues in relation to the affective variables. Namely "your thoughts about mathematics" (group A, 16 statements); "your thoughts about learning mathematics" (B, 13 statements); "your thoughts about your own abilities in mathematics" (C, 10 statements); and "your own experiences" (D, 3 statements). The following 7 groups reflected the activities in the lessons, questions like "how often does it happen in the mathematics lesson", teaching tools, handling new theme, teachers' assessment, etc.

### *Justification of the chosen instrument*

The research questions aim to illuminate students' affective domain, especially students' beliefs, attitudes, and emotions towards mathematics. There were several reasons for using a Likert scale in the study. First, one of the aims was to have a comparative survey, and this more or less predetermined the instrument used (see the results of this comparison in Kislenko, et al., 2007). Secondly, and more generally, Likert type scales (Likert, 1932) are widely used to measure people's attitudes, preferences, images, opinions and conceptions, in general (Göb, McCollin & Ramalhoto, 2007; Wu, 2007). In particular, there are several studies about affect in mathematics education that use a Likert scale questionnaire as the main instrument or one of the instruments (e.g. Ma, 1997; Nurmi, Hannula, Maijala & Pehkonen, 2003; Pehkonen, 1994; Pehkonen & Lepmann, 1994; Pehkonen & Törner, 2004, Vale & Leder, 2004). Thirdly, as dealing with a large sample size a questionnaire seemed to be an appropriate data collection tool within the confines of limited resources.

### *Criticism towards the instrument*

As pointed out earlier, affective variables, like attitude and belief, are hardly definable concepts, and to use questionnaires to investigate these complex notions has been criticised as being too narrow and simple. All this criticism has been thought over and taken into account when preparing the study and deciding about the methods. One of the limitations of the Likert scale instrument has been pointed out by Kloosterman (2002), who notes that

> Such scales [Likert scales] can give researchers information about the beliefs that students hold but they are severely limited in their ability to explain how such beliefs formed or how beliefs are likely to influence the action. (p. 249)

Thus it is possible to answer to the question "what" but not easy to answer the questions "how" and "why". Richard Pring (2000) expresses a general critique towards surveys when he disagrees with the researchers who try to quantify aspects that are not quantifiable. He particularly dislikes the strict scientific/mathematical paradigm that is applied inappropriately when measuring understanding of human beings. Moreover, he notes that

> Surveys which tot similar responses to the same question might in fact give a very distorted picture of how different people really felt about or understood a situation. And this becomes even more insidious where children's understandings, knowledge and attitudes are given numerical scores, and these then compare with others' scores or grades, as though it is the "same thing" being spoken about. For some strange reason, this problem is rarely acknowledged, and thus, under the urge to quantify, we reduce to an arithmetical unit the complexity of children's struggle "to make sense" or to understand.
> (Pring, 2000, pp. 54–55)

These limitations have to be taken into account when using questionnaires. In this study, where I wanted to investigate if the structure and characteristics of affective variables were similar to the one found in the KIM-study, the only possible instrument was a similar questionnaire. Another reason was that I needed to reach a larger group of respondents, so interviews were not possible.

## Data gathering procedure

The questionnaire was web-based, and was made available to the students on an Internet webpage. Each student was given a unique code that he/she used to log into the questionnaire page. Most students answered the questionnaire during their spare time. The kind of answering procedure might have excluded some students from the study, as there existed a conscious risk that those who answered to the questionnaire might have been more positive towards schooling. It is possible that the data was gathered from the students who might have been more diligent, and more responsible towards schoolwork, and therefore not been the representative of the whole cohort in the classes. Thus, it is possible that the answers are biased due to the answering procedure. The overall response rate was more than 75 % (326 codes were given out and the analysis was done based on the answers of 245 respondents), which can be considered as good.

## Data analysis

### *Justification of the methods*

The following analysis is done taking into account the appropriateness of the methods using a Likert scale questionnaire. There are two parts in the analysis; first, a factor analysis together with the correlation coefficients for structuring the items and detecting the relationships between the factors; and secondly, descriptive techniques, like, frequencies and cross-tables. The correlation coefficients presented are Spearman's, and as this is a non-parametric method it is suitable for the ordinal scale (Field, 2005). Different descriptive techniques are also considered to be appropriate for this kind of instrument (see e.g. Fan & Yeo, 2008; Kislenko & Grevholm, 2008). Still, one has to be aware that using factor analysis on the data collected through Likert scale is problematic as the items in the Likert scale are discrete in their nature but factor analysis assumes that the scale of the observed variables is continuous (Byrne, 2001). Moreover, the normality of the data, which is another assumption of the factor analysis, is also questionable. One of the characteristics of a Likert scale is that verbal labels are symmetrical around a neutral middle, and response levels are anchored with consecutive integers (Uebersax, 2006). Therefore, first, the scale is not continuous, and second when the aim of the study is to determine respondents' opinions it seems awkward to build up a questionnaire, which assumes that a considerable amount of answers are "neutral" or "undecided"; and this is, in turn, contradictory to the idea of the answers being normally distributed. Thus, the logic behind the structure of a Likert scale is inconsistent with the normality assumption (Clason & Dormody, 1994; Wu, 2007).

But despite the violations of the assumptions I followed Kim and Mueller's (1978) suggestions, where they noted that when "the researcher's goal is to search for clustering patterns" then using factor analysis on data containing variables with a limited number of categories may be justified.

