Operationalizing the analytical construct of contextualization

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This article elaborates on the construct of contextualization, which constitutes a constructivist contextual view on learning. Principles of constructivism and contextualization are operationalized into a set of four analytical categories that teachers and researchers can use in organizing their thinking about teaching and learning mathematics. The categories are discussed and verified throughout the design and analysis of a classroom compatible learning activity, which is thought to promote probabilistic reasoning. The article discusses suggestions for developing the operationalization and, thus, encourages future efforts that further explore the explanatory power of contextualization and its analytical categories.

The goal of research in mathematics education is to understand the nature of mathematical thinking and teaching, and to use such an understanding to improve mathematics instruction (Schoenfeld, 2000). However, research in mathematics education is exposed to criticism due to its missing payoffs in practice (Burkhardt & Schoenfeld, 2003; Sfard, 2005), and voices can be heard urging for a bridging of the gap between research and practice (Groth & Bergner, 2007). Burkhardt and Schoenfeld (2003) discuss several causes for this gap and suggestions for reducing it. One thing they point to is the character and use of theory in mathematics education. The purpose of theory is to provide a structured set of lenses through which a learning activity can be observed and analyzed (Niss, 2007). However, Burkhardt and Schoenfeld (2003) argue that most theories that have been applied to education are quite broad, lacking the specificity that helps to guide and understand the design and

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Nilsson, P. (2009). Operationalizing the analytical construct of contextualization. *Nordic Studies in Mathematics Education*, 14 (1), 61–88

analysis of learning activities. Cobb et al. (2003) adhere to this view, claiming "General philosophical orientations to educational matters – such as constructivism – are important to educational practice, but they often fail to provide detailed guidance in organizing instruction" (p. 10). In his discussion on the dialogical approach, Ryve (2008) also problematizes the shortcomings of adopting overly global constructs in the analysis of mathematical learning. Specifically, he encourages a discussion on the need for analytical categories of context for properly analyzing learning activities in mathematics.

The analytical construct of *contextualization* accounts for a constructivist contextual view on learning (Wistedt & Brattström, 2005), which has helped to account for students' ways of dealing with learning tasks in a variety of settings (Halldén, 1999; Scheja, 2002; Ryve, 2006; Pettersson, 2008). In my own research I have used the construct of contextualization in studying students' reasoning about situations demanding an understanding of randomness and probability (Nilsson, 2007; Iversen & Nilsson 2007; Nilsson, submitted). However, analytical principles and categories that have been used in previous studies on students' processes of contextualizations have often been quite broad and are not specific to learning mathematics.

Against this background, this article aims to operationalize the theory of contextualization into a set of analytical categories for the design and analysis of learning activities in mathematics.

The article is organized as follows: The next section presents some basic principles of constructivism. The aim is not to give an exhaustive account of constructivism; however, it is necessary to introduce certain principles in order to understand the analytical construct of contextualization, which is the theme of the second part of the next section. The subsequent section deals with operationalizing the construct of contextualization into a set of analytical categories. This operationalization is then discussed and illustrated within the frame of the design and analysis of the *Game of totals* (Nilsson, 2007), which constitutes a learning activity compatible with the realities of a classroom, designed to encourage probabilistic reasoning. In Nilsson (2007), the object of the investigation was the students' probabilistic reasoning. Such elements will naturally be part of the current paper as well; however, based on the theoretical construct of contextualization, the issue here is deriving analytical lenses for organizing our thinking about teaching and learning mathematics. The Game of totals and the students' ways of interacting with the game are used as means of illustration and verification.

Theoretical considerations

A constructivist perspective

The principles of constructivism rest on Piaget's theories on cognitive development (Cravita & Halldén, 1994), which relate to Darwinian evolution and biological growth. An organism's biological growth can be said to occur if it contributes to increasing the organism's possibility to survive. In a similar manner, the occurrence of cognitive growth becomes a consequence of our struggle to understand the world in order to cope with and improve our life conditions. Hence, cognitive growth is viewed as the processes in which we strive to adapt our thought patterns in order to increase our capacity to act in and understand the world (von Glasersfeld, 1995).

From a constructivist perspective, there is no access to an objective reality independent of our knowledge of it. Our knowledge of the world is constructed actively by us as learners and is not passively received from our surroundings: "[...] knowledge does not result from a mere recording of observations without a structuring activity on the part of the subject" (Piaget, 1980, p. 23). Hence, the world we are striving to organize and understand is called the world of our experiences (von Glasersfeld, 1995).

An individual's striving for adaptation concerns his/her intention to continually establish and maintain *equilibrium* (Glasersfeld, 1995). Say, for instance, that you engage in a situation and find that elements of it do not make sense to you. A case of *disequilibrium* has appeared in your experience. Equilibration is the generic term for the elimination of perturbations: It is through equilibration that one tries to re-establish balance, equilibrium; "[...] expanding equilibration [...] means an increase in the range of perturbations the organism is able to eliminate" (von Glasersfeld, 1995, p. 67). Hence, that behavior is goal-directed by nature, in terms of one's intentions to understand the world and eliminate disturbances in experiences, should be considered in the design of learning activities.

From such a perspective, knowledge can be characterized as a tool, instrument or resource (von Glasersfeld, 1989). Such an instrumental metaphor of knowledge gives us reason to reflect on the *viability* of knowledge. The ability to establish and maintain equilibrium is a question of the viability of an actor's knowledge. So, whether knowledge is abandoned or preserved depends to a great extent on its viability in helping us understand a phenomenon or achieve goals in a certain situation (von Glasersfeld, 1995).

