Future activities in the Nordic graduate school in mathematics education network?

Another summer school

The sixth summer-school organised by the Nordic graduate school in mathematics education took place in Søminestationen, Holbæk, in Denmark in the third week of September. Thirty doctoral students participated and four professors acted as their supervisors in the working groups. The group leaders were Morten Blomhøj, Barbro Grevholm, Marta Menghini and Mogens Niss. The students came from Finland (4), Denmark (5), Norway (10) and Sweden (11). This year no students from Iceland or any of the Baltic countries took part, but among the students we still find several other national roots, such as Albanian, Bosnian, Dutch, Mexican, and Kenyan. The main part of the programme in the summer school consists of 13 hours of working group activities. Here students present and discuss their research questions and the background for them, the theory they use, the methods and methodology, the data collection and analysis, the results of their study and the implications of them. Each group had 7–8 members so there was plenty of time to have deep conversations about all these issues. Many of the students witnessed how important the suggestions from the group leader and the fellow students were to them.

The two workshops

Two workshops were offered. One was about how to read a scientific paper and how to write one. A paper had been sent out in advance for all to read and in the session the participants could discuss what they experienced when reading the paper, what they discerned during this reading and how they interpreted the paper. A number of different ways of putting emphasis during the reading emerged and some interesting comparisons took place. Few of the readers observed structure during

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Grevholm, B. (2009). Future activities in the Nordic graduate school in mathematics education network? *Nordic Studies in Mathematics Education*, 14 (3), 85–92.

the first reading but, when discussing how to write a paper, issues of structure became visible. The participants discussed the intention of the abstract, the introduction and background, the theory section, how to present methods and methodology. The selection of what data to present is crucial, and so is the issue of how to be transparent about the analysis of data. Presentation of results and the implications are of course highly important. After this work through a typical structure for a paper in a scientific journal all participants got the task to sketch their next paper and a number of interesting drafts emerged from this work.

The other workshop dealt with actual data analysis of authentic video material from a doctoral study. This workshop was led by Uffe Jankvist and built on his experiences from video analysis. Here participants could work hands on with a realistic situation. Another point of this workshop was to illustrate to the doctoral students how one can be inspired by and apply (elements of) a theory in mathematics education (in this case Sfard's theory of commognition) to analyze the actual data.

The four lectures

Each morning students could enjoy a lecture by one of the group leaders or an invited researcher. Mogens Niss was first talking about *What is quality in a PhD dissertation?*. He structured his discussion in two parts: The quality of the *underlying research* and the quality of the *dissertation as a report* of the research conducted, and of its results. For the first part he emphasised the quality of the reserch questions, being clear, deep, of scholarly interest, original and researchable. Further he mentioned the quality of the research design adopted, and the corresponding methods of investigation and discussed this in depth. Focus was then on the quality of the results obtained, mentioning for example their range, applicability and generalisability. Next Mogens turned to discuss some frequently encountered problems with quality of research and finally two aspects of the report of the work done: scientific/scholarly quality and communicative quality. The concluding discussion about what could or should lead to a rejection of a dissertation might have been scaring to listen to for some of the participants. The final remarks placed the responsibility on the student by observing that every PhD project is unique and puts high demands on the student's independence and originality and every PhD student must "think from scratch", even though supervised by the most experienced and respected of supervisors. Mogens has promised to write a paper for Nomad based on his presentation so doctoral students and supervisors can look forward to a more elaborated text on this important issue.

The second lecture was held by Morten Blomhøj about *Five major challenges in mathematics education research and how to relate to them in a PhD project*?. The five were the challenge of keeping the meta-reflections on mathematics education research alive, supporting the interplay between research and the development of teaching practices, accumulating theoretical knowledge and avoiding isolation among sub-paradigms, defining and strengthening the relations to the supporting sciences, and integrating mathematics education in general liberal education. Morten elaborated on what these challenges could mean and why he claims that this is important. He then discussed what to do in a PhD project concerning each of these five challenges and gave concrete suggestions of possible actions and why they could be important. The interested reader can find more about the challenges in the proceedings from the ICMI Jubilee conference in Rome in 2008 documented in the book: *The first century of the International Commission on Mathematical Instruction (1908–2008). Reflecting and shaping the world of mathematics education*.

The third lecturer was Marta Menghini presenting *From geometrical figures to definitional rigor: Teachers' analysis of teaching units mediated through van Hiele's theory*. She sees this as a curriculum design project and the work is based in the fact that teachers realize that pupils entering upper secondary school, even if they know names and shapes of many geometric figures, are not familiar with their properties and are not always able to point out specific differences expressed in the definitions. In the project they used a conceptual framework for teacher enhancement, a didactical framework called *From perception to definitions* and a mathematical framework: *Exclusive and inclusive definitions*. The design of the project and the methods were presented and illustrated by exemplars of pupils and teachers work. Among the conclusions Marta Menghini mentioned that making the first levels of van Hiele explicit allowed teachers to clarify their teaching aims. The development of the passage from exclusive to inclusive definitions, and the role of the latter in starting deduction, became clear to them. van Hiele's theory is a framework for different experiences, such as "a pupil's knowledge acquisition is local and temporary", "definitions may be inclusive for some figures and exclusive for others", and "a pupil may be at a certain level for some points and at a different level for others". In the discussion she commented on the Italian school of mathematics education research and its tradition to work together with teachers in mathematics classrooms, as illustrated in her presentation.

