Gender and strategy use in proportional situations: an Icelandic study

Olof Bjorg Steinthorsdottir and Bharath Sriraman

This study was conducted to investigate the influence of contextual and number structures on individuals' use of strategies in solving missing value proportion problems, and to examine gender differences in strategy use. Fifty-three eighth graders in one school in Reykjavik, Iceland, participated in this study. Twenty-seven females and twenty-six males were individually interviewed as they solved sixteen missing value proportion problems. The problems represented four contextual structures. No gender differences were identified in the overall success rate. However, girls were more successful than boys in handling associated sets and symbolic problems, and boys were more successful than girls in part-part-whole problems. Moreover, the data suggest that the contextual structures influence females' choice of strategy more than that of males.

Mastery of proportional reasoning is a key requisite for success in learning higher mathematics (Lesh, Post & Behr, 1988). Attempts to define the domain of proportional reasoning have led to the delineation of various rational number constructs. Within these constructs, researchers have identified variables that contribute to an individual's ease or difficulty in solving problems. Problem context and number structure are among these variables. In fact, they exist side-by-side in proportional problems, with both exerting an influence on an individual's use of a solution strategy. We have surmised that a problem's contextual structures and number value, along with the individual's understanding of a given mathematical situation influence their choice of strategy. The first author designed a study that examined these variables in a population of

Olof B. Steinthorsdottir, *University of North Carolina at Chapel Hill*
Bharath Sriraman, *The University of Montana*

Steinthorsdottir, O. B. & Sriraman, B. (2007). Gender and strategy use in proportional situations: an Icelandic study. *Nordic Studies in Mathematics Education*, 12 (3), 25–55.

fifty-three eighth graders. The project attempted to determine which contextual and number structures are critical to an individual's use of particular strategies to solve missing value proportion problems. In addition, this study examined the existence of gender differences in strategy use and understanding. An individual's understanding is demonstrated by the strategies he or she uses (Carpenter et al., 1999). These strategies can be categorized in order of mathematical sophistication.

Scholarly research related to gender and mathematics is not currently published as frequently as it was in the 1980's or the 1990's (Steinthorsdottir & Sriraman, 2007a). This decline in frequency raises the question of whether there is not the need to investigate gender differences at this time and age. At ICME 10 (*International congress of mathematics education*) held in Copenhagen in 2004, *Topics study group 26* dealt with gender and mathematics education and 15 papers were presented. Two studies from Scandinavia showed interesting results indicating that gender differences remain. In particular, a study of 9th and 11th grade students in Sweden showed that they still viewed mathematics as a male domain (Brandell, Nystrom & Sundqvist, 2004). Another study from Finland reported that teachers held different beliefs about girls and boys in their classroom, believing that girls tended towards routine procedures whereas boys used their power of reasoning (Soro, 2004). These findings, along with numerous other studies from the U.S (Wiest, 2004), South Africa (Mahlomaholo & Sematle, 2004), Australia (Forgasz, 2004) and Iran (Pourkazemi, 2004), suggest that not much has changed in terms of society's dominant conceptions of mathematics and gender. In addition to the aforementioned studies, PISA documented statistically significant gender differences in achievement favoring boys for both 2000 and 2003. In 2000, statistically significant gender differences in achievement were found in 29 of 41 participating countries. In 2003, statistically significant gender differences in achievement were found in 27 of 41 participating countires. One country, Iceland, had statistically significant gender differences in achievement *favoring girls*.

In PISA, many items in the content strands of 'change and relationship' and uncertainty implicitly assume facility with proportional reasoning. Sriraman and Lesh (2006) claim that proportional reasoning and estimation are two of the most useful types of elementary mathematical thinking relevant for modeling situations. Finally, one of the seminal influences in the field of mathematics education, Zoltan Dienes, has repeatedly emphasized the value of proportional reasoning in cultivating mathematical thinking (see Dienes, 2000, 2004; Sriraman & Lesh, 2007). Therefore, it is of great interest to both the Nordic countries and mathematics education communities to carefully examine students' understanding of proportion and the nature of gender differences among Icelandic students in proportional situations (Steinthorsdottir & Sriraman, 2007b).