Piaget (1985) highlights assimilation and accommodation as two fundamental processes involved in knowledge formation. Assimilation is the process by which new experiences are incorporated into already existing mental structures: ”[...] we construct our knowledge of our world from our perceptions and experiences, which are themselves mediated through our previous knowledge” (Simon, 1995, p. 115). Accommodation, on the other hand, refers to instances when existing structures are not able to make sense of new information. For sense to be made of the information – that is, for equilibrium to be established – previous knowledge structures may be re-organized.

Previous research on learning mathematics has mainly focused on the process of *accommodation*, i.e. on students' misconceptions (Smith et al., 1993) and how they may be changed through teaching aimed at stimulating so-called 'cognitive conflicts' (Posner et al., 1982). With the analytical construct of contextualization we are striving to account for students' sense-making activities; how and why a certain way of reasoning takes form and what it contains in terms of mathematical potential. This does not ignore the idea of accommodation. However, it has proven crucial to emphasize the principle of assimilation, in terms of understanding the major impact students' current points of reference and goals of direction may have on how they understand and deal with new sensations and sources of inspiration (Halldén, 1999).

Contextualization – a constructivist contextual view on learning

There are two specific sources of the development of the analytical construct of *contextualization*. The first refers directly to the process of assimilation. Halldén (1999) argues that little interest has been directed to two of the conditions for learning proposed by Posner et al. (1982); that a new concept must be *intelligible* and appear initially *plausible* (for the learner). Halldén shows the importance of taking into consideration these two conditions when analyzing learning as well as how the construct of contextualization could be a useful tool in such an analysis. The second source of the development of this contextual view on learning refers to the criticism from sociocultural researchers that constructivist theories do not adequately take into account the situational character of cognition (see, e.g., Ryve, 2006). On account of this, research conducted within the frame of contextualization has focused not only on the conceptual requirements of a given task but also on beliefs about the immediate setting as well as the appreciation of the discursive rules and social requirements (Scheja, 2002).

Meaning-making in personal contexts

To allow for an adequate understanding of contextualization, it is necessary to explain what is meant by *context* in this model. In sociocultural research on learning, context refers to the physical and discursive setting in which a learning activity takes place (Janvier, 1989; Säljö, 2000). In constructivist research, context does not refer to the spatiotemporal setting of the learning activity but to a mental device, shaped by personal interpretations of the activity (Cobb, 1986).

To speak about students' processes of contextualization is to speak about how learners struggle to render a phenomenon or concept intelligible and plausible in personal contexts of interpretation (Caravita & Halldén, 1994). This idea rests on the principle that we always experience something in a certain way, from a certain set of premises and assumptions (Säljö et al., 2003). Talking about how students contextualize a phenomenon is a way of organizing and conceptualizing the learners' view of the phenomenon and what this view implies for their understanding of and subsequent interaction with the phenomenon.

Modeling students' meaning-making in terms of contextualization has proven particularly useful as it covers the idea that conceptual elements are not pure, isolated items that are added to already existing elements in a strictly linear manner. Instead, it adheres to a view on understanding as being in the form of a comprised system of linked, interrelated and coordinated knowledge elements and bits of information (diSessa & Sherin, 1998; Petersson & Scheja, 2008). Connected to this, the notion of context also emphasizes principles of guiding and framing, which makes us alert to how different knowledge elements make the activation of others either more or less likely (cf. Shelton, 2003). Content-related principles and ideas are brought about and assimilated on the basis of how they fit into the construction of a network of interpretations (Halldén, 1999), of which situational and social elements are a part (Janvier, 1989). Sense-making, in terms of rendering a phenomenon or a task intelligible and plausible, thus involves creating consistency and coherence in personal contexts of interpretations, that is, in the way the phenomenon is experienced by the learner (Caravita & Halldén, 1994). From this contextual view, conceptual understanding is not only regarded as context-dependent, in the sense that context is only there to support the development of conceptual structures; context is considered an integral part of the students' conceptual understanding. Learners develop personal contexts, of which conceptual principles are a part, helping them organize and overcome dissonances in their experiential world.

A view on learning in which conceptual understanding is considered from the perspective of contextualization makes us attentive to the fact that learning difficulties do not have to imply conceptual limits of a specific subject matter. The problem may be that the students have contextualized the study situation in a way that causes principles of and relationships between mathematical ideas to not appear relevant or meaningful to them (Caravita & Halldén, 1994). Learning in such a view concerns coming to understand why a contextualization seems relevant to the learning activity at hand. We may talk about learning and understanding as processes of *contextual awareness* (Wistedt 1993; Nilsson & Iversen, 2008), in which students reflect on the premises and (implicit) assumptions of their reasoning (Marton et al., 1992). Hence, learning is viewed as the extension of the learner's conceptual repertoire as well as a differentiated organization of contextualizations (Caravita & Halldén, 1994).

Operationalizing the construct of contextualization

The theoretical discussion above concerns the general principles of knowledge building. That conceptual understanding develops through a process of contextualization does not tell us anything about what kinds of contextualizations students actually develop. However, the model makes us aware of underlying structures in their thinking. It gives us reason to look at certain aspects of the design and observation of a study activity. The construct of contextualization supplies us with ideas about how certain thinking processes should be supported, in order to establish conditions for students to develop their understanding of mathematics. A basic idea of contextualization is that learning mathematics pre-supposes that the learner develops contextualizations, networks of interpretations, in which a mathematical treatment appears relevant and meaningful.

