Practical activities in mathematics teaching

Mathematics teachers' knowledge based reasons

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The current article assumes that mathematics teachers' understanding of mathematics and professional beliefs are integrated into their professional knowledge. The focus of the article is on teachers' knowledge based explanations and reasons for choosing practical activities in the teaching of mathematics. Based on interviews of eight mathematics teachers in Norwegian elementary school (where the pupils are 6 to 16 years old), the article analyses and discusses relations between mathematics teachers' professional knowledge and choice of using practical activities. The findings give grounds for suggesting that both disciplinary and didactical knowledge have an impact on teachers' choice, and that inexperienced teachers do not have clear knowledge based explanations or reasons for using practical activities at the level of experienced teachers. However, the inexperienced, yet acknowledged teacher will, regardless of high or minimal disciplinary knowledge in mathematics, develop a more thorough and clear opinion on about practical activities as the pedagogical content knowledge deepens through experience.

Throughout history mathematics as a school subject has been taught in remarkably similar ways around the world (Volmink, 1994), with an almost canonical form when it comes to teaching activities and curricular content. Pupils' level of success depends on their ability to acquire and master the mathematical content which is presented by the teacher (Mellin-Olsen, 1991; Johnsen Høines, 2002). Over the last 30–40 years traditional approaches to teaching mathematics have been challenged, and questions have started to emerge about how teaching is planned and delivered. One of the noticeable changes is the anticipation of an

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increased application of practical activities[1] in school mathematics (e.g. NCTM, 1989, as cited in Meira, 1998; KUF, 1996; KD, 2006), and the fact that a considerable amount of mathematics teaching in school includes using practical activities. However, there are different opinions regarding this development (e.g. Bell, 1993; Gardiner, 2004), and quite a lot of mathematics teaching is still delivered in a way we choose to label as traditional (Alseth, Breiteig & Brekke, 2003).

Little is known about the reasons why some teachers choose to include practical activities whereas other teachers do not. We have chosen to look at what we call *acknowledged*[2] teachers of mathematics, and the research question is: *How do acknowledged teachers of mathematics explain and give reasons for choosing practical activities in their teaching, and to what extent is this related to their professional knowledge?* New insights on this topic will strengthen the focus on the relationship between disciplinary knowledge of mathematics and didactical freedom and provide new arguments based on research evidence into the discussion about practical activities in teaching of mathematics.

Theoretical background

Criticism regarding an increased emphasis on practical activities has been put forward, claiming that such priorities has lead to a decreased emphasis on basic mathematical skills and mathematical training (Grønmo, 2004a; Olsen & Grønmo, 2006). Moreover, the use of practical activities is not always sufficiently followed by a clear focus on learning and subject demands (Kjærnsli, 2004) or a structured and organized summing up of the lesson (Swan et al., 2000; Klette, 2003). Researchers report (Meira, 1998; Klette, 2003; Kjærnsli, 2004) that due to various reasons much time and effort spent on practical activities seem to be in vain. From a similar perspective Spillane (2000) claims that the potential learning outcome from a practical activity does not always seem to be within reach, mainly due to limited beliefs held by the teacher regarding the learners' abilities. A procedural perspective tends to replace a more holistic teacher perspective on learning and teaching. On the other hand, research also suggests that it is easier for pupils to learn mathematics when the content has a meaningful focus, as is the case with practical activities (Greeno, 1988, as cited in Meira, 1998; Bell, 1993; Bones, Stedøy & Wæge, 2006).

Knowledge as a base for teaching

Based on Shulman's (1987) *scholarship in content disciplines*, which Shulman identifies as one of the sources for the teacher's knowledge

base for teaching, three elements of content knowledge are emphasised regarding mathematics:

- disciplinary knowledge,
- didactical knowledge, and
- beliefs about mathematics and the teaching of mathematics.

Disciplinary knowledge

Teachers need to know the subject they teach (Gudmundsdottir & Shulman, 1987). Without such knowledge the lessons might be in danger of being experienced by the pupils as mechanically directed guiding tours (e.g. Thompson, 1992; Clarke, 1997). The teacher leads the pupils technically through the content of the lesson, without looking for possibilities to leave the textbook suggestions for content priorities. Personalised adjustment of the content to individuals or groups of pupils is also left to suggestions from the textbook as an established safety for an impregnable, but timid teaching performance. The textbook becomes the authority of the classroom (Lerman, 1993; Clarke, 1997). Similar patterns have been identified on several occasions with respect to different subjects (e.g. Gudmundsdottir & Shulman, 1987; Thompson, 1992).