### *Items description*

Based on the research questions the statements (42 altogether) from the first four groups were considered in the analysis. These were the same statements as used in the KIM study factor analysis (Brekke, Streitlien & Wiik, 2004). The statements in group A were 5-point Likert items from "totally agree" to "totally disagree"; in groups B and C 4-point Likert-type items (the choice "undecided" was excluded). Group D statements used the choices from "never" to "very often" i.e. 4-point Likert-type items. In order to make comparison easier the items rating were kept similar to the KIM questionnaire, and the items were not homogenized.

### *The concept of factor and factor analysis*

In the beginning of the factor analysis the researcher has different questions or items, which are more or less theoretically founded. These questions and items are called variables, and the idea is to gather different variables into a group that is called a factor. With other words, "when a group of variables has, for whatever reasons, a great deal in common, a factor is said to exist" (Child, 1970, p. 2). Therefore, the researcher uses a statistical technique called factor analysis for identifying these groups of variables (Field, 2005). When the groups are detected then confirmatory factor analysis could be used for testing these groups, now called factors. Finally, based on the theoretical foundations, exploratory and confirmatory analysis, and, moreover, common sense, the factors are created and ready to be named.

### *Exploratory factor analysis*

Exploratory factor analysis gave the suggestion of a five-factor solution in my study, at the cost of eliminating several items, which did not belong to any of these five factors. Looking closely at the variables, which were eliminated, and based on common sense they seemed to constitute another factor and this was quite striking as it was not coherent with the suggestion given by the computer program (suggestion of a 5-factor structure) nor was it coherent with the structure presented by the KIM study analysis (the KIM study presented a 5-factor structure as well (Brekke et al., 2004)). The sixth factor, which I have named the MAD-factor (Mathematics as a Absolute Discipline), was tested in a confirmatory factor analysis (see discussion later), and this factor had the strongest fit indices i.e. this factor had the strongest structure, and there could be found strong theoretical evidence in relation to this factor in the literature. Two studies introduce the statements, which are in close accordance with the items in my sixth factor (see the list of items below). In the chapter by Alan Schoenfeld he presents "Typical Student Beliefs about the Nature of Mathematics" (1992, p. 359). These 7 statements illustrate students' beliefs about mathematics, including statements that are coherent with many statements in my sixth factor. Such as "mathematics problems have one and only one right answer"; "there is only one correct way to solve any mathematics problem – usually the rule the teacher has most recently demonstrated in the class"; "ordinary students cannot expect to understand mathematics; they expect simply to memorize it and apply what they have learned mechanically and without understanding"; "students who have understood the mathematics they have studied will be able to solve any assigned problem in five minutes or less" (Schoenfeld, 1992, p. 359).

In the survey carried out by Christine Brew and her colleagues one group of items described epistemological beliefs associated with mathematics being an absolute discipline, and the items were following: "mathematics does not involve values and opinions"; "mathematics is not creative"; "in mathematics you are either right or wrong"; "mathematics is best learnt by students working alone"; "more than one right answer in mathematics is confusing"; "mathematics is about learning rules" (Brew, Riley, Walta & Itter, 2006). They called it a MAD-factor as the items reflect the belief that mathematics is an absolute discipline (Brew et al., 2006).

Comparing my sixth factor items with these two results I decided to adopt the wording from Brew and colleagues (2006), and call the factor a MAD-factor, an abbreviation of *Mathematics as an Absolute Discipline* as this factor have a resonance with the factor described in Brew et al. (2006) and Schoenfeld's (1992) statements that are as well related to mathematics as a discipline.

### *Confirmatory factor analysis*

Confirmatory factor analysis (CFA) is considered to be appropriate when a researcher has some knowledge of the factor structure (Byrne, 2001). Therefore, I ran a CFA using software AMOS 4 based on the KIM study factors, and factors from the exploratory analysis. If the fit indices were not good enough, a new structure was tested out. Finally, after several considerations 6 factors were extracted that all had a reasonably good fit. Five of them were almost consistent with the KIM factors and the sixth one was an extra factor constituted by the items the KIM study did not consider. I decided to call these five factors *Interest*, *Hard-working*, *Self-confidence*, *Usefulness*, *Insecurity* (using similar names as in the KIM study, except the last one where the KIM used Security) and *MAD*.[5]

## Results

Frequency tables and cross-tables are supposedly the most informative presentation of the ordinal data (the means and standard deviations of the items and factors are excluded from the analysis as these weren't considered as illustrative to this type of data; for an extensive discussion see Kislenko & Grevholm, 2008). The following paragraph presents the frequency tables for every separate factor in order to give the general picture of the results together with the correlation-graph, and introduces the differences between gender and grade (the corresponding cross-tables of different groups are presented in Appendix A and B, respectively[5]).

### *Interest*

The first factor was called *Interest* which can be interpreted as "feelings of enjoyment" (Ma, 1997). The percentages of the answer of every item are presented in table 2.

Table 2. *Interest*

| | Totally agree | Partially agree | Undecided | Partially disagree | Totally disagree |
|---|---|---|---|---|---|
| A2 Mathematics is exiting and interesting | 18.5 | **39.5** | 12.3 | 21.7 | 8.0 |
| A3 Mathematics is one of the subjects I like the least | 22.5 | 18.8 | 11.6 | **26.4** | 20.7 |
| A8 Mathematics is one of the subjects I like the best | 16.7 | 21.0 | 15.6 | 19.9 | **26.8** |
| A9 I never get tired of doing mathematics | 7.2 | 12.0 | 13.4 | 33.3 | **34.1** |
| A10 I like to do and think about mathematics also out of school | 3.3 | 14.1 | 20.3 | 29.0 | **33.3** |
| A13 Mathematics is boring | 22.1 | **28.3** | 13.4 | 22.8 | 13.4 |
| C3 Mathematics does not suit me | 17.4 | 19.2 | - | **32.6** | 30.4 |

*Note*. Numbers are in percentages.