Based on constructivism and the construct of contextualization, a set of four interrelated categories seems crucial to consider in the analysis of learning activities. The categories do not claim to present a complete list of important aspects to consider in an analysis of processes of contextualization, but rather have a local focus in the sense that they attend to learning and conditions for learning in direct connection with an activity. The categories should be considered tools that teachers and researchers could use for organizing our thinking about teaching and learning mathematics within the frame of contextualization. Given these considerations, the following categories appear to be central in the design and analysis of mathematical learning activities.

1. *Students' problem encounters and the mathematical potential of the problems*. Constructivism implies that behavior is goal-directed: "Humans naturally seek to understand interactions" (Jonassen et al., 2000, p. 107)". That learners constantly interpret and try to understand experiences should be used in teaching and taken into account when analyzing learning.

   A problem that students encounter should be meaningful to them, i.e. it should be both intelligible and plausible (Posner et al., 1982). The problem should originate from the students' interest in understanding the situation they act within. Hence, to promote meaningful learning the students should be challenged to develop and solve problems they have formulated from their own needs and willingness to understanding their experiences. We can connect this line of reasoning to the *devolution process* of the theory of didactical situations (Brousseau, 1997). A devolution process aims at making the student responsible for his/her own learning. Having the student develop and engage in problems that are meaningful to him/her may serve as a good starting point for such a process.

   However, it is not enough that students develop problems for which they are responsible. Research on students' contextualization has shown that they often engage in problems different from what the teacher intended (Halldén, 1988, Wistedt, 1993). Hence, this first analytical category concerns the problem context of the students: the problem the students develop, the mathematical content involved and the potential of the problem context for deepening the mathematical treatment of the situation. Hence, the situation should be explorative and demand investigations rather than simply requiring the application of a ready-made method.

2. *Issues of familiarity*. The principle of assimilation stresses that new knowledge has to be compatible with pre-knowledge to some extent. A study activity should seem initially intelligible and plausible to the learner (Posner et al., 1982). If students are familiar with a phenomenon they may have an intuitive understanding of it, from which they can develop subsequent explorations and contextualizations. However, it is also important to challenge what students may take for granted. A situation may be familiar in a way that a mathematical treatment does not become present, even if this would be preferable from a normative view (Wistedt, 1994). Situations that challenge students' taken-for-granted conceptions may inspire them to contextualize the situations differently and perhaps more mathematically relevantly.

3. *Variation in contextualizations*. This analytical category refers to the following three subcategories:

    a. *Students' mathematical possibilities*. The current contextual view implies that learners' conceptual resources are dependent on the personal context within which they are operating. Different contextualizations support different solution strategies and bring into play different sets of knowledge elements. Hence, to offer students the possibility to apply different solution strategies, thereby also providing the researcher with more material for exploring the repertoire of the students' mathematics, one has to establish a situation that encourages the students to vary their ways of contextualizing a learning activity.

    b. *Connections between contextualizations*. Mapping and coordinating different ways of reasoning about a learning object is essential to conceptual development. By reflecting on and coordinating different ways of reasoning about a phenomenon, learners may avoid the constraints imposed by one idea and be able to utilize the opportunities for reasoning and perception from another (Carey & Spelke, 1994; Parnafes & diSessa, 2004).

    c. *Discussion and argumentation*. Discussion can encourage students to take a more reflective stance regarding their mathematical reasoning and can require them to consolidate their thinking by verbalizing their thoughts (Weber et al., 2008). If students interpret a learning situation in different ways they may be challenged to debate whether a particular strategy is appropriate. Provoking students to be explicit about the preconditions of their reasoning may facilitate processes of contextual awareness (Wistedt, 1993; Pettersson & Scheja, 2008).

4. *Reflection on viability*. In the present contextual view on learning, cognitive growth is viewed as a consequence of our struggle to understand our experiences of the world. We develop and accept ideas that seem functional to us, in our aspiration to establish consistency and coherency in our experiences. According to contextualization, a major aim of a teaching activity in mathematics is to have the students organize personal contexts in which mathematical elements and relations make sense, i.e. are perceived as being of help in organizing their experiences. Thus, the activity should support instances in which the students are challenged to explore

the implications and viability of their contextualizations. This could mean that the activity explicitly requires from the students that they perform predictions, which are based on their contextualizations. The situation should then offer feedback to the students, against which they can judge the efficiency of their thinking models and predictions (Thompson, 1985). Moreover, if the students find it necessary to reconsider a certain line of reasoning, the activity should challenge the students anew, to make predictions and try out the sustainability of their thinking. Hence, the activity should be iterative to some extent. In iterations, students are offered the possibility to reflect on the viability of a previous line of reasoning and to use this experience in subsequent interaction.

Illustration and verification of the analytical categories

The purpose of this section is to discuss the four categories within the frame of a certain learning activity, based on the Game of totals (Nilsson, 2007). The section begins with a description of the rules of the Game of totals and the setting of the data collection. The analytical categories are then connected to the design of the activity. The section ends with a presentation and analysis of a transcript from the activity of two students playing the game.

The Game of totals – rules of the game and setting of the activity

The Game of totals is based on the total of two dice and aims to offer students the opportunity to reason about probability when they are dealing with compound random events. The game consists of a playing board, two dice and a set of markers. In Nilsson (2007), the playing board was numbered from 1 to 12 (figure 1).