However, research also reports that extensive dependence on the textbook in mathematics teaching is recognizable independently of the teacher's level of disciplinary knowledge in mathematics (Lerman, 1993; Streitlien, Wiik & Brekke, 2001). Some disciplinarily highly qualified teachers choose to organize their teaching in a traditional manner (Boaler, 1997).

Teachers who organize mathematics teaching from a more progressive point of view include, to a larger extent, more practical activities than teachers who represent a traditional approach. The level of disciplinary knowledge of mathematics varies among these teachers as well. Practical activity influenced teaching is offered by both disciplinarily highly qualified teachers (e.g. Holden, 2003; Wæge, 2007), and disciplinary low qualified teachers (e.g. Klette, 2003; Kjærnsli, 2004). It is therefore necessary to accept the fact that teachers of mathematics vary greatly in their level of disciplinary knowledge, and that the teacher's ability to see and make the mathematical potential in an activity visible, is influenced by his/her disciplinary knowledge in mathematics.

Didactical knowledge

Disciplinary and didactical knowledge are closely connected and constitute a major part of teachers' pedagogical content knowledge (Shulman,

1987). Moreover, reflection on practice within a specific context is a key element of teachers' professional knowledge and leads to a kind of didactical growth. The teacher's disciplinary knowledge is essential in this process, however, the teacher's formal and practical understanding of how pupils learn, and the impact of this understanding on how mathematics ought to be taught, are also of vital relevance to teaching (e.g. Koehler & Grouws, 1992; Andrews & Hatch, 2000; Hill, Rowan & Loewenberg Ball, 2005).

Beliefs about mathematics and the teaching of mathematics

Beijaard et al. (2000) point to teachers' beliefs as crucial for the development of practical knowledge and claim that beliefs and knowledge are tightly interwoven. Pajares (1992) ascribes beliefs almost a filtrating effect towards new impulses and identifies several features which characterise beliefs. Based on Pajares' (1992) identification of commonalities of understanding of beliefs, Beijaard et al. (2000, p. 262) consider beliefs to be:

- Highly individual, deeply personal and seem to persist.
- Formed by past experiences.
- Represent an individual's understanding of reality enough to guide thought and behaviour and to influence learning.

Several researchers look at how mathematics teachers' teaching is influenced by beliefs (e.g. Fennema & Franke, 1992; Hoyles, 1992; Thompson, 1992; Pehkonen, 2003; Sztajn, 2003). The teacher's beliefs about mathematics and teaching of mathematics also colour the decisions made by the teacher (Thompson, 1992; Lerman, 1993; Pehkonen, 2003). Both Thompson (1992) and Pehkonen (2003) refer to this in a thorough manner, describing how didactical decisions are guided by beliefs rather than knowledge. The teacher's beliefs about using practical activities might, for example, be coloured by experiences with mathematics as a pupil (e.g. Thompson, 1992; Lerman, 1993; Andrews & Hatch, 2000), the interaction with the pupils in the mathematics lessons (e.g. Hoyles, 1992) or how the teacher interprets educational reforms and the curriculum (Sztajn, 2003).

Handal and Lauvås (1987) introduce the term *practical theory of teachers*. This is the teacher's personal and dynamic construction of experience, knowledge and values of relevance to teaching, and Handal and Lauvås use it to identify factors which guide teachers' planning and the actual teaching in a desired direction. One of the factors they identify is labelled "transmitted knowledge, experience and structures" (ibid., p. 11),

or in other words, acquired disciplinary or didactical knowledge on which the teacher relies when teaching.

Knowledge based choice of practical activities

To recap some of what has been said so far, both Shulman (1987) and Handal and Lauvås (1987) point to a complex composition of factors which influence teachers' reasons for making specific organizational and content choices. The teacher's disciplinary and didactical knowledge which are integrated parts of the professional knowledge, are fundamental factors when it comes to teaching (Gudmundsdottir & Shulman, 1987; Shulman, 1987), and influence the teacher's choice of using practical activities in mathematics teaching. Furthermore, the teacher's beliefs about mathematics and the learning of mathematics influence the choice of using practical activities. The beliefs are affected by the teacher's opinion of mathematics as a school subject and how it ought to be taught and developed. Thus the teacher conveys a message to the pupils about what is important and not important in mathematics. Therefore, the teacher needs to be conscious and confident about the mathematical content and how to plan and how to teach, and at the same time be aware that subconscious beliefs make an impact on how the content is presented.