Approximately 60 % of the students agreed that mathematics is exciting and interesting. Nevertheless, slightly more than 50 % of the students claimed that mathematics is boring. Altogether, about 43 students out of 276 (15.6 %) agreed that mathematics is boring and interesting at the same time. The interrelationship of these statements showed that 86 % of students who agreed that mathematics did not suit them found mathematics boring, whereas 63 % of "bored"-students found that mathematics did not suit them. So it seems that if students had acknowledged that mathematics is not suitable for them they were certain that mathematics was boring but "bored in mathematics" students were not so sure that mathematics does not suit them. Certainly, it is obvious that being bored in mathematics is a result of many different aspects, and the unsuitability of mathematics could only be one amongst several.

In general no gender difference was found between the replies of boys and girls (see Appendix A [5]). Nevertheless, boys tended to be more radical in their opinions as they "totally agreed/disagreed" more often. With the statement "I like to think about mathematics also out of school" 24 % of boys and 11 % girls' agreed (girls tended to be more uncertain).

Based on the comparison between the level-groups it appeared that students from course Y (the students more interested in humanities) showed more negative interest towards mathematics than students from other level-groups (see Appendix B [5]). They most often totally disagreed with the positive statements (A2, A8, A9, A10) and totally agreed with the negative ones (A3, A13). Forty one percent of Y-course students totally agreed that mathematics is boring and altogether the agreement rate in this group about this item was approximately 60 %. X-course students (the students more interested in mathematics and natural sciences) were the ones who found mathematics boring to the least degree, at the same time these were 9$^{th}$ grade students who agreed that the most of mathematics is exiting and interesting (83 % compared to 66 % and 42 %).

### *Hard-working*

The second factor was called *Hard-working*, another appropriate concept would be "effort in mathematics" (Kloosterman, 2002).

Table 3. *Hard-working*

| | Totally agree | Partially agree | Partially disagree | Totally disagree |
|---|---|---|---|---|
| B5 I have to solve many tasks to become good at mathematics | 37.3 | **42.0** | 13.4 | 7.2 |
| B7 I have to work hard in mathematics even if I do not enjoy it | 37.7 | **41.7** | 12.7 | 7.6 |
| B8 To be good at mathematics is dependent on hard work | 40.6 | **43.5** | 10.9 | 5.1 |
| B9 I have to solve many tasks to remember the method | 35.1 | **44.9** | 14.1 | 5.8 |
| B11 It is my responsibility to learn mathematics | 39.5 | **46.4** | 10.5 | 3.3 |
| B13 I have to ponder a lot in mathematics | 32.2 | **54.0** | 9.1 | 4.3 |
| C4 If I want to be able in mathematics I have to spend plenty of time solving tasks | 33.3 | **50.4** | 12.7 | 3.3 |
| C6 I can become clever in mathematics if I learn all the rules | 28.3 | **48.9** | 19.2 | 3.6 |

*Note.* Numbers are in percentages.

Based on table 3 it is clear that students acknowledged that hard work is needed in learning mathematics. Eighty four percent of students claimed that to become good at mathematics hard work is necessary, and 86 % agreed that they have to ponder a lot in mathematics. It is evident that learning rules i.e. rule-memorization plays an important role in the students' perception of mathematics as more than 77 % agreed that they can become clever in mathematics if they learn all the rules. This is consistent with the answers of statement in the MAD-factor B1 "the most important in mathematics is to know many rules" (70 % agreement, see discussion below).

Gender-comparison did not detect any significant differences, even though girls were generally more radical in their opinions than boys (except statements B11 and C6). Level-comparison gave no considerable differences either. This is true except for the item B5 (importance to solve many tasks), with what 30 % of 9$^{th}$ graders disagreed compared to 15 % (X-course) and 10 % (Y-course). Nevertheless, approximately 80 % claimed to become clever when learning all the rules. Thus it seems that for them it is more important to learn rules than to solve many tasks.

## *Self-confidence*

The third factor was called *Self-confidence* and it mainly considered items about one's abilities in mathematics.

Table 4. *Self-confidence*

| | Totally agree | Partially agree | Undecided | Partially disagree | Totally disagree |
|---|---|---|---|---|---|
| A4 Mathematics is difficult | 21.0 | **44.2** | 11.6 | 18.1 | 5.1 |
| A14 Mathematics is easy | 7.2 | 27.2 | 20.7 | **31.2** | 13.8 |
| C1 I am able in mathematics | 19.6 | **44.6** | - | 22.1 | 13.8 |
| C2 I can solve most of the mathematical tasks if I concentrate | 31.5 | **45.7** | - | 17.8 | 4.7 |
| C8 Mathematics is easy for me | 8.7 | 34.4 | - | **35.1** | 21.4 |
| C10 It is bad luck if I do not do well on a mathematics task | 12.7 | 31.9 | - | **36.2** | 18.1 |

*Note*. Numbers are in percentages.