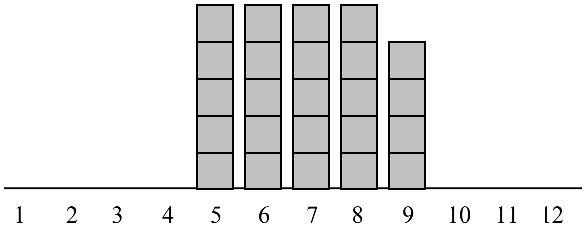

Figure 1. *The second setup of group A*

Two teams compete in the game, in which they are to distribute, based on the total of two dice, a set of markers among the 12 numbers on the playing board. If one or both teams have at least one marker in the area marked with the sum of the dice, they are allowed to remove exactly one marker from this area regardless of which team rolled the dice. The team that first succeeds in removing all markers from the board wins the game.

The activity was carried out in the students' ordinary classroom. Eight seventh graders (12 and 13 years old) were divided into four groups of two students each. The observation took about 70 minutes, of which ten minutes were spent on presenting the game rules.

The group discussions, which were tape-recorded and fully transcribed, took place in each corner of the room. When the groups had finished their discussions the teams entered the playing board, placed in the middle of the classroom, and started playing against one of the other teams. In order to capture the diversity of the students' reasoning, they were asked to refrain from commenting on the game while at the playing board. However, spontaneous reactions during play were videotaped.

The observer was present during the session, playing the role of an active observer. The observer introduced the game to the students and sometimes intervened during the small-group discussion, asking the students to clarify their reasoning.

Connecting the design of the activity to the four analytical categories

The activity does create opportunities for categories one, two and three in a rather natural way. In the small group, the students were encouraged to discuss and argue for winning strategies. It is a rather open activity in which the rules of the game constitute overall conditions for the students' explorations. The students are responsible for the activity. There are no pre-given questions or problems; the situation requires investigations and problematizations from the students. At the age of 12–13 years, students in the Swedish school system have never been formally educated on compound random phenomena. Hence, there is no ready-made strategy for them to adopt in this situation. As such, the situation is open to their probabilistic reasoning and variation thereof. In addition, the idea of using dice as random generators is based on the fact that it offers opportunities to generate interactive relations between everyday experiences and a certain desired mathematical aspect (Truran, 2001).

The students were not able to base any of their decisions on how their opponents bet, as each team's small-group discussion took place in a separate corner of the classroom. Such a setting would also facilitate variation, as the groups were not able to simply imitate other groups' approaches.

With specific reference to promoting reflections on viability, the fourth category, and to the intended mathematics of the game, we turn to examine more closely the design of the four dice setups that were arranged for the game (Nilsson, 2007).

Possible probabilistic structures

A system of four specifically designed dice was arranged for the game. The dice were designed to bring to the fore several aspects of probability and simultaneously give the students the opportunity to encounter small differences in mathematical structure between different situations. The overall guiding principle of the system was to challenge the students to base probability predictions on sample space composition. It was assumed that the idea of the game, depleting one's quota of markers, would provide information about students' ideas regarding the implicit probability distribution of the total of two dice. Specifically, it was assumed that the game would challenge the students to perform additional thinking when distributing their markers, i.e. encourage them to reproduce the probability distribution of the total of two dice with the number of markers.

Empirical evidence is understood to encourage students' probabilistic reasoning (Steinbring, 1991). However, we conjecture that if students' notions of, for instance, equiprobability (Lecoutre, 1992) are to be challenged, the part-part relation between the observed frequencies has to stand out noticeably. When rolling two ordinary numbered dice a number of times this might not be the case, as the probabilities between consecutive totals are too similar (e.g., the probability for 6 is $5/36$, whereas the probability for 7 is $6/36$, leading to a difference in probability of only $1/36$). Below, a design is presented in which the probabilities between consecutive totals differ by as much as $1/4$. To further encourage the students to reflect on a certain line of reasoning by matching it to empirical evidence, they were asked to start their next discussion by reflecting on how they would place the markers if they played the game again with the same pair of dice.

The dice were presented to the students in the following order:

Round 1 (yellow set) – The faces were marked with one and two dots, distributed as (111 222) and (111 222). The symmetry of the dice reduces the calculations of the 36 possible outcomes (ordered pairs) to the four equally probable outcomes (1,1), (1,2), (2,1) and (2,2).

Based on sample space composition, the probability (P) for the three possible totals 2, 3 and 4 appear as $P(2) = P(4) = 1/4$ and $P(3) = 1/2$.

Round 2 (red set) – Included two different dice, each with a distribution of two outcomes among the faces as (222 444) and (333 555). The aim of the design was to let the students encounter a compound event in which an outcome, in this case the sum 7, could be arrived at in two distinctly different ways (2 + 5 and 3 + 4). The dice give four equally likely outcomes: (2,3), (2,5), (3,4) and (4,5). The probabilities of the totals are $P(5) = P(9) = 1/4$ and $P(7) = 1/2$.

Round 3 (blue set) – The totals possible is the same as for the yellow set; however, in the blue set the four sides were marked (1111 22) and (1111 22). The aim was to challenge the students' notion of sample space and the way this influences the outcomes of a random phenomenon. The design gives $P(2) = 4/6 \cdot 4/6 = 16/36 = 4/9$, $P(3) = 2(4/6 \cdot 2/6) = 16/36 = 4/9$, $P(4) = 2/6 \cdot 2/6 = 4/36 = 1/9$.

Round 4 (white set) – These dice were a mix of the red and the blue sets. The dice displayed (2222 44) and (3333 55), giving the probabilities $P(5) = P(7) = 4/9$ and $P(9) = 1/9$.