Mette Andresen from the NAVIMAT (national science centre for didactics of mathematics) in Copenhagen gave the fourth lecture about *Recent developments in school mathematics roles and relations*,

Mathematics in multi-disciplinary teaching projects, Danish science gymnasiums (DASG) *and NAVIMAT*. The centre is rather new and a presentation can be found at www.navimat.dk. The research perspective is "Potentials for mathematics education of multi-disciplinarity". The reasons for this work of multidisciplinarity is that it is prescribed in the Danish upper secondary schools' curriculum since 2005 in mathematics. The curriculum demands support of students' knowledge about "important aspects of the interplay between mathematics and culture, science and technology" and students are supposed to know how mathematics adds to understanding, formulating and treating problems in different subject areas". Further students must know about mathematical reasoning and the aim is to enable students to competently take a position on the applications of mathematics and to pass further education. Mette then described a number of collaboration projects with teachers in this spirit of multidisciplinarity and the outcome of them. Finally she talked about how the centre initiates professional development for teachers instead of giving courses to them. The main reasons for this part of the work are to locate the teachers' learning in the institutional setting, to account for the collective learning of the teacher group, and to relate teachers' activity in professional development sessions and in the classroom.

The networking

Another important feature of the summer schools is the networking that takes place. Doctoral students have time to get to know each other and discuss research projects, and as the local conditions were excellent this could happen both in the social activities, like the excursion, walks in the forest and meals and at late evening dances. Students have during the time slots in the programme for informal conversations the opportunities to meet individually with the group leaders and discuss their projects. This was used extensively and thus a network is built of doctoral students and more experienced researchers. Now that the funding from NordForsk to NoGSME has come to an end the hope is that this network will be strong enough to survive from its own power. One initiative came already during the summer school, a Swedish subgroup of students decided to organise a local conference in Karlstad to unite forces and learn with help from each other.

Seventh summer school in 2010

The application sent in spring 2009 to NordForsk for support to a summer school in May 2010 was granted and thus there will be another chance

for new doctoral students to experience such an event. It will take place in Dømmesmoen in May 25–30, 2010. The first announcement was sent out in October 2009. We already have a promise from Professor Jo Boaler in Sussex that she will be one of the group leaders and half promises from some other outstanding researchers to come. Dømmesmoen is a course- and conference centre, part of the University of Agder campus beautifully situated in Grimstad, on the south coast of Norway. Doctoral students are welcome to register for the summer course (7.5 ECTS).

The 11th international conference in the Baltic countries

Peteris Daugulis, who is chairing the Organizing committee of TM2010, has announced that the 11th international conference *Teaching mathematics: restrospective and perspectives* will take place on May 6–7 at Daugavpils University, Daugavpils, Latvia. More information about this yearly Baltic conference that circulates in the Baltic countries is posted on the web at http://www.de.dau.lv/tm2010

Four new doctoral dissertations in the Nordic countries

Lovisa Sumpter presented her thesis O*n aspects of mathematical reasoning. Affect and gender* at Umeå university on June 1. She explores two aspects of mathematical reasoning: affect and gender. Studying upper secondary students' reasoning, when solving tasks, she revealed that, when not guided by an interviewer, algorithmic reasoning was predominant. The reasoning was based on memorising algorithms, which may or may not be appropriate for the task. She then continued to investigate students' different strategy choices and conclusions. Beliefs about safety, expectation and motivation were important in the decisions made during task solving. Her third study investigated upper secondary teachers' conceptions about gender and mathematical reasoning. Findings indicate that the teachers attributed gender symbols including insecurity, use of standard methods and imitative reasoning to girls and symbols such as multiple strategies, guessing and chance-taking to boys. Results from the final study show that students share the teachers' rather traditional view on femininities and masculinities. A surprising result was that, when students were asked to reflect on their own behaviour, this result was not repeated. The result implies that girls and boys share many of the same core beliefs about mathematics. Sumpter concludes that still much work is needed to create learning environments that provide better opportunities for students to develop beliefs that guide them towards well-grounded mathematical reasoning.