Methods

As previously mentioned the focus of this study is: *How do acknowledged teachers of mathematics explain and give reasons for choosing practical activities in their teaching, and to what extent is this related to their professional knowledge*? In order to get access to teachers' reasons for choosing practical activities, a qualitative study recognized as a hermeneutical study (Grønmo, 2004b) was carried out. Eight teachers of mathematics from different Norwegian elementary schools were interviewed. With the teacher's focus on practical activities in the teaching of mathematics as the contextual frame, an interpretation of the teachers' explanations, reasons and intentions for choosing practical activities was applicable. A holistic understanding of teachers' choice of practical activities for mathematics teaching is thereby developed (ibid.).

In Norway most teachers in elementary school have graduated from the Norwegian teacher education programs. Until 1975 mathematics was a mandatory part of a four year program for students who did not go through upper secondary school before they entered the program. Students who went through upper secondary school entered a two year program which did not include mathematics. In 1975 mathematics

became optional for all teacher education students as part of a reform which implied that the program became a three year program for all students, and that the students were required to either have finished upper secondary school (three year program), or exhibit equivalent competence basis (Skjelmo, 2007). The decision to make mathematics optional for all was based on opinions saying that students received sufficient mathematical training during their mathematical studies in upper secondary school to teach mathematics in elementary school (e.g. Breiteig & Venheim, 1993). Mathematics was made a mandatory part for all teacher education students in 1992 (15 ETCS[3]). Hence, teachers could still graduate in 1994 (three year program) and be certified to teach mathematics in elementary school without any ECTS' in mathematics. In 1998 the mandatory part was increased to 30 ETCS.

The eight teachers in this study were recruited according to the following criteria:

- As a group they represent teaching experience from lower and upper grades in Norwegian elementary school.
- The group includes teachers of both genders with various levels of formal education in mathematics[4], and with a varied level of practical teaching experience.
- All teachers are recognized as acknowledged teachers in the school where they work (see note 1).

Since the sample of teachers was relatively small, access to each teacher's opinions and impressions were considered essential to maintain the validity of the study. A semi-structured interview made it possible to compare given responses (Dysthe, 2002; Kvale, 2006).

The teachers were interviewed by one of the researchers, and each interview lasted for approximately 75 minutes. The interviews followed the same interview guide, and in addition to some demographic information about the teacher, the guide aimed at eliciting teachers' opinions on three main focuses:

- mathematics and school mathematics,
- teaching of mathematics, and
- future teaching of mathematics.

The teachers received the interview guide one day in advance (by e-mail). All eight interviews were recorded and transcribed. The first phase of

analysisconcentrated on extracting essences of meaning from the transcriptions (Kvale, 2006). The material was categorised according to the interviewed teachers' level of disciplinary knowledge and teaching experience:

1. Inexperienced teachers with low disciplinary knowledge.
2. Experienced teachers with low disciplinary knowledge.
3. Inexperienced teachers with high disciplinary knowledge.
4. Experienced teachers with high disciplinary knowledge.

A hermeneutical approach was applied in the analysis (Winter, 1989; Grønmo, 2004b), with an awareness of possible interpretational challenges (Tillema, Orland-Barak & Mena Marcos, 2008). In order to focus on the main research question for the article the analysis within each category focused on each teacher's explanations and beliefs about using practical activities.

Findings

The findings are presented with regard to disciplinary knowledge, didactical knowledge and beliefs as a knowledge basis for teaching. Sequences from interviews are presented in order to illustrate beliefs and reasons regarding practical activities in teaching mathematics within each category of teachers. In all sequences "R" represents the researcher and "T*n*"[5] the teacher who is interviewed.