In the first two rounds the students were asked to distribute 24 markers, and in the last two rounds they played with 36 markers. There are several reasons for the choice of number of markers. First, based on the results of earlier studies there was reason to assume that the students would adopt equiprobability thinking regarding the three possible totals (Lecoutre, 1992). The choice of numbers of markers would enable them to express such an approach exactly: 8 respectively 12 markers distributed on each of the three possible totals. Second, the choice of markers would also enable the students to imitate the underlying sample space structure of the setups. Third, it was also important that the number of markers be large enough so the game would produce distinct differences between the frequencies of the totals, with the possibility to challenge the viability of the students' reasoning and provoke them to reconsider their approaches.

In the third and fourth settings the underlying probability distribution corresponds. The idea behind the design of these dice was to further challenge the students' different ways of modeling the underlying sample space when making probability estimates. If, for instance, students' equiprobability responses are assimilated to the fair distribution of a single die (Pratt, 2000), how will they react to a warped design?

The case of Tom and Louise

The work of Tom (T) and Louise (L) (group A), throughout the four rounds of the activity, is shown below. Tom and Louise's work is in many respects representative of the activity as a whole, and focusing on only one pair of students allows us to illustrate the analytical potential of the categories in greater detail. After the presentation of the four rounds, we turn to analyze the students' activity by means of the four analytical categories.

Round 1 – The yellow set

This is a new game situation for the students, and during the first round the group has difficulty making sense of it. Particularly, they struggle to understand how they would be able to get the higher numbers on the playing board with the dice they are using. Group A asks the observer (O) to clear things up:

T: Look if I now roll ... a three, how can we come over here then [points to the high numbers on the board]?

O: You don't have to put on all. It's a little trick.

L: Okay, so we use these [dice] and the ones we're playing against use their own dice?

O: They have exactly the same kind of dice as you do.

T: But if we put them [the markers] over here [six and above] you will not be able to get there.

O: So you have to do smart setups and you have found out that it is not so wise to stake on some [referring to the numbers on the playing board].

Even with the observer's concluding support, the group is still puzzled about how to distribute the markers. After a period of silence, Tom suggests that they distribute their markers on some numbers. Louise continues:

L: But you can't distribute here [referring to the high numbers on the board], or can you?

T: Why?

L: I don't know.

The rest of the discussion is a bit unclear. We do notice, however, that the observer reminds the group that they are playing based on the totals of the two dice. Even so, the group decides to choose two numbers each with higher totals and to distribute six markers on each of the four numbers chosen (figure 2).

Round 2 – The red set

During their first match, the group realizes almost immediately that they will lose; their competitors had markers only on possible outcomes. Based on this experience, in the second round group A develops what I describe as an extreme value approach (Nilsson, 2007):

L: [Laughs] Okay, it can be nine at the most!
T: Yes, and so at minimum it can be ...
L: ... three plus two.
T: ... two, ... five!
L: Yes.
T: Then we take ... then we take from two to five then!
L: Two to five? It can be nine at the most!
T: Yes of course, five to nine I mean.

Based on the extreme value approach, the possible outcomes constitute all totals between the lowest and highest possible totals. Without discussion, the students start to distribute their 24 markers uniformly among the five identified outcomes. They do not explicitly motivate why nine should only have four markers (figure 3).

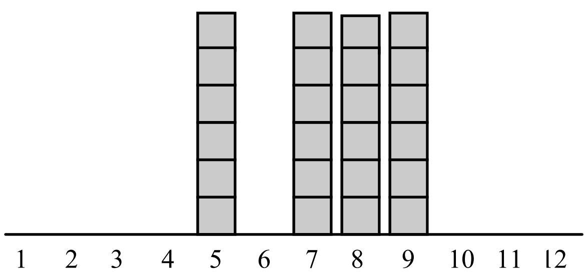

Figure 2. *The first setup of group A*

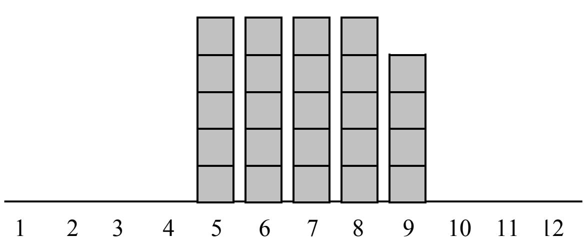

Figure 3. *The second setup of group A*

Round three – The blue set

After only a few throws, the group recognizes their failure in round 2. Louise introduces the third round:

L: [Laughs] How much can it be? Now we have to think. Think now.

The group members now recognize the importance of correctly identifying possible and impossible events for the total. Tom inspects each single die and discovers that there are only one and two dots on the sides.

T: It can go from one to four, cause 2 + 2 is 4, 1 + 2 is 3 and 1 + 1 is 2. From two to four.

L: Yes ... okay, then we should stake on?

T: Just check that there is no empty square.

Their attention is completely focused on controlling the sample space. The observer sees that the group's strategy again tends to be equiprobabilistic and therefore asks the students if they can see any differences between these dice and the yellow ones.

T: Wait, there is a little difference among the twos.

L: There are few more twos on that one [referring to the yellow die].

O: Does that matter?

T: Yes, we should probably put more on two and not so many on four. Look, there are a lot of ones and not so many twos.