Influence on students' strategies

As mentioned above, an extensive literature has examined the influences of both number and contextual structures on students' strategy use. In outlining his theory of the multiplicative conceptual field, Vergnaud (1994) discussed the need to critically analyze and classify problem situations to better understand the cognitive tasks associated with what it means to do mathematics (Vergnaud, 1994). Based on our assumption that the contextual structures influence one's ability to solve missing value proportion problems, they can be used as one way to analyze and classify problems. Our belief that contextual structures are defined by the problem's inherent meanings of quantity is also influenced by the classification of addition and subtraction problems associated with the research based on the *Cognitively guided instruction* model (Fennema et al., 1996).

The concept of ratio and proportion as applied by young people has been widely studied. Piaget and his collaborators identified proportionality within their stage of formal operational reasoning (Inhelder & Piaget, 1958). In particular, children were found to demonstrate an intuitive understanding of proportionality before they could deal with problems quantitatively. Some of Piaget's results have been criticized, however, for their use of complex physical tasks to assess proportional reasoning, thereby underestimating the influence of problem contextual structures (Sriraman & English, 2004). In fact, research has shown that the student's degree of familiarity with a problem type affects problem difficulty. For example, Tourniaire (1986) reported that mixture problems are more troublesome to students than other contextual structures of proportional problems. Furthermore, the location of the missing element in relation to the other three numbers in a proportion has an influence on children's thinking (Tourniaire & Pulos, 1985).

In an effort to examine the influence of problem type on solution strategies, Lamon (1993a) developed a semantic framework for classifying proportion problems. Lamon grouped problem situations into four categories: *well-chunked measures*, *part-part-whole*, *associated sets* or *stretchers-shrinkers*. In individual clinical interviews with students, Lamon found that various semantic problem types elicited different levels of sophistication in solution strategies. It is not obvious from her study, however, if and how she controlled the number structure used in her problems or what influence number structure had on her results.

In addition to looking at the contextual structure of the problem, researchers have asked if the number structure plays an important role in strategies used by students to solve proportional problems. Students deal more easily with numbers between one and thirty, than numbers less than one and greater than thirty (Hart, 1981). Also, working with whole numbers is easier for students than working with fractional numbers (Bell, Fischbein & Greer, 1984). Unit ratios, especially 1:2, facilitate solutions more so than do fractional numbers (Noelting, 1980a, 1980b). Tourniaire and Pulos (1985) outlined three difficulty factors associated with number structure: presence or absence of integer ratios, placement of the unknown number, and numerical complexity – that is, the size of the numbers used and the size of the ratios.

Karplus, Pulos and Stage (1983) defined the relationships of those quantities used in proportion problems by focusing on whether an integral relationship exists within the ratio or between ratios. 'Within' refers to the multiplicative relationship in the ratio , while 'between' refers to the multiplicative relationship between the ratios in the problem For example, the problem $\frac{2}{4} = \frac{12}{x}$ has integer multiples both within the ($2 \cdot 2 = 4$) ratio and between ($2 \cdot 6 = 12$) ratios. A noninteger ratio, on the other hand, is when the multiplicative relationship within a ratio or between ratios is not an integer.

Abramowitz (1975) identified four kinds of number structures. The first, termed differences (equal/unequal), refers to the presence or absence of a repeated difference between the measurements used. For example, equal differences would be $\frac{4}{6} = \frac{6}{x}$, where the difference between 6 and 4 ($6 - 4 = 2$) is the same within the ratio and also between ratios. An example of unequal differences would be $\frac{4}{6} = \frac{10}{x}$, where the difference between 6 and 4 ($6 - 4 = 2$) is not the same as the difference between 10 and 4 ($10 - 4 = 6$). The second number structure, size (larger/smaller), refers to whether the unknown number is larger or smaller than the known number. The third is order, that is the order in which the quantities are presented in the problem.

The fourth category, type (simple/complex/multiple), indicates whether there are integer or non-integer relationships in the ratios and whether the answer is an integer or non-integer. These views from the literature indicate that among the crucial numerical factors in problem difficulty is the presence or absence of an integer relationship and of an unknown integer.

Gender and mathematics

Research on gender and mathematics conducted in the last thirty years provides evidence for the existence of gender differences (Lubienski & Bowen, 2000; Steinthorsdottir & Sriraman, 2007a). Some research also implies the need to look at gender differences from a different viewpoint to provide us with a deeper understanding of the situation (Fennema, 1995). Studies conducted on children's strategy use indicate that the choice of strategies when solving complex mathematics problems does differ between girls and boys. Studies have shown that girls tend to use more conventional strategies that relate to commonly taught algorithms whereas boys use more non-conventional strategies such as invented algorithms (Carr & Jessup, 1997; Gallagher & De Lisi 1994; Marshall & Smith, 1987; Fennema et al., 1996).