Approximately 65 % of the respondents found mathematics difficult against 35 % who thought mathematics in general is easy. Forty three percent meant that mathematics is easy for them. "Both-sides agreement" gave a result (mathematics is easy *and* difficult) where 34 % of students who agreed that mathematics is easy agreed that mathematics is difficult, and 18 % of students who agreed that mathematics is difficult agreed that mathematics is easy as well. The fact that several students found mathematics difficult and easy at the same time might be explained by the idea that students' self-confidence can vary between different topics within mathematics i.e. in some areas in mathematics students feel more confident than others (Hart, 1989; Kloosterman & Clapp Cougan, 1994; Kloosterman, 2002). Even though a majority of the students noted mathematics to be difficult 65 % agreed that they are able in mathematics and that they can solve most mathematical tasks when they concentrate (77 %).

There were no gender differences identified in students' self-confidence in mathematics. In general, Y-course students' self-confidence was lower than that of 9$^{th}$ grade students and X-course students and this could be seen in every statement. A significant difference was found in the item C1 "I am able in mathematics" with what 57 % of Y-course students disagreed against 37 % (9$^{th}$ grade) and 30 % (X-course students). Moreover, only 56 % of Y-course students acknowledged that they can solve most of the mathematical tasks whereas more than 80 % of 9$^{th}$ grade and X-course students were certain about their abilities in solving tasks. Another interesting outcome was that even though 23 % of Y-course students totally disagreed that mathematics is easy approximately 42 % totally disagreed that mathematics is easy for them, which seems to be consistent with the idea that mathematics is an elite subject i.e. only special and extremely intelligent people can do it (Nardi & Steward, 2003).

### *Usefulness*

The fourth factor was called *Usefulness* where the statements reflected an utilitarian view of mathematics.

Based on table 5 it is obvious that students acknowledged the importance of mathematics as 91 % agreed that mathematics is important and 81 % agreed that mathematics is useful in their lives. Students' acknowledgement that mathematics is useful in their lives did not directly imply that they needed it in order to study further (64 % who claimed mathematics to be useful agreed that they need it after school) whereas around 90 % of students who thought they need mathematics after school acknowledged it's usefulness as well. While only 38 % of students claimed that mathematics helps them understand life in general, of these students more than 95 % also agreed that mathematics is useful which would suggest these beliefs are correlated. The pattern was not as strong when the reverse was considered. Explaining, of the 81 % who agreed that mathematics was useful, less than half of these students (45 %) agreed that mathematics helps them to understand life in general (A11). Thus it appeared that mathematics could be considered useful for lives even though this may not help the students to understand their lives better. A

Table 5. *Usefulness*

| | Totally agree | Partially agree | Undecided | Partially disagree | Totally disagree |
|---|---|---|---|---|---|
| A1 Mathematics is important | **63.4** | 27.5 | 5.1 | 2.9 | 1.1 |
| A5 Mathematics is useful for me in my life | **48.6** | 32.6 | 7.6 | 7.6 | 3.6 |
| A6 It is important to be good at mathematics in school | 36.2 | **45.3** | 10.1 | 6.5 | 1.8 |
| A7 I need mathematics in order to study what I would like after finishing school | **35.1** | 23.2 | 24.3 | 9.1 | 8.3 |
| A11 Mathematics helps me to understand life in general | 10.5 | **27.9** | 26.1 | 19.9 | 15.6 |
| A15 I do not need to know mathematics | 6.1 | 8.6 | 14.7 | 29 | **40.8** |
| A16 Good mathematical knowledge makes it easier to learn other subjects | 20.3 | **41.3** | 21.7 | 11.6 | 5.1 |

*Note*. Numbers are in percentages.

reason for this could be that students see that their lives consist of some parts where mathematics will not be useful for them.

No gender differences were detected in this factor. The expectation that Y-course students' answers would differ from the answers of students at other groups was not fulfilled. There were slight but not significant differences between the levels. Only 24 % of Y-course students disagreed that they needed mathematics in order to continue to their chosen further education, and only 5 % disagreed that mathematics is important, and approximately 20 % disagreed that mathematics is useful in their lives. This leaves seven Y-course students out of 10 who acknowledged the utility of mathematics in their lives despite the general assumption that mathematics does not have a part in their future profession or future study plans. Interestingly, these were X-course students, who are supposed to be more interested in natural science subjects and more focused in their studies of mathematics, who totally disagreed the most that mathematics helps them to understand life in general.

It was evident that 9$^{th}$ graders compared to others were more radical in their answers. The total agreement rate in every statement was notably higher amongst 9$^{th}$ graders than in the other two groups. For example, 75 % of 9th graders totally agreed with mathematics being important, and 66 % that mathematics is useful compared to 59 % and 40 % (X-course) and 46 % and 33 % (Y-course), respectively.

### *Insecurity*

The fifth factor called *Insecurity* might also be called ”Mathematics Anxiety associated with Learning” (MAL) (Brew et al., 2006) as all items were related to the learning of mathematics.

Table 6. *Insecurity*

| | Never | Seldom | Often | Very often |
|---|---|---|---|---|
| D1 I am afraid of making mistakes when I do mathematics | 18.5 | **43.1** | 29 | 9.4 |
| D2 I become nervous when we have tests in mathematics | 18.1 | **38.0** | 24.3 | 19.6 |
| D3 I am afraid to show my teacher that I do not understand mathematical problems | **44.6** | 35.1 | 14.1 | 6.2 |

*Note*. Numbers are in percentages.

