Book Review

Matematik for lærerstuderende. Delta: fagdidaktik

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Jeppe Skott, Hans Christian Hansen og Kristine Jess (2008). *Matematik for lærerstuderende. Delta: fagdidaktik*. Samfundslitteratur, Frederiksberg. ISBN 978-87-593-1340-4

Delta is one of four books in a series written for the mathematics teacher educations in Denmark. Delta aims at introducing and discussing theories from mathematics education that are developed for understanding teaching and learning in classroom practices. While other parts of the series deal with theories of teaching and learning in specific areas of mathematics such as algebra or geometry, Delta focuses more on generic theories developed in mathematics education.

As stressed by the authors, these theories are presented in close connection to practice and throughout the book mathematics as well as mathematics education problems are integrated in the theoretical discussions. The authors also notice that the close connection between theory and practice in the respect of building theories and developing the mathematical classroom practice is something that characterizes mathematics education as a research field.

Overview and comments on Delta

Delta is structured into four parts and fourteen chapters. An overview of the four parts is presented below:

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1. Learning mathematics, chapters 2–4
2. Teaching in mathematics, chapters 5–9
3. Schools in didactics of mathematics, chapter 10–11
4. Meta-perspectives in mathematics and teaching of mathematics, chapter 12–14

Chapter 1 serves both as an extensive introduction to, and partly a summary of, the kernel of the book constituted by theories of learning and teaching in mathematics presented in chapters 2–9. In serving as an introduction it discusses mathematics in schools as a subject including both products and processes by referring to Niss and Jensen (2002) as well as drawing on the Standards (NCTM, 2000). I view such a discussion as absolutely essential for prospective teachers and am glad that it is put right at the top of the text. The rest of the chapter deals with short presentations of perspectives on learning and teaching mathematics, which could serve as introductions as well as summaries of forthcoming chapters.

Chapters 2–4 constitute the first part of Delta focused on theories on learning. This first part is structured using two metaphors of learning; the acquisition metaphor and the participation metaphor (Sfard, 1998). Chapter 2 introduces principles and key concepts within constructivism and radical constructivism such as assimilation, accommodation, reflective abstraction, and mental schemes by foremost using the writings of Piaget and von Glasersfeld. Constructivism and Radical constructivism here serve as important theoretical examples of the two most used theoretical perspectives from the acquisition metaphor.

The participation metaphor is represented by the work of Sfard (2006), Vygotsky (1986), and Lave and Wenger (1991). Here, and representative for the whole book, theories are seriously discussed by introducing principles, key concepts, how those concepts are related to each other, and possible misinterpretations of concepts. Let me take the discussion of zone of proximal development (ZPD) as an illustrative example. First, the discussion of ZPD is carefully conceptualized in a wider scope of the work of Vygotsky by first discussing language use in practices, everyday and scientific concepts, and the concepts of learning and development. As the authors make clear, it is absolutely essential to understand how Vygotsky relates the concepts of learning and development to each other in order to understand the scope of ZPD. In the book three different views of the relation between learning and development are introduced before the conceptualization of Vygotsky is made explicit.

From this platform Vygotsky is cited specifying how ZPD could be seen as a link between learning and development and how that is related to interactions between children and more knowledgeable individuals. The authors then continue to make distinctions between overall and content-specific developments and how that relates to the development, rather than learning, of the decimal number system. As an extra ingredient, distinctions introduced earlier and originating in the work of Lampert are connected to the ZPD stressing important aspects of mathematical teaching and learning. After summarizing aspects of ZPD the authors highlight two common misinterpretations of ZPD and how such misinterpretations may affect the mathematical classroom practice. From my point of view, the way of handling ZPD is representative for how other complicated constructs are discussed in the book and I am impressed by the clarity and analytical depth of the presentation. At the same time, it does not seem to be too complicated for prospective teachers even though they probably have to work systematically with the text in order to fully grasp it.