Gender differences in proportional reasoning have also been identified (Tourniaire & Pulos, 1985). In these cases, boys outperformed girls. Karplus et al. (1983) found no gender differences except those on noninteger problems, which favored males. Also, they reported that content type related to gender. Linn and Pulos (1983) focused on the source of the gender differences. Their conclusion was that intelligence, spatial visualization, cognitive style, or formal reasoning could not explain the gender differences. Consequently, motivation or attitudinal factors are more likely to be the source of gender differences.

Three main sources are available to examine the gender differences in performance in Iceland. These are the *Icelandic standardized test* (National institute for educational research) given to all students at the end of tenth grade (Olafsson, Halldorsson & Bjornsson, 2006), the *Third international study*, TIMSS (Beaton et al., 1996), and PISA 2000 and 2003 (OECD, 2004). The Icelandic standardized test had shown no gender difference in overall achievement until recently, when the gender differences began to favor girls (Olafsson et al., 2006). TIMSS, meanwhile, reported no gender differences in overall achievement. When looking at the content areas, very small differences exist, with girls doing better than boys on algebraic problems and boys doing better than girls on problems centered on proportion.

In PISA 2003 (OECD, 2004), statistically significant gender differences in achievement favoring boys were found in 27 of 41 participating countries. The only country in PISA 2003 which had statistically significant gender differences in achievement *favoring girls* was Iceland. Studies such as the one we are reporting on in this paper and document gender differences in learning trajectories for particular mathematical concepts offer one way of explaining such gender differences.

Proportional reasoning strategies

The test problems in this study were designed to help understand how different contextual and number structures may influence students' choice of strategies. The research on proportional problem solving suggests that there are three categories of strategies used in reasoning proportional relationships: qualitative, build-up, and multiplicative. These strategies represent different levels of sophistication in thinking about proportions. Research with preadolescent students indicates that their representation of situations involving ratios and proportions occur on an informal, qualitative basis long before they are capable of treating the topic quantitatively (Tourniaire, 1986). The qualitative reasoning strategy, however, is based on informal or intuitive knowledge of relationships without numerical quantification (Kieren, 1993). This informal knowledge includes a visual understanding of ratios and proportions. This is seen in young children as they express comparisons among quantities using words such as 'bigger' and 'smaller' or 'more' and 'less' to relate to the quantities in question. This qualitative reasoning is characteristic of young children but does not disappear when more formal strategies develop. In fact, qualitative reasoning continues to be used in proportional problem solving even by people who have the ability to reason proportionally (Behr, Harel, Post & Lesh, 1992, 1994; Kieren, 1993; Resnick & Singer, 1993).

Build-up strategies (based on repeated addition) require quantification of the ratio relationships and are more sophisticated than qualitative reasoning (Tournaire, 1986). They involve applying one's knowledge of addition or subtraction to the proportion in question. To use the strategy, a child notes a pattern within a ratio and then iterates it to additively build up to the unknown quantity. A build-up strategy is often observed during childhood and adolescence, when it appears to be the dominant strategy (Tourniaire & Pulos, 1985). The build-up strategy can be used successfully to solve problems with integer ratios but can lead to error if applied to non-integer ratio problems.

Multiplicative strategies acknowledge the covariance of ratio quantities and can be applied to both integer and non-integer problems. This is a more complex form of reasoning than that based on addition (i.e. build-up strategy). Multiplicative strategies are grounded in understanding that the two ratios in a proportion are equal. Two types of multiplicative strategies have been identified: within strategies and between strategies (Karplus et al., 1983; Noelting, 1980b). The between strategy (also called a scalar strategy) is based on applying the multiplicative relationship within one ratio [1] to the second ratio to produce equal ratios. The within strategy (also called a function strategy) is based on determining the multiplicative relationship between the corresponding parts of the two ratios and creating equal ratios.