Inexperienced teachers with low disciplinary knowledge

Teachers with a basic level of disciplinary and didactical knowledge, limited experience and a lack of confidence when it comes to teaching mathematics, were quite positive to using practical activities. They claim that the possibility to see mathematics as practical and useful is a strong argument. Statements such as "mathematics is in everything!" suggest enthusiasm when it comes to including practical activities in the teaching, based on elements of usefulness, fun and creativity.

The following sequence stems from an interview with a female teacher who has taught mathematics mainly in grades 1 to 4 (pupils aged 6 till 10 years old) for about two years, and occasionally at higher levels. In the interview she was clear about how she personally experiences mathematics as a difficult subject. However, she also revealed beliefs about mathematics as an applied subject which requires inspiring teaching and

meaningful content. Moreover, she prioritizes to focus on application and meaningfulness of mathematics through practical activities in her teaching:

R: How do you see your own teaching practice?

T1: I am ... ehh ... if you think methods? ... or how I teach it? ... then I think that on my behalf it is extremely important...ehhh...with the concrete approach ... the practical approach. To go from working very concrete, very visual ... ehhh ... use the senses one actually can activate, so to speak. I am not ... I am not that bookish, I feel that working ... the book is a working tool to me and the pupils, and I think that working ... they ought to work in their books as well, but to me it is equally important to put the book aside, because I feel that when the pupils get that book in front of them, mathematics instantly becomes a much more boring subject. Because then they are in a way supposed to sit and work in that book ... But if one can make mathematics teaching more ... more fun! What I mean is a bit meaningful, and that one works concrete and visually, and with things that you can grasp and feel, and thing like that ... that is important!

She emphasises the importance of activating pupils and that mathematics should be experienced as interesting and fun. Mathematics as a subject is not ascribed any self motivating qualities. The teacher is supposed to motivate for learning, and practical activities are included because they are perceived as illustrative for the usefulness of mathematics and play an important role in maintaining and developing pupils' motivation for the subject.

Experienced teachers with low disciplinary knowledge

Increased experience as a teacher of mathematics plays a role in how critical the teacher is when it comes to including practical activities. In the following sequence an experienced female teacher who has taught mathematics in primary school (pupils aged 6 till 12 years old) without formal education in mathematics for the last two decades, elaborates on how experience has made her more selective in choosing various activities for teaching mathematics. It is of relevance that this teacher went to school herself before practical activities became a prioritized part of school mathematics, and that she did not add mathematics as an optional part of her teacher education. This means that she has developed both interest and an acceptance for using practical activities through her own teaching.

R: To what extent do you use practical activities in your teaching?

T2: I can use it ... I try to put in ... I do have some faith in seeing and shopping and touching things, and not just calculate ... look at numbers, but rather that they use everything from dices and play games ... play hopscotch or ... or use multi-base material [base 10] and rhyme and nonsense verses. But at the same time they must be allowed to use books ... some pupils feel ... they are skilful in their books, so to speak. That is, they find it satisfying to calculate in books.

Even though the teacher did not have any formal courses in mathematics during her teacher education, it seems that her disciplinary level of mathematics in informal ways has developed from practice. Teaching relevant mathematical content for years, together with the impact of interpreting different national curriculums and a constantly developing didactical knowledge and personal beliefs about teaching of mathematics, has served as her "textbooks" in her own learning of mathematics and teaching of mathematics.

To this teacher practical activities have become a naturally integrated part of the mathematics lessons, but the use of practical activities must, in her opinion, be part of a clear aim for the lesson. There are more to mathematics than practical activities per se. Theoretical approaches to mathematics are viewed as meaningful and serve as future learning aims for the pupils. The opening for alternatives is therefore essential. The teacher is clear about the possibilities which are available for teaching mathematics and that these possibilities ought to be included selectively in the lessons. An activity is not a goal in itself (Meira, 1998), it has to serve specific learning goals in mathematics.

Inexperienced teachers with high disciplinary knowledge

The inexperienced, but formally highly educated mathematics teacher seems quite hesitant when it comes to including practical activities. Responses from two male teachers who have taught mathematics in lower secondary school (pupils aged 12 till 16 years old) for two (T3) and four (T4) years respectively, support this impression. They both have at least 60 ETCS in mathematics, which take their disciplinary knowledge of mathematics far beyond what is required in order to teach mathematics in elementary school in Norway. To them teaching according to the textbook seems to be a considerable didactical challenge in itself, although the level of the mathematical content is trivial compared to their disciplinary knowledge. Alternatives to the textbook suggestions or additional activities are not prioritized. The two teachers seem quite confident on how mathematics is to be taught, but at the same time they

are in a way bewildered on how they can elaborate their introduction of mathematical content to include practical activities:

R: What is mathematics to you as a subject?