Taking table 6 as a basis it seems that students felt secure in mathematics. Students on average were seldom afraid of making mistakes in mathematics and around 80 % of respondents were seldom or never afraid to show their teacher that they do not know how to solve a mathematical problem. Nevertheless, about 20 % of the respondents noted that they very often become nervous when they have tests in mathematics.

Gender comparison revealed that boys claimed to be less afraid of making mistakes and perform in a mathematics test than girls. Altogether there was a 15 % difference between boys and girls' ”never/seldom” agreement answer to the statement D1, and 18 % to the statement D2. Moreover, ten percent more girls than boys noted that they very often become nervous in test situation. Based on the level-comparison it appeared that X-course students were less anxious than others. Only 6 % of Y-course students claimed never to be afraid of making mistakes (approximately 20 % in other levels), and 28 % were often afraid to show their teacher that they do not understand mathematical problems (c 11 % at the other levels). The most anxious towards tests in mathematics appeared to be 9th grade students as approximately 3 students out of 10 in the 9th grade very often become nervous when they have tests (only 15 % at other levels). One of the possible reasons could be that the 9th graders are not as used to having tests as older students because they perform on tests more seldom than others.

## *MAD*

The sixth factor was called *MAD*. The same items that constitute this factor can be found in several studies where they have used, for example, notions like "an existence of a math mind" (Kloosterman & Clapp Cougan, 1994; Kloosterman, 2002) or "elitism" in relation to mathematics (Nardi & Steward, 2003).

Table 7. *MAD*

| | **Totally agree** | **Partially agree** | **Partially disagree** | **Totally disagree** |
|---|---|---|---|---|
| B1 The most important in mathematics is to know many rules | 20.3 | **51.1** | 21.0 | 7.6 |
| B2 It is important to be fast finding a right answer in mathematics | 8.7 | 30.8 | **35.5** | 25.0 |
| B3 There is just one right answer in mathematical tasks | 11.2 | 20.3 | **39.5** | 28.6 |
| B4 When I do mistakes in mathematics it shows that I do not have enough knowledge in mathematics | 6.9 | 26.4 | **40.2** | 26.4 |
| B6 Right answer is more important than the procedure I have used for finding the right answer | 8.7 | 17.0 | **37.0** | 36.6 |
| C7 It is innate to be good in mathematics | 12.3 | **33.7** | 25.7 | 27.5 |

*Note*. Numbers are in percentages.

Based on table 7 it was evident that knowing rules had an important place in learning mathematics as more than 70 % of respondents agreed that the most important in mathematics is to know many rules. Controlling the relationship between the statements "it is innate to be good" and "mathematics is difficult" (factor *Self-confident*) it appeared that 70 % of students who thought mathematical abilities to be innate agreed that mathematics is difficult (only 32 % of these students agreed that "mathematics is easy" A14). Looking closely at items B2, B3, B4, B6 and statements from the *Typical student beliefs about the nature of mathematics* presented by Schoenfeld (1992) (statements like "mathematics problems have one and only one right answer"; "there is only one correct way to solve any mathematics problem", etc.) it seemed that students in my study did not consider time as an important factor when solving problems, nor were they sure that there is only one answer to a mathematics task (c 70 % disagreed with the item B3). More than 66 % of students expressed that it does not show their lack of mathematical knowledge when they make mistakes and almost 3 out of 4 valued the procedure for finding an answer more than the answer itself.

There were 15 % more boys than girls who claimed that it is important to find an answer quickly but no other gender differences in this factor were identified. Some differences between the levels were found. For example, Y-course students agreed most strongly with "being fast" claim (57 % compared to 29 % ($9^{th}$ grade) and 35 % (X-course)). X-course students held most strongly the idea that there is just one right answer to a mathematics task (agreement with 37 % of certainty compared to 26 % ($9^{th}$ grade) and 32 % (Y-course)). Ten percent points more of the Y-course students (22 %) compared to $9^{th}$ grade (8 %) and X-course students (13 %) totally agreed that mathematical ability is innate. At the same time, more than every second X-course student (54 %) – $9^{th}$ grade (39 % ) and Y-course (49 %) – agreed altogether that mathematical ability is predetermined.

It was evident already in the factor *hard-working* that $9^{th}$ graders were the ones valuing learning rules the most. Total agreement with the item C6 was 38 % compared to 25 % (X-course) and 14 % (Y-course). The tendency was visible as well when looking at the answers to the item B1 "the most important in mathematics is to know many rules" as 27 % of the $9^{th}$ graders totally agreed with the item compared to 15 percent (X-course) and 13 (Y-course) percent.

### *Relationships between the factors*

For detecting the relationships between the factors Spearman's coefficients (Spearman, 1910) were calculated (figure 1). It appeared that there were two factors that were related to the existing four factors. These were *hard-working* and *self-confidence*. *Self-confidence* itself correlated positively strongly to the factors *interest* and *usefulness*, which correlated together as well. It means that students who claimed to be more self-confident acknowledged more the usefulness of mathematics and interest towards mathematics. The need to work hard had a positive relationship with *usefulness*, *insecurity*, and *MAD*; and a negative relationship with *self-confidence*. It can be interpreted that the respondents who acknowledged the need for hard work in mathematics publicised their higher insecurity and belief in more formalistic mathematics, and disbelief that they are