Although there appears to be no explicit systematization in their strategy, this change in perception stimulates the group away from distributing the markers uniformly among the possible outcomes. When the students notice that there are more sides displaying one dot than sides displaying two dots, they distribute the markers for the blue setting with 23 markers on two, 8 markers on three and 5 markers on four.

Round 4 – The white set

The group begins this round by restricting possible outcomes by means of the extremes. They determine that the highest outcome is nine and the lowest is five. They then turn to the distribution on the single die and its implications for the distribution of markers.

L: Okay, then look here. One three, two threes ... there are four threes and two fives [Holding up one of the dice].

T: Okay, here are four twos ... wait ...

L: And two fours.
T: We have to have a lot of fives.
L: Yes ... no, not a lot of fives.
T: Yes, but look, I have two here, I have four twos here ...
L: Yes, that's right.
T: ... and you have four threes.

Louise suggests that they should place a high number of markers on five. She then returns to the sample space for the resulting totals. In the discussion, Louise holds the die with four sides marked one and two sides marked three and Tom the die with four sides marked two and two sides marked four.

L: It can't be six, for example. It can't be six!
T: I'll draw a line over that then. Are you sure of that? Do I not have a four?
L: You don't have a three and not a one?
T: I have a two ... it can be ...
L: It can be seven, so it can.
T: Wait, you have a three and I have a ... no ...
L: It can't be eight!
T: Yes.
L: No, it can't be eight.
T: But it can't be seven either!
L: Yes, three plus four.
T: You had that, aha.

When adding up the three possible outcomes, Tom concludes that the totals are the same as for the red set of dice. He proceeds:

T: And, the most [markers] here then, on five. How many on five?
L: It is 36 divided by ...
T: Maybe, twen ..., sevente ..., eighteen maybe. Eighteen on five!

Louise fills in eighteen markers on five on the playing board. The next outcome to be discussed is seven:

T: If we put ten on seven, then we are up to 28.
L: Hmm, but how big ... ?
T: I have four. Wait, how many threes do you have?
L: A lot, four!

T: I have two fours on this one.

L: It will be … the most it can be … one, two three …

Louise does not develop her last line of reasoning any further. However, the discussion results in the group thinking they should put more markers on five, resulting in a setup of 22 markers on five, eight markers on seven and six markers on nine.

Category 1 – Students' problem encounters and the mathematical potential of the problems

This episode reflects the natural curiosity and willingness to understand their experiences that children develop when captured by an activity. In the students' attempts to develop winning strategies, two overall problem contexts emerge throughout the four rounds.

During the first two setups, most of the students' activities were understandable if we ascribed to the students the efforts of trying to discern impossible totals from possible ones (cf. Fischbein, Nello & Marino, 1991). To accomplish this, several students adopted a strategy that considered the extremes of possible totals. In the identification of the lowest and highest possible totals, all possible totals were viewed as being constituted by these extremes and the numbers between them.

In the two first rounds, all four groups come up with setups reflecting equiprobability thinking (Lecoutre, 1992). To understand this, we have to take into account that the students' focus is not on the chance of the totals, but on possible totals. The students bring to the fore, and thus base their judgments on, single outcomes of resulting totals. In the problem at hand, the students have not made different representations of resulting totals available. Of course, I had reason to believe that they would have difficulties taking into account the order of the dice, i.e. the difference between the outcomes (1,2) and (2,1) in the first setting (Fischbein et al. 1991). But, still focusing only on possible totals, none of the four groups reflect on the distinct different ways to arrive at seven in the second round, i.e. 5 + 2 and 3 + 4. Consequently, having the distribution of markers allocated to a context in which each outcome is represented only once, it is possible to understand why the students consider each total to be equally likely to appear.

For Tom and Louise, and one other group, the problem of identifying possible totals is still the main subject of investigation at the beginning of round 3. They again come to speak in terms of equiprobability. The observer makes the choice to intervene in the situation as he notices that these two groups are not reflecting explicitly on the design of the third pair of dice on their own. From this point on, all groups are implicitly or explicitly aware of how the single dice in the third and the fourth rounds differ from the two previous settings. What role does this kind of information play in the students' reasoning?

In the two last rounds, the students notice that they can construct the totals in a different number of ways. Thus, they start to consider the differences in rate at which the piles of markers on possible outcomes should vanish. The students perform some kind of matching procedure between the distribution of individual dice and the number of representations of the totals, which can be spoken of in terms of a *number model* (Nilsson, 2007). This strategy reflects a kind of additive thinking; the totals are evaluated against the amounts of numbers of the single dice, taken together. Taking a concrete example: Students in round 3 perceive that they should be able to get more of the sum two than of the sum four since there are considerably more ones than twos on the single dice. This model could be viewed as a generalization of the *fairness resource* (Pratt, 2000). By this resource, the chances of the totals are considered to be equal since the outcomes of the single dice are equally probable. However, when the features of the dice are made available to the students, number model thinking shows that the students, rather spontaneously, find it necessary to consider and reflect on the number of favorable outcomes when reasoning about the chances of compound events.

Category 2 – Issues of familiarity

In the beginning, we note that the students are struggling to understand the game. They are particularly confused about the relationship between the dice they are currently using and the numbers on the playing board. What it actually is that is troublesome for the students is not easy to put one's finger on. It may be an issue of a low as well as high degree of familiarity. The game as such is not familiar to the students; they have not played it before, and are not completely clear about its rules.