Error strategies in proportional reasoning have been documented in the literature. Two types of error strategies have been frequently observed by researchers. The first error strategy is when students ignore part of the information given in the problem. For example, a student might solve the problem by comparing just two of the numbers in the problem (Hart, 1981; Karplus et al., 1983). A second type of frequently used error strategy is the ratio difference. In this strategy, students use the difference between the numbers within a ratio or between ratios and then apply this difference to find the unknown (Hart, 1981; Inhelder & Piaget, 1958; Tournaire & Pulos, 1985). The ratio difference is often used as a fallback strategy when dealing with a non-integer ratio (Karplus et al., 1983; Tournaire & Pulos, 1985).

Development of strategies

Inhelder and Piaget (1958) proposed a developmental sequence of proportional reasoning. First, they argued, students use only part of the information given in the problem to form a qualitative response. Following, the students are capable of understanding some of the numerical relationships, such as how the differences change with changes in the size of the numbers. On the other hand, students fail to recognize that the numbers form an equal ratio. Next, in the so-called pre-proportional stage, students understand some of the relationship between the two ratios and develop an efficient strategy, such as the build-up strategy. Finally, students reach the proportional reasoning level, where they understand both the scalar and functional relationships between the two ratios.

Noelting's (1980a, 1980b) answer to Piaget's developmental sequence was to report on the somewhat different developmental stages in proportional reasoning. Noelting's developmental sequence was built on a child's understanding of integer and non-integer relationships within and between ratios. Since many researchers believe that true proportional reasoning is defined as multiplicative, the change from using build-up strategies to multiplicative strategies is considered a benchmark. Knowledge of the factorial structure of numbers influences this change (Resnick & Singer, 1993). Experience with factorial number structures permits use of multiplicative reasoning in proportional situations.

Methodology

This study was conducted to investigate the influence of contextual and number structures on an individual's use of strategies in solving missing value proportion problems and to examine gender differences in strategy use. Fifty-three eighth graders in one school in Reykjavik, Iceland, participated in this study. Twenty-seven females and twenty-six males were individually interviewed in January as they solved sixteen missing value proportion problems. The problems represented four semantic structures: *well chunked* (W–C), *part-part-whole* (P–P–W), *associated sets* (A–S), and *symbolic* (S–P). For each contextual structure there were four problems, each representing a distinct number structure: *integer-integer* with an *integer* answer (I–I–I), *integer-noninteger* with an *integer* answer (I–N–I or N–I–I), *noninteger-noninteger* with an *integer* answer (N–N–I), and *noninteger-noninteger* with a *noninteger* answer (N–N–N).

Creation of problems

Lamon's categories of contextual structures were used (Lamon, 1993a). The well-chunk category refers to situations in which two extensive measures (e.g., dollars and items) are compared to the results in an intensive measure or rate (dollars/item). The part-part-whole category refers to situations in which ratios compare two subsets of one whole set. The two subsets are easily understood to be parts of one whole (Lamon, 1993a). Mixture problems (e.g., orange juice-water mixture) would fit this category. The associated sets category refers to a problem type in which two elements are compared and their relationship is defined by the problem itself. The two elements have little or nothing in common without the connection being made in the problem setting. The fourth problem category is called symbolic problems. In this category, two ratios are presented in mathematical symbols and compared without context. The ratios are all in the form of an equation such as $\frac{3}{7} = \frac{x}{28}$.

We consider the views from the literature as summarized in Abramowitz' (1975) four categories of number structure: (1) differences (equal/unequal), (2) size (larger/smaller), (3) order, and (4) type. In this study we applied these four structures to equal missing value proportional problems. The first variable *difference* is constant; all problems have unequal fractions, with the differences between and within ratios unequal. This prevents the use of ratio difference strategies. The second variable *size* is also constant, with the unknown number always larger than the known. The third variable *order* is another constant, with the place of the unknown number always the same. The fourth variable *type* identifies as the one variable we allowed to remain.