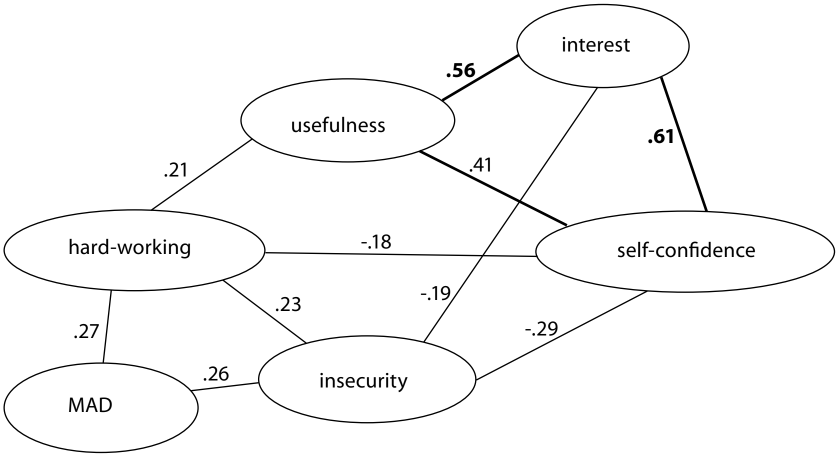


Figure 1. *The correlations between the factors. The connection weights are Spearman's coefficients (2-tailed; p < 0.01)*

good in mathematics. *Insecurity* correlated positively with *MAD* also, but negatively with *interest* and *self-confidence*. Therefore, the students with higher insecurity in lessons showed lower interest and self-confidence in mathematics.

## Conclusion

### *In what way are the affective variables structured?*

The idea to use a factor analysis for structuring a data came from Field (2005) who explains that factor analysis as a technique has three main uses from which one is to "understand the structure of a set of variables" (p. 619). Therefore, using factor analysis seemed to be appropriate for my research questions and the data collected. Based on the analysis the structure of students' views of mathematics was coherent with the structure that Brekke et al. (2004) found using almost the same questionnaire seven years ago with one additional factor, which might give a positive signal about the appropriateness of the instrument as the structure remained stable over different populations. The additional factor was called MAD, where the statements represent the view that mathematics is an absolute discipline (Brew et al., 2006) in relation to the idea that the items were consistent with a formalistic view of mathematics (Pehkonen, 1994).

### *What are the general tendencies in pupils' answers?*

Already two decades ago students claimed that mathematics is important, difficult and based on rules (Brown et al., 1988). Together with memorizing rules mathematics was noted to be useful by students (Dossey, Mullis, Lindquist & Chambers, 1988). Years later several studies in Finland have revealed similar outcomes: students viewed mathematics as important and highly useful subject, and at the same time classified mathematics as boring and demanding rather than interesting and exciting (Hannula & Malmivuori, 1996). The same trends have been noted in the KIM study in Norway also (Brekke et al., 2004; Brekke & Streitlien, 2005). Based on my study all these features are still remarkably distinguishable.

Six students out of ten thought mathematics is exciting and interesting while every second student found it boring in my study. In several cases students noted that mathematics could be both interesting and boring, which seems contradictory. One of the explanations might be in the nature and content areas of mathematics. For example, in his study Kloosterman (2002) found out that students' enjoyment of mathematics varied by topic area within mathematics (different interest towards algebra and geometry). Therefore, some topics might seem interesting to students and others might not. The fact that half of the respondents thought mathematics is boring is coherent with the results of other researchers (e.g. Nardi & Steward, 2002, 2003). The majority of students who participated in the study carried out by Nardi and Steward (2003) expressed that mathematics is an irrelevant and boring subject. One of the reasons for this might be the abstract nature and heavy symbolic representation of mathematics (e.g. Tikly & Wolf, 2000). Another might be engagement in mathematics lessons. In most lessons in the study by Nardi and Steward students were not deeply and meaningfully engaged in mathematics and this implied a situation where students saw mathematics lessons as ”boring”, ”tedious”, ”a null period”, and mathematics itself as ”grey”, ”ugly”, ”depressing”. They did what was expected but actually did very little real thinking (Nardi & Steward, 2002).

As mathematics has been a highly respected discipline in school for centuries, it implies wide-spread understanding in a society of mathematics as an important and valuable subject. Students in my study are no exception as they found mathematics highly important and useful in their lives. For many people mathematics becomes an important subject when they use it themselves or see how others use mathematical knowledge for finding solutions for everyday real-life problems. For example, students in the Kloosterman and Clapp Cougan (1994) study mentioned jobs, sports, and college as places were they expected to use mathematics or places where people they knew used mathematics. Together with the importance, students in my study noted the need for hard work in mathematics. Students acknowledged that learning mathematics requires much effort, which in some studies has been classified as one of the factors that a "good mathematics teaching" includes (e.g. Akinsola & Tella, 2007). It has been pointed out that not only the importance of mathematics is related to the claimed effort needed in mathematics but also to the actual effort that students apply when learning mathematics (Ma, 1997; Kloosterman & Clapp Cougan, 1994). This means that if a student acknowledges the importance of mathematics he/she probably puts more effort into learning mathematics than the one who does not value mathematics.

Most of the students claimed that they feel secure in mathematics lessons. However, students became nervous when they have tests in mathematics. This is coherent with the results of other studies where it has been pointed out that mathematics anxiety surfaces most dramatically when the person is or is perceived to be under evaluation (e.g. Tooke & Lindstrom, 1998). One of the reasons for this fear of assessment might be the grade as an outcome from tests. But it is not possible to clarify if students get nervous because of the anticipated low mark or if they get a low mark because they get nervous in tests. The latter was agreed by the students from Nardi and Steward (2003) study, who believed that "nervousness/anxiety in tests affects their performance" (p. 358).