Chaper 4 continues by introducing yet another approach that to some extent tries to combine, or at least takes influence, from both metaphors. We are here introduced to the work of Cobb and colleagues and it is perhaps more fair to say that their approach has not only been influenced by the two metaphors but instead contributed substantially to their developments. I am not going into details about which aspects of the work of Cobb and colleagues that are elaborated on but in 34 pages their theoretical ideas and findings are related to mathematical problems, transcripts, and challenging tasks that all seem to be helpful for the development of usable knowledge for the prospective teachers.

The second part of the book (chapters 5–9) discusses aspects of teaching mathematics. Chapter 5 elaborates on aspects of being a teacher including what teaching could be, teaching with understanding, and teachers as reflexive practitioners by drawing on research from mathematics education, such as the work of Ball and Lampert, and from research carried out in other traditions such as Schön (1991) and von Glasersfeld (1995). Chapter 6 deals with how to introduce and work with mathematical tasks and problems within the mathematical classroom and contains a lot of examples for the prospective teachers to work on. Chapter 7 continues to discuss aspects of classroom practice and focuses on the classroom communication by presenting, among other things, ways to ask mathematical questions, evaluate responses, the IRE-model, reflective shifts in classroom discourse. The chapter presents a thorough discussion of theories combined with practical examples of how prospective teachers could help pupils become more active in the mathematical classroom.

In addition the chapter introduces and discusses theoretical ideas that not only help prospective teachers act in practice but also reflect upon practice.

In the beginning of chapter 8 the focus of the book is again widening to include broader discussions of aims of education and the aims of mathematical teaching. The very first discussion in the book of competences and areas in mathematics is developed and this serves as a ground for suggesting how mathematics teachers could plan lessons in relation to goals and content referring to, among others, Gomez (2002, 2006).

Chapter 9 discusses assessments in the mathematical classroom by introducing important distinctions (such as summative and formative assessment or validity and reliability in assessment), presenting historical aspects, and analyzing practical examples. As in the other chapters, the structure of the text is clear and important concepts and distinctions are properly introduced and related to relevant research. In fact, I felt that I learned a lot reading this chapter and hypothesize that many teacher educators, at least in Sweden, will feel the same.

Part three of the book comprises two chapters (chapter 10–11), each introducing a particularly influential research tradition within mathematics education. In chapter 10 the work of Freudenthal serves as a basis for elaborating on the tradition of *Realistic mathematics education* (RME) and in chapter 11 key constructs of the "French school" are introduced by using, first and foremost, the publications of Brousseau. The work of Cobb is also shortly summarized but since that work has been referred to extensively throughout Delta the authors are not going into deeper discussion here.

The fourth and final part of the book is headed *Meta-perspectives on mathematics and mathematics education* and deals with historical aspects of school mathematics in Denmark, arguments for and against including mathematics within the school practice, and a more philosophical elaborations on the ontology of mathematics. I view those aspects as more and more important for mathematics teachers to be knowledgeable in and could imagine that the chapter could be useful in discussing the relevance of studying mathematics with, for instance, pupils and their parents as well as with colleagues.

Summing up

In the introduction of Delta it can be read that there are two overall aims with the book. First, the prospective teacher should become acquainted with recent and important theories about learning and teaching mathematics. Second, they should be given opportunities to see how those

theories could be useful for understanding and reflecting about mathematics classroom practices. As I have indicated above I certainly think that the authors have succeeded. Delta deals with important aspects of mathematical teacher knowledge by using an impressive amount of research introduced and related to practical tasks. Delta is an ambitious book potentially very useful for mathematics teacher educators in helping prospective teachers developing proficiency in teaching mathematics. In addition, the theoretical depth of Delta qualifies it as a potentially very useful artifact in establishing the mathematics teacher education on a research basis. All in all, I am waiting and hoping that it could be translated into Swedish so I can start recommending it to mathematics teacher educators.

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