T3: (quiet for some seconds) ... First of all it is something which can be done concrete and practical. When I went through my teacher education I really enjoyed the element of practical activities [in mathematics], like when the teacher pulled up a leek, and cut it up with diameter and circumference in mind, and ... that I really liked. So ... such small, simple, fine practical activities are fine, I think.

R: Mmm.

T3: But then, when you are supposed to teach on your own, then it is not so easy to come up with ... how are we supposed to go into this? Then it would have been nice ... to maybe get a tip from somewhere, for instance from a booklet, for inspiration ... for practical activities related to the textbook which is used for the class you are teaching.

T4: ... there is something with ... there is something saying ... do the pupils really get something out of it? That is, is it just stuff and nonsense, or do they get something out of it? And it is much easier in a class context ... like I mentioned earlier [in the interview] to go through new subjects, the pupils work with the new content, and then a summing up.

R: Mmm.

T4: Right? And it is very structured and straightforward. Nice and easy.

The theoretical presentation of mathematical content and discussion with the class, followed by the pupils' work with exercises related to the topic at hand, is the familiar and safe recipe. In order to fulfil the demands of the subject and the curriculum, no risks are taken. The possibilities to include practical activities are therefore limited, although the awareness of what practical relevance may offer is present:

T3: In mathematics in lower secondary school there ... well ... I believe that everything which we [teacher and class] have been through up till now should not be problematic to explain practically ... when I develop my imagination a bit further ...

Optimism is related to an expected development of teaching, and an increased emphasis on practical approaches is looked upon as an enriching part of the development. However, such teaching methods have not yet been frequently included in the teaching, and it is a challenge to increase the use of practical activities:

R: To what extent do you use practical activities in your teaching?

T3: It is when new subjects are introduced maybe. Then I try to do it a bit practical. But the fact that the practical approach becomes so extremely theoretical ... that is ... a challenge. One should, to a much larger extent, bring all kinds of helping devices to the classroom of course, and let the pupils try for themselves as well ...

R: The practical becomes theoretical?

T3: Yes.

R: Can you try to elaborate a bit on that?

T3: Yes. The practical part is more or less done by the teacher. You ... you ... find an example and go through it on the blackboard.

R: So you explain the example, for instance with drawings on the blackboard?

T3: Yes, it becomes kind of a demonstration, and then it ends there, instead of the fact that the pupils could find ... or I could prepare for their exploration with something tangible or concrete ... and they could suggest a formula or hypothesis, and so on ...

The fact that it is seen as a challenge to avoid the theorising of the practical approaches probably stems from the disciplinary knowledge of mathematics and beliefs about mathematics as a subject which carries both theoretical and applied relevance. It is in the disciplinary mathematics tradition to strive towards theoretical results which may be adjusted and applied to practical problems. The bewilderment can therefore be related to didactical knowledge, its starting point and how pedagogical content knowledge is continuously developing.

Experienced teachers with high disciplinary knowledge

Three experienced male teachers who have taught in lower secondary school (pupils aged 12 till 16 years old) (T5 and T7) and primary school (pupils aged 6 till 12 years old) (T6) for more than two decades express similar opinions regarding the impact of disciplinary knowledge on the teacher's choice of practical activities. They explain how the knowledge base for teaching makes an impact when a practical activity is included in the delivered teaching:

R: What is your opinion regarding practical activities in mathematics teaching?

T5: Some of them are really positive, but ... ehhh ... I also feel that you should be very ... you ought to examine them critically before you use them. (...) And I will also allege that some of what is used ... they might be very enjoyable, and the pupils are eager, and it might be quite funny, but I must say that I do not always feel that the mathematical content in the practical activities ... puts down the root with the pupil ...

T6: Several teachers want to teach mathematics, because ... well we have a textbook. You just have to look, exercise 1 and 2 and 3 and 4, and most teachers can manage that within the primary school curriculum. But they cannot manage to teach in an interesting and exiting manner or do a lot of practical things with mathematics because they do not have that understanding themselves.