It is reflected from several statements from different factors that students highly valued knowing many rules in mathematics. This result is coherent with the result of Kloosterman's study where he found out that "many students do feel mathematics is a set of rules to be mastered" (2002, p. 260), and with the study done by Nardi and Steward where several students seemed to "experience mathematics as a set of rules" (2003, p. 354). Several studies carried out in different times and different countries have found a strong evidence of students thinking that mathematical abilities are innate (Kloosterman & Clapp Cougan, 1994; Nardi & Steward, 2003; National Research Council, 1989; Schoenfeld, 1992). This can be illustrated with the saying from one student in the study of Kloosterman and Cougan (1994) "Some just weren't born to do math" (p. 383). The idea of the existence of "math mind" surfaces in my data as well, as 46 % of students agreed with the fact that it is innate to be good in mathematics.

### *Are there differences from the perspective of gender?*

Studies dealing with the beliefs and attitudes in mathematics are widely investigated from the perspective of gender (e.g. the anxiety in mathematics researched by Bernstein, Reilly & Cote-Bonanno, 1992; Campbell

& Evans, 1997; Woodard, 2004). The only significant difference in my study appeared in the factor *insecurity*. Boys in general claimed to be less afraid of making mistakes than girls. A similar trend appeared to the answers reflecting the nervousness in tests situations in mathematics. It might be a sign that girls might need some additional support in the lesson. Therefore, it is a suggestion for the teaching practice to be aware of the fact that there is a higher insecurity amongst girls than boys in some aspects of the learning situation.

### *Are there differences from the perspective of particular grades?*

In the literature the studies, which investigate what happens with students' beliefs and attitudes in mathematics when they progress through the grades, have concluded that students confidence drops (Dossey et al., 1988; Brekke et al., 2004), and negative interest reinforces (Brekke et al., 2004; Mullis, Martin, Gonzalez & Chrostowski, 2004). Students in higher grades claim mathematics to be less useful and more difficult than students in lower grades (Brekke et al., 2004). It might be that older students have a more realistic picture of mathematics than younger as the content of mathematics becomes more challenging and abstract in higher grades, which makes it harder to see the usefulness of mathematics for everyday life activities.

Three groups of students were under consideration in this study, and the grouping of the students was not only based on their age but as well based on the courses they have attended. These were the total cohort of the $9^{th}$ grade students, the students in their first year in upper secondary who had chosen the deep mathematics course that is meant for students who are going to relate mathematics to their future profession, and the students in their first year in upper secondary who had taken the mathematics course meant for students more interested in humanities. The study could detect certain characteristics of every group of students.

For the ninth graders mathematics seemed to be more exciting and they also believed in the usefulness of mathematics. At the same time knowing rules and solving many tasks were important for them. It could be explained with the aspect that the content of mathematics in lower grades comprises themes that demand more drilling than in older grades, e.g. the developed skill to operate with fractions needs lots of practice. Moreover, as the as the curriculum for upper secondary is more demanding then there simply is not so much time for drilling as in lower grades.

The students from the deep mathematics course agreed least that mathematics is boring and were less insecure in lessons. As mathematics is supposed to be one of their majors subjects and it is highly likely that they had performed rather well in mathematics (the lower performers does not usually choose mathematics as their first choice) then it is quite understandable.

The students from Y-course were least excited about mathematics and their self-confidence was remarkably lower. One of the reasons could be that their excitement was addressed towards other subjects that were more relevant for them in relation to further career. Secondly, the lower self-confidence could be the outcome of poorer performance in mathematics in earlier grades because they had been performing better in subjects related to humanities, which in turn influenced the course they had chosen.

### *What characterises the relationship between the factors?*

Interest and self-confidence in mathematics were positively related to the usefulness of mathematics, and these three factors constitute the core of the beliefs. A positive view towards mathematics encompassed the belief that mathematics is interesting and enjoyable, is easy and accessible, and is important and useful for life. Interestingly, usefulness was positively related to effort and self-confidence but at the same time presumable effort and self-confidence were related negatively. It meant that claiming mathematics to be important was in connection to the acknowledgement that learning mathematics requires hard work, and that one is able in mathematics. But looking for the direct relationship, it appeared that the effort students claimed to put in learning mathematics decreased when students' self-confidence increased. Therefore, being self-confident in mathematics (mathematics is easy; I am able in mathematics) is connected with the higher excitement in mathematics lessons, the higher acknowledgement of the importance and usefulness of mathematics, lower insecurity in learning mathematics, and less effort that is needed in learning mathematics. Despite the fact that confident students claim to put less effort in learning mathematics, it has been detected in several studies that confidence is the factor that is positively strongly related to mathematics achievement (Fennema & Sherman 1977, 1978; Kapetanas & Zachariades, 2007; Mullis et al., 2004; Nurmi et al., 2003). Therefore, there might not be a real need to emphasise the need for the hard work in mathematics for better performance, it is more important to foster students' higher self-confidence in mathematics that was positively related to interest in mathematics. But talking about practice in Norway a recent study showed that teachers emphasise hard work as important for learning mathematics (Hundeland, 2009). The relationship between the statements "mathematics is interesting" and "able in mathematics", which also has been found in several other studies (e.g. Kloosterman & Clapp Cougan, 1994; Ma, 1997; Nardi & Steward, 2003), might give a hint that students performance in mathematics might improve when the enjoyment towards the subject improves. This important and interesting finding could justify design studies where one aim is to improve students' interest in mathematics and investigate the consequences. One of the important aspects for endorsing the interest toward the subject is the achievement of understanding i.e. students have strong positive feelings towards mathematics when they achieve understanding (Kloosterman, 2002; Nardi & Steward, 2003). A crucial question to explore is if students' interest would increase if they perceive that they understand mathematics better? The slogan teaching for understanding has been around for some time but have research evidence been found for the implications of such an aim? Moreover, when students found mathematics less interesting they often also found it difficult, and this could result in students' deepening lower engagement in mathematics lessons (Nardi & Steward, 2002), and in worst case scenario dropping out of mathematics courses as pointed out by Ma (1997).