However, we may also find arguments that it is a relatively high degree of familiarity that causes the students to act as they do in the first two rounds. That learning should depart from a student's experience and pre-knowledge is crucial in the present view on learning. Hence, a high degree of familiarity with and pre-understanding of an activity are desired, to develop and establish consistency in a context of interpretations. However, the problem may be that an activity's features entice students to consider factors irrelevant to the intended mathematics (Caravita & Halldén, 1994, Wistedt, 1994). In the current case, a high degree of familiarity refers to how the students perceive the game activity and allocate it to a context, which centers on their everyday experiences with dice games. In the first two rounds, there is no discussion about chance. The focus is instead on resulting totals. In everyday games, however, one is rarely expected to make fine-grained analyses of the probability of different outcomes. Players often want to know what numbers of the dice should be thrown (or should not be thrown) to reach (or avoid) a certain area on a playing board. Hence, the focus in many everyday game situations is on the possible totals, articulated, for instance, in terms of ”I'd better not roll a ...” (recall, for instance, the game of Monopoly). Keeping to the idea that the activity centers on students' familiarity with everyday dice games, we are also in a good position to understand why the group adopts the extreme value approach. With ordinary dice, gaps among the totals never appear. Hence, the students may not have had experience with dice situations in which there are gaps in the set of the totals and, consequently, they interpret the set of possible totals of the present game to be without gaps as well.

Category 3a – Variations and students' mathematical possibilities

Tom and Louise's activity explicitly shows the crucial importance of taking students' contextualizations of a phenomenon seriously, in order to do justice to and make sense of their understanding. For instance, as the students place markers on impossible events, we could claim that Swedish students, 12–13 years old, are not able to calculate $1+2$, $3+4$ and so on. However, taking into account the students' way of contextualizing the game we find that this is not the case. When Tom and Louise vary their way of contextualizing the game in round 2, we find evidence that the students not only are able to perform these simple calculations, but also show that they understand the difference between possible and impossible events.

However, regarding possible and impossible events, we again notice how a contextualization seems to hold the students back. By means of the extreme value approach, they place markers on impossible events in round 2 as well. The extreme value approach establishes no motives for them to take a closer look at the dice. However, when the group becomes aware of the strategy's shortcomings and makes the features of each single die available for consideration, the question of what should count as possible outcomes is no longer an issue.

We may adopt a similar argument regarding the students' ability to evaluate differences in chance of the totals. When they become aware of the distributions of the single dice, they show the ability to evaluate difference in chance in terms of favorable outcomes.

Category 3b – Variations and connections between contextualizations

A leading principle in the design of this activity was to facilitate possibilities for students to develop and connect between theoretical/combinatorial and frequency reasoning about probability. During the activity, however, we notice that frequency information is given low priority. For instance, none of the teams carries out samples within the small group, to be used for elaborating on what regulates the totals in the long run. Time should not be considered crucial in the absence of samples. There was naturally not enough time for 1000 throws; however, there was enough time to conduct samples up to at least 50 throws.

The absence of samples may be explained by how the students perceive norms and discursive roles of the activity; do they perceive the situation as permitting them to try the dice before the game or would that be cheating?

Frequency information is mainly in play when competing against another team and, thereby, in relation to the evaluation of a thinking route. I will return to this in the next section.

With reference to the fairness resource (Pratt, 2000) and the number model (Nilsson, 2007), studies on people's reasoning about the total of two dice indicate that the physical shape of ordinary dice more or less orients students to take into account only the numbers displayed on the individual dice. Hence, the case may be that the regularities (familiarity) of the dice in one way or another constrain the students' reasoning. Hence, a natural question would be: How would the students react if they played the game with asymmetrically shaped dice?

Category 3c – Variations and discussion and argumentation

The students discuss matters during the game. The question, however, is to what extent their discussions promote learning with understanding in terms of enhancing their contextual awareness.

Issues of conflicting views do not truly appear in the students' discussions. We find no instances in which Louise advocates a view different from Tom's, or the other way around. The members of the group seem to agree on the overall principles of how the game should be approached. When the students seem to be at odds, this mostly concerns the calibration of number of markers on the outcomes.

However, even if the students do not display great differences in contextualizations, we do find instances requiring clarification of and argumentation regarding a thinking route.

The first occasion occurs in the beginning of the activity when the observer is asked to approach group A's table. Note, however, that it is Tom who asks the observer for clarification. In an attempt to keep from giving too much information, the observer never puts the students in a position where they are asked to explain how they think. This might support the students in determining on their own what should count as possible outcomes in the first two rounds. Actually, Louise does ask Tom at the end of round 1 why he wants to place markers on high numbers. However, Tom avoids answering her question, probably because he does not know the answer, by posing a counter-question. Louise's question is not based on a strong personal idea. This is probably why she drops it instead of demanding that Tom clarify his approach. A complementing interpretation of what Louise does with Tom's reaction is that she perceives him as not having a good explanation.

In rounds 3 and 4, several occasions of implicitly or explicitly formulated questions appear. However, it is not that the students are disagreeing or adopting different views about underlying principles of the game; it rather concerns making sure that they are working with information appropriate to their thinking models. Taking a concrete example: In the third round, Toms asks "Are you sure of that?", when Louise claims that they are not able to arrive at the sum of six. At this moment the students know what should count as a possible outcome in the game. It is not what the question is about, but rather a kind of control. They are not making any premature conclusions. Nevertheless, we do get a glimpse of why a learning situation should promote questioning and argumentation: Tom's question leads Louise to externalize and make her thoughts explicit, and rather immediately he is convinced.