Kajsa Bråting's dissertation took place at Uppsala university on June 5. The title of the thesis is *Studies in the conceptual development of mathematical analysis*. In the thesis the development of certain mathematical concepts are considered from a historical and didactical point of view. In particular, Bråting has studied the conceptual development in analysis during the mid-19th century, for instance, concepts such as functions, continuity, convergence and infinite series have been investigated. The use of basic concepts was studied in connection with two important theorems: Cauchy's sum theorem from 1821 and Cauchy's theorem on power series expansions of complex valued functions from 1840. In the thesis she also investigates the role of visualizations in mathematics from a historical and didactical perspective. The visualizations have been considered on the basis of historical examples as well as on her own empirical studies. The thesis consists of a "kappa" and three papers, two of which are published in scientific journals. The first paper deals with a new look at E. G. Björling and the Cauchy sum theorem and the second with visualisations in mathematics. The third manuscript is about E. G. Björling's view of power series expansions of complex valued functions.

Claire Berg's thesis *Developing algebraic thinking in a community of inquiry: collaboration between three teachers and a didactician* was defended at university of Agder on June 12. The study includes three teachers from lower secondary school and a didactician from a university in Norway (Claire). The thesis offers an account of the relationship between the participants' development of algebraic thinking and the processes related to the creation and development of a community of inquiry. In addition, the thesis presents elements of the relationship between the teachers' development of algebraic thinking and their thinking in relation to their teaching practice. The theoretical framework was elaborated according to the criteria of relevance and coherence. In order to conceptualise the participants' development of algebraic thinking within the community of inquiry, Berg started from Wenger's theory of community of practice and expanded it in order to include both the dimension of inquiry and Vygotsky's ideas of mediation and scientific concepts. Methodologically, she classifies the study as a case study, within a developmental research paradigm. The results of the study indicate that the participants' development of algebraic thinking is deeply interwoven with the processes related to the creation and development of the community of inquiry. It seems that the participants' confidence in the community was developing gradually while the confidence in the subject-matter was related to the nature of the mathematical tasks with which the participants engaged. In addition, the study shows how the teachers engaged in a process of both investigating critically their own teaching practice as a

consequence of their collaborative engagement within the community of inquiry, and of envisaging possible implications for their future teaching practice. Furthermore, insights are offered into the doctoral student's own development both as a didactician and as a researcher and how these relate to research outcomes. Berg claims that the thesis contributes to a better understanding of issues related to collaboration between in-service teachers and a didactician from a university, while focusing on the development of algebraic thinking. Implications are also suggested concerning the way algebra could be addressed in schools.

Uffe Thomas Jankvist defended his thesis *Using history as a 'goal' in mathematics education* on August 28 at Roskilde University. The work is an analytical and empirical study of using history of mathematics in mathematics education. The analytical part consists in proposing two categorizations based on a literature survey, one for the arguments of using history (history as a tool and history as a goal) and one for the approaches to doing so (the illumination, the modules, and the history-based approaches), and then analyzing the interrelations between these "whys" and "hows" of using history. A modules approach is chosen to fulfil the purpose of using history as a goal in the new Danish upper secondary mathematics programme. Two historical modules are designed and implemented in a particular upper secondary class. The purpose of the empirical study is to see whether students at upper secondary level are (1) capable at engaging in meta-issue discussions and reflections of mathematics and its history, (2) if these discussions and reflections in any way are anchored in the taught and learned subject matter (in-issues) of the modules, and (3) if such modules in any way may give rise to changes in students' beliefs about mathematics (as a discipline) or the development of new beliefs. Based on videos of the implementations, students' essays, mathematical exercises, questionnaires, and follow up interviews, the conditions on and ways in which the students are able to carry out and engage in meta-issue discussions and reflections are analyzed and discussed and so are the levels of anchoring of these in the related in-issues. In particular, four different levels regarding the students' discussions about meta-issues are identified: the non-anchored, anchored comments, anchored arguments, and anchored discussions. It is found that modules like the ones designed in the present study may cause some changes in students' views of mathematics on a content specific level as well as in the way the students hold their beliefs. In particular it is found that the students' beliefs seem to grow in consistency and that the students' desire to justify and exemplify their beliefs increases over the one year period of the study.

The four theses are very different. Sumpter's is the only one so far reported in the NoGSME network that relates to gender, Bråting's is in history of mathematics, Berg's is focused on algebraic thinking, and Jankvist's is about use of history of mathematics in the teaching of mathematics. Two of them include historical aspects but in different ways. Maybe one common trait could be seen in the quest for conceptual learning and avoiding mere superficial memorization and use of mechanical algorithms? The three theses in didactics of mathematics are clearly aiming for implications of how to improve mathematics teaching and learning.

Meanwhile Erkki Pehkonen has reported about some Finnish dissertations defended lately that have not been mentioned here. We will try to present them in the next issue of Nomad.

The final meeting of the NoGSME board

On December 14 the NoGSME board will hold its final meeting and hand over responsibility for many of the NoGSME activities to the NoRME board. A final report will be sent to NordForsk and the final part of the funding will be concluded and reported. We are convinced that NoRME will take good care of the NoGSME spirit and that both participating doctoral students and supervisors are eager to initiate future cooperation within the NoGSME network. It has been a great adventure to run this Nordic graduate school in mathematics education and to get to know so many supervisors and doctoral students and help them with different kinds of support in their scientific work. Now all must share the responsibility for the future activities.

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