Reaching understanding and confidence in mathematics are not the only characteristics for the better achievement in mathematics. It has been mentioned that students' beliefs about mathematics as a subject play an important role in students' performance and success in mathematics classrooms (e.g. Schoenfeld, 1992; Stodolsky, 1985). Students' beliefs regarding mathematics might act as an obstacle for solving non-routine problems in mathematics (Schoenfeld, 1985; Silver, 1985), and students' ideas of what is important in mathematics might determine the place where they put their effort when learning mathematics (Kloosterman, 2002). For example, if students value knowing lots of rules, then memorizing rules might become their priority; or when valuing the answer over the procedure, they might put more emphasise on getting the answer (it is often possible to get an answer without actually understanding the solution) than understanding the solving procedure. Based on my study, it seemed that the factor in question (MAD) was rather separated from other factors. It was not directly connected with three so-called core factors (interest, self-confidence and usefulness), therefore it seemed to stand separately from their influence. The factor was positively related to hard-working and insecurity. Students agreeing to a formalistic view of mathematics acknowledged higher insecurity in mathematics and the need of hard working in mathematics. This result was not so surprising because, first, when looking at the wordings of the statements in factors hard-working and MAD, there seems to be a resonance between the statements. For example, ”I can become clever in mathematics if I learn all the rules" (factor hard-working), and "The most important in mathematics is to know many rules" (factor MAD). Secondly, the formalistic view of mathematics is related to the rote learning, and when looking at the items of the factor hard-working more closely it was evident that several of them are related to rote learning also. For further implication, it has been mentioned already before that emphasising the need for hard work might not be the aspect one should strongly support in mathematics, and this evidence is found here also, as it actually might not help to improve the view of mathematics being exciting, creative and fun.

The novel outcome of the study is a relationship found between Norwegian mathematics teachers' and students' understandings of what is important in learning mathematics as it has been reported that Norwegian teachers believe that learning by rule comes first, and then the understanding follows (Espeland, Goodchild & Grevholm, 2008; Hundeland, 2009), which is similar to the students' answers in this study. This could be a sign for teachers that their understanding about what is important in mathematics might carry over to students' understanding. Nevertheless, students in my study most often disagreed that a right answer is more important than the procedure used, so it might be possible that together with valuing rules (which is not considered a negative factor as long as it does not become the core of learning mathematics) students at the same time value the solving procedure as well.

Therefore, to support the idea of mathematics being an interesting subject and reachable to every student (mainly possible with the help of different ways of presenting the content and using imaginative teaching' approaches) could be the main points for further implication as the acknowledgement of the importance of mathematics, which comes together with the acknowledgement of hard-work in mathematics, were already visible in students' answers.

## Acknowledgement

The project LCM is supported by The Research Council of Norway within its KUL program *Kunnskap, utdanning og læring* (Knowledge, education and learning, Norges Forskningsråd, project no. 157949/S20).

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## *Notes*

1 KUL – Kunnskap, utdanning og læring, translated as Knowledge, education and learning.

2 KIM – Kvalitet i Matematikkundervisningen, translated as Quality in Mathematics Teaching.

3 For clarification, in Norway, every student has to complete a one year-course in mathematics in upper secondary school. Course 1M is the most basic course for those who do not plan to continue to higher education (university, for example). Students planning to continue to higher education must in addition complete 1X or 1Y. Course 1X is designed for students who are interested to study mathematics or subjects related to mathematics deeply further on, exploring elements of theoretical mathematics in more depth. Students more interested in the humanities usually follow the 1Y course (Norwegian directorate for education and training, 1997).

4 The language used in the questionnaire was Norwegian, and the English translation can be found at http://www.tlu.ee/~kirstik/PhDsurvey/

5 Factor loadings based on exploratory analysis are presented in Appendix C, and fit indices based on confirmatory analysis in Appendix D. All appendixes (A, B, C, and D) are available at NOMAD's webpage http://ncm.gu.se/media/nomad/kislenko_appendix.pdf

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# Sammanfattning

Elevers uppfattningar om och attityder till matematik är viktiga eftersom de är relaterade till resultaten av lärandet i matematik. Utgående från en studie som genomfördes i Norge 2005 verkar det som om elevers syn på matematik skulle kunna relateras till sex faktorer: *intresse*, *att arbeta hårt*, *självförtroende*, *användbarhet*, *ängslan* och *MAD* (matematik som en absolut disciplin). Trots att eleverna saknar intresse för matematik kan de erkänna dess användbarhet, betydelse och att man måste arbeta hårt med matematik. Nästan hälften av eleverna tror på medfödd förmåga att lära matematik.