Category 4 – Instances of reflection on viability

The structure of the game activity requires the students to explicitly formulate predictions about winning the game, through the distribution of markers. In playing against another team, the students are offered opportunities to reflect on such predictions by means of empirical evidence. In the play, the students are not only informed about how well their own model works, but are also exposed to how well the setup of another team works. Such experiences and feedback may play an important role for students in changing and developing an understanding of a phenomenon (Thompson, 1985).

The empirical evidence, which appears during the first two matches, immediately challenges the viability of Tom and Louise's first two setups. After only a few throws, they understand that they will lose. Based on these experiences, they reconsider their approaches for the next game. However, what the game interaction did challenge was Tom and Louise's understanding of possible and impossible outcomes with reference to the resulting totals, but the game did not challenge the viability of the students' equiprobabilistic approaches, applied in the two first rounds. From the outcomes of these rounds, there appears to be no reason for the students to question their equiprobabilistic approaches. If they had played the game with uniformly numbered single dice again, they would probably distribute their markers uniformly again. As said above, the reason the students turn away from equiprobability thinking is the way they interpret the effects of the askew distribution on the single dice.

To summarize, the number model indicates that the students are able to understand the role of the number of favorable representations in the probability of events. However, when the single dice are distributed uniformly, the totals of the dice are understood to be of a uniform kind as well. The current study verifies that such a belief is rather strong. Hence, our analysis of viability motivates why we should pay serious attention to increasing the power of empirical evidence in the game, in order to challenge equiprobability thinking and to encourage students to reflect on the structure of the underlying sample space. Accomplishing more distinct frequencies should therefore be of decisive importance in re-designing the game.

Concluding discussion

This article adds to a larger project with the purpose of theoretically deriving and empirically testing analytical tools, for organizing our thinking about teaching and learning mathematics. This enterprise is fueled by voices arguing for the need to bridge the gap between research and practice (Groth & Bergner, 2007). The current project particularly strives to specify and operationalize global theoretical principles into more useful and concrete lenses for investigating mathematical learning activities (Burkhardt & Schoenfeld, 2003; Niss, 2007).

Applying the analytical construct of contextualization in learning mathematics means that we reject learning as a linear process. New mathematical principles are not isolated elements that are simply added to already existing elements in a strictly hierarchical order. Conceptual ideas are activated and prioritized on the basis of how well they fit into a context of interpretations (Halldén, 1999). The purpose of the present article was to operationalize this constructivist contextual view on learning into a set of analytical lenses for organizing thinking about teaching and learning mathematics.

Four analytical categories were outlined and tested within the frame of the design and analysis of a classroom compatible learning activity.

How the categories help in organizing and motivating the design of the Game of *totals* (Nilsson, 2007), with its four specifically designed pairs of dice, is described. The categories are then applied to the analysis of two students interacting with the game.

In line with the network metaphor of contextualization, we note how Tom and Louise's way of associating the game to everyday dice game activities appears crucial to the problems they encounter and the strategies they develop. Such an observation points to the reflexive dependence of the categories. However, there appears to be no reason to define a hierarchical order between the categories in advance. For instance, later in the activity we notice how other aspects take the role of leading the students' reasoning.

The outcomes of the current article provide reasons for future efforts to elaborate and adjust the categories. A particular issue in this work would be to reflect on the character and meaning of category two, issues of familiarity. At present, the focus of this category is rather general. As associations and recognition may appear at different levels and in relation to different knowledge elements, the explanatory power of the category would probably increase if it were divided into a set of sub-categories. A good starting point would probably be to consider the division between *conceptual*, *situational* and *cultural frames of references*, used in previous analyses of students' processes of contextualization (e.g., Halldén, 1999; Scheja, 2002; Wistedt & Brattström, 2005; Ryve, 2006; Pettersson, 2007; Nilsson, 2007; Iversen & Nilsson, 2007). The need to refine the lenses through which we investigate mathematical teaching and learning is also in line with and supports Ryve's (2008) suggestion regarding the need to develop and extend the range of contextual resources for the proper analysis of mathematical activity.

At game level, the analysis also supports reasons to reflect on issues of familiarity and, particularly, whether the game should be more 'unfamiliar' to the students. Making the game more unfamiliar, everyday-oriented interpretations may be constrained, stimulating reflections on the specifics of the actual situation. The outcomes of the analysis also imply the need to further reflect on how the activity could better encourage variations in reasoning and generate challenging frequencies against which predictions could be tested and thinking models objected to for reconsideration and variation.

Given these considerations, the present article encourages future research to develop and refine the current set of operative categories for organizing our thinking on teaching and learning mathematics. There is a need for elaborated examples, in which the explanatory power of contextualization and its analytical categories are investigated and developed through its confrontation with a teaching episode.

Acknowledgement

The author would like to thank Andreas Ryve for providing valuable feedback on previous versions of this article.

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Sammanfattning

Denna artikel elaborerar kontextualiseringsmodellen, som är en konstruktivistisk kontextuell modell för lärande. Principer avseende konstruktivism och kontextualisering operationaliseras i fyra analytiska kategorier, som lärare och forskare kan använda för att organisera undervisning och lärande i matematik. Kategorierna är diskuterade och verifierade genom utformningen och analysen av en klassrumsliknande aktivitet, som syftar till att stimulera resonemang om sannolikhet.

Förslag på hur operationaliseringen kan utvecklas diskuteras och artikeln inbjuder, i anslutning till sådana förslag, till framtida insatser, där kontextualisering och de analytiska kategoriernas förklaringsvärde ytterligare utforskas.