Therefore, a vital issue is that the teacher is disciplinary confident and skilled enough to see and act spontaneously and expand on mathematical possibilities when they occur.

They also present similar opinions about the role practical activities ought to play in teaching mathematics. Based on disciplinary confidence and skills, and a continuously increased experience as teachers of mathematics, these teachers find it more and more interesting to include practical activities in their teaching, however, only when they find it appropriate. The two following sequences illustrate their restrained enthusiasm towards basing the teaching on practical activities:

T7: Totally independent of my own skills I mean that it is [the abstracting from a practical activity to theoretical content] really important, because it ... is important that the pupils do not experience mathematics only as a lot of enjoyable happenings, instead of a lot of boring exercises. That would leave us in separate trenches, in my opinion, because these enjoyable happenings are supposed to help the pupils understand, and motivate them to work with mathematics ... but the mathematical competence is the ability to apply the terms and the calculation techniques, or in other words the theoretical competence. It is supposed to show itself in the ability to use mathematics, both related to written traditional exercises and related to day-to-day situations. And the situations they meet in their lives do not seem to be more similar to plastic pieces or other activities than they are to the written exercises which pupils' used to work with. So I am really concerned that if the mathematics is supposed to be limited to the classroom, then it is indifferent to me if they are related to plastic pieces or other pupil centred activities. But if the mathematics does not move outside, so that the pupils can solve challenges on their own, then I believe that the result will be equally poor. If one is bored to death or amused to death, so to speak, becomes irrelevant.

R: To what extent do you use practical activities in your teaching?

T5: I always have a bit of bad conscience for using it too rarely, but at the same time I have a quite clear opinion about introducing big projects where the pupils are supposed to play their way to increased mathematical

> competence. I simply do not believe in such an approach. But on the other hand, if you can include mathematics when you as a teacher become aware that there is mathematics in an activity which the class is supposed to work with, that is something else. My present class was, for instance, supposed to build a turf hut for the pupils at the primary levels in our school. In relation to that specific practical activity it was easy and meaningful to introduce mathematical challenges for the pupils. How can we calculate the area of the turf hut? How should we saw the *bakhun*[6] parts which we were given to build the walls with? And so on. Of course you have to use such opportunities ...

The evolution of beliefs about practical activities

In figure 1 we present a suggestion to how beliefs about practical activities seem to be related to the teacher's disciplinary and didactical knowledge in mathematics teaching. It shows how a pattern of beliefs about practical activities, based on understandings of our findings, seem to evolve for acknowledged teachers with regard to a continuous development of disciplinary and didactical knowledge.

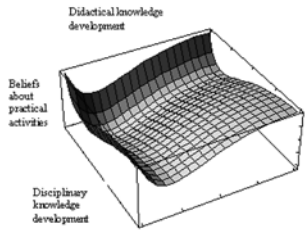

Figure 1. *Beliefs about practical activities in mathematics teaching for knowledge based reasons.*

The beliefs about practical activities in teaching of mathematics are positive, mostly for motivational reasons. Starting in the upper left corner, Figure 1 shows a rapidly decreasing faith about practical activities, based on disciplinary and/or didactical knowledge development. This may, for instance, be related to how the initial dependence on practical activities develops, or a reflection of affiliation to a disciplinary tradition paradigm with a lack of didactically based development of mathematical application. Through didactical and/or disciplinary knowledge development the figure shows that the beliefs about practical activities in teaching mathematics become more positive and a relevant part of the teacher's mathematics teaching. Moreover, the figure illustrates that this is not the case

with all teachers. A teacher who does not develop his/her pedagogical content knowledge on the basis of teaching experience and/or increased disciplinary knowledge, will not look to practical activities as a highly relevant or broadening element of teaching.

Discussion

When asked to explain their knowledge based choices of practical activities, the acknowledged mathematics teachers in this study tend to focus on the usefulness of mathematics. The teachers' with a low level of formal education in mathematics explain the use of practical activities by referring to didactical and psychological dimensions such as interest, motivation, variation and fun. The mathematical content plays a minor role when decisions about activities are made. They see many possibilities for practical activities, but the analysis of the context of the choice is somewhat superficial. This aligns with Klette's (2003) and Kjærnsli's work (2004), which reports on mathematics teachers who are both eager and willing to include practical activities, even if the arguments for using such activities are weak and the aim of the activity is vague and mottled to such an extent that not even the teacher has the overview. Initially, the faith in practical activities and openness towards what might be recognized as practical activities of relevance to learning mathematics seems to be at its peak among acknowledged teachers with the lowest level of disciplinary knowledge of mathematics. The poorer the knowledge is, the higher the thoughts about practical activities in teaching mathematics.

The development of teachers' personal theory of practice (Handal & Lauvås, 1987) and pedagogical content knowledge (Shulman, 1987) leads to a more critical and restrictive perspective towards including practical activities in teaching. This development, rooted in the cumulated experience as mathematics teacher and personal didactical development of teaching, may lead to change of core beliefs (Korthagen & Vasalos, 2005) about teaching mathematical content, and about how the content might be arranged in the best possible manner to enhance learning (Beijaard et al., 2000). Our findings suggest that some teachers will (re-) develop or maintain their faith in practical activities as they develop their pedagogical content knowledge, as illustrated through figure 1.

Teachers who have a deeper disciplinary knowledge of mathematics with short teaching experience have been found to be the teachers least able to find space and priority for practical activities in their teaching. A likely explanation is that these teachers carry with them a disciplinary knowledge based sense of responsibility for the subject based on their extensive education in mathematics (Gardiner, 2004). However,

they claim that they would like to use practical activities more often, but that they experience a lack of vigour when it comes to exploring the possibilities in teaching of mathematics (Pehkonen, 2007). They need to be given the opportunity to develop self-confidence about how to take advantage of disciplinary knowledge of mathematics in didactical ways. Furthermore, these teachers explain that when practical activities have been tried out, they have not proven to function well compared to traditional teaching. The result has then normally been that the activity perspective has been put aside, based on impressions saying that traditional teaching enhances learning in a better way. It is important to realise that new ways of teaching will not immediately function just as well as the way teachers are used to teach. Changes have to be introduced from time to time, and teachers need to master a variety of teaching approaches. It seems that even if the teacher holds positive beliefs about practical activities, the teacher needs to be persistent and self-aware with a strong will to integrate practical activities. If not, the experience with preliminary, and perhaps not too successful practical activities, may lead to a teaching approach of which practical activities rarely are part of the planned and delivered teaching.

In this article we have defined acknowledged teachers to be *teachers who are viewed as competent mathematics teachers by the principal and earn respect from colleagues, pupils and other groups of relevance within the working environment*. The acknowledged teachers of mathematics who have a high level of disciplinary knowledge of mathematics and a developed pedagogical content knowledge, are positive towards using practical activities when teaching mathematics. Their self-confidence in teaching mathematics is indisputable, and they are aware of the knowledge based conditions which have to be fulfilled if they are supposed to consider using a practical activity. The activity has to offer new perspectives or at least concretise the mathematical content in a way which enhances learning beyond the possibilities of traditional teaching. In that sense they do not look mainly to the usefulness of mathematics or didactical dimensions, but rather consider the mathematical content as the primary basis for using a practical activity. The activity is used only when appropriate.

Figure 1 suggests a symmetric development when it comes to beliefs about practical activities based on increased disciplinary and/or didactical knowledge. As long as the teacher continues to develop his/her pedagogical content knowledge, prolonged experience is likely to lead to an increased consideration of practical activities as a relevant impact factor in the teaching, independent of the level of formal disciplinary knowledge of mathematics (e.g. Gudmundsdottir & Shulman, 1987). Increased disciplinary knowledge of mathematics, and by that an increased ability

to see practical applications for theoretical mathematical content, seems to bring along similar changes. However, increased disciplinary knowledge does not necessary implicate a developed didactical knowledge and an improved learning environment for the pupils (Hill, Rowan & Loewenberg Ball, 2005). The teacher who develops both formal disciplinary knowledge and didactical knowledge will be able to see and use possibilities for appropriate practical activities and invest time and effort to include the activities in what Spillane (2000) refers to as a holistic perspective to teaching.

Conclusions

The aim of this article has been to present teachers' knowledge based explanations and reasons for choosing practical activities in teaching of mathematics. The method used in the study has been interviews with what we have defined as acknowledged teachers of mathematics, teachers who enjoy professional appreciation and respect within their personal teaching context.

The limitations of the current study ought to be mentioned. We cannot generalise the findings from the study because of two main reasons. First of all, the concept of "acknowledged teachers" used in this article is a new concept developed for the purpose of the article. Secondly, the number of teachers interviewed is rather small. However, having said this, we believe that the study presents some qualified hypothesis, and can serve future studies examining teachers' choice for using practical activities in the teaching of mathematics.

First of all, our findings show that experienced acknowledged teachers do not embrace a practical activity as something fulfilling in its own. A practical activity is considered relevant only when it seems appropriate in order to concretise the mathematical content at hand, and proves to be equally or more useful than other approaches. According to the acknowledged teacher of mathematics with a developed pedagogical content knowledge, mathematics is primarily about content, not form.

Secondly, our suggestion of a possible evolvement of beliefs about practical activities related to the teacher's disciplinary and didactical knowledge in mathematics teaching (figure 1) needs to be tested on a larger scale in order to make it reliable as a basis for conclusions about a possible pattern of continuous development.

Third, and finally, our findings suggest that explanations and reasons given by teachers seem to be strictly related to their beliefs and self-awareness about their own disciplinary and/or didactical knowledge in mathematics. A noticeable distinction between experienced and

inexperienced teachers is that the inexperienced teachers do not have clear knowledge based explanations or reasons for using practical activities, in the same way as the experienced teachers (e.g. Andrews & Hatch, 2000). The inexperienced, yet acknowledged teacher will, regardless of high or minimal disciplinary knowledge in mathematics, develop more profound professional explanations and reasons for choosing practical activities as the pedagogical content knowledge increases through experience (Goldsmith & Seago, 2007). The inexperienced teacher needs support and space to try out practical activities as a teaching approach (Clarke, 1997), independent of his/her level of disciplinary knowledge. It is a matter of changing established and developing new beliefs about mathematics and mathematics teaching, and gaining confidence in and self-awareness of personal teaching ability (Handal & Lauvås, 1987; Shulman, 1987). Based on such experiences the teacher will hopefully evaluate in a more balanced way the inclusion of practical activities on a knowledge basis, and explain or give knowledge based reasons for choosing such activities in his/her teaching.

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Notes

1 A practical activity includes all forms of engagement where the pupil uses physical concretes while carrying out the activity at hand.

2 We define *acknowledged* teachers of mathematics as follows: Teachers who are viewed as competent mathematics teachers by the principal and earn respect from colleagues, pupils and other groups of relevance within the working environment.

3 European Credit Transfer and Accumulation System

4 Four of the teachers graduated from the teacher education program later than 1994, and therefore have at least 30 ETCS in mathematics. Two of the teachers graduated between 1978 and 1990, and did not choose to study mathematics in their teacher education. One of them chose not to because of the high disciplinary level of mathematics in upper secondary school. Respectively, the last two teachers in the study graduated from the teacher education program before 1975 and followed a university program.

5 $n = 1, 2, 3, 4, \ldots$ representing the interviewed teachers.

6 The Norwegian word "bakhun" was used in the interview, and it represents the first-cut parts which are sawed of a log at the sawmill, and therefore are curved on one side.

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Sammendrag

I artikkelen antas det at matematikklæreres forståelse av matematikk og deres profesjonelle oppfatninger er integrert i deres profesjonelle kunnskap. Fokuset i artikkelen er på læreres kunnskapsbaserte forklaringer og grunner for å velge praktiske aktiviteter i matematikkundervisning. Basert på intervjuer med åtte matematikklærere i norsk grunnskole (hvor elevene er 6 til 16 år gamle), analyserer og diskuterer artikkelen sammenhenger mellom matematikklæreres profesjonelle kunnskap og valg av praktiske aktiviteter. Resultatene antyder at både faglig og didaktisk kunnskap påvirker lærerens valg, og at uerfarne lærere ikke har like tydelige kunnskapsbaserte forklaringer eller grunner for å bruke praktiske aktiviteter som erfarne lærere har. På den annen side vil den uerfarne men anerkjente lærer, uavhengig av faglig kunnskapsnivå i matematikk utvikle en grundigere og tydeligere mening om praktiske aktiviteter etter som den pedagogiske fagkompetansen utvikler seg gjennom økt erfaring.