By allowing the *type* to remain a variable, we looked specifically at how the number structures in the ratios used in the problem influence the use of strategies within each contextual structure. We used four types of numerical relationships in the problem. The first was a simple whole multiple, or an integer-integer ratio with the unknown an integer. In the first type, the integer multiplier was found both within the ratio as well as between the ratios (i.e. $\frac{10}{5} = \frac{30}{x}$). The second was an integer-noninteger ratio or a noninteger-integer ratio with the unknown an integer (i.e. $\frac{2}{8} = \frac{x}{24}$ and $\frac{5}{7} = \frac{x}{28}$ respectively). Test problems were created in these categories in such a manner that the integer relationship exists either within a ratio or between ratios. By comparing equally difficult problems, one hopes to see a preference in strategy use. Karplus et al. (1983) made a clear distinction between problems having a within integer ratio and those having between integer ratios. The third type is a complex multiple, or a noninteger-noninteger ratio with the unknown an integer ($\frac{6}{13} = \frac{33}{x}$). The fourth type is also a complex multiple, or a noninteger-noninteger ratio, with the unknown a non-integer ($\frac{3}{4} = \frac{x}{17}$). We consider this last type to be the most difficult one and included it to determine if individuals are consistent with their use of strategies. Within each of the four contextual structures outlined, we wrote four missing value proportional problems using the number structures described above (see table 1 and appendix A).

Table 1. *Example of problems*

	Example	Number structure
Associated sets	Staff members at the Dog Motel Hvutti estimated that 10 dogs eat 5 kg of dry food each day. There are 30 dog guests at the motel today. How many kg of dry food are needed to feed all the dogs?	I – I – I $\frac{10}{5} = \frac{30}{x}$
Well-chunked	A group of people is planning to backpack in Iceland this summer. When planning, they estimate that in 5 hours they can walk 7 km. If they walk at the same rate, how many hours will it take them if the hike is 28 km?	N – I – I $\frac{5}{7} = \frac{x}{28}$
Part-part-whole	Hal was mixing fruit drink for his birthday party. According to the recipe, there are 2 cans of Sprite to every 8 packs (jugs) of orange juice. For the exact same taste, how many cans of Sprite would be needed if Hal used 24 packs of orange juice?	I – N – I $\frac{2}{8} = \frac{x}{24}$
Symbolic problems	No context	N – N – I $\frac{6}{13} = \frac{33}{x}$

Categorization of strategies

We used six categories of reasoning strategies: no conceptualization, ratio differences, qualitative, build-up, combined , and multiplicative strategies. This classification forms a hierarchy of reasoning sophistication. No-conceptualization was used if the subject made no attempt to solve a problem or used a number randomly and could not provide a reasonable explanation. See table 2 for the descriptions of the categories.

Sample

The population for this study, which included 27 females and 26 males, consisted of eighth-grade students in one school in Reykjavik, the capital of Iceland. At the time of the study, the school was one of the largest compulsory schools in Reykjavik. Families with a wide range of incomes, ranging from low-income and single parent families living in government-supported housing projects to upper-middle class families, live in the neigbourhood.

Table 2. *Students' strategies for solving missing value proportional problems*

Strategies	Description of strategy
No conceptualization	This student made no attempt to solve a problem or used numbers randomly and could not provide a reasonable explanation of their thinking.
Ratio difference	This student calculated the difference between the numbers in the known ratio and used this information to create a second ratio with the same difference.
Qualitative	This student considered the numerical relationships and used estimation to quantify the problems.
Build-up	This student calculated a unit ratio or used a given ratio to build up additively in an attempt to reach a target number (the known number of the second ratio).
Combined	This student used multiplication to get near a target number (the known number of the second ratio), but resorted to build-up, ratio difference, or qualitative thinking to adjust for non-integer multipliers.
Multiplicative	This student applied only multiplicative reasoning either within or between measure spaces to achieve a solution.

The school had four eighth grade classes, each with a different mathematics teacher. They were all mixed classes, with students ranging from those with learning disabilities to those with outstanding academic performance, and had approximately the same number of females and males. Two of the four classes were randomly selected to participate in the study. All the students in the two classes, except for one, gave their permission to be interviewed. The one exception asked his parents for permission not to participate in the study because of a learning disability. Other students with learning disabilities or difficulties did participate and their results were included in the data analysis.

Interview procedure

Each of the 53 students was interviewed individually by the first author (native of Iceland) and audiotaped. The students were asked to solve 16 missing value proportional problems (see appendix A). In the process of solving the problems, field notes where taken to capture the student's work. Individual students required between forty to eighty minutes to complete the 16 problems. The sixteen problems were presented in a random but predetermined order during individual interviews with each student. In some cases, when it was clear that a student was struggling and getting frustrated with a particular problem, they were provided assistance but the result was marked as unsolved for further analysis.

Students were provided with paper and pencil to write as they felt necessary. They were repeatedly encouraged to describe their thinking as they solved each problem, whether by writing, drawing, communicating orally, or using a combination of these. They were not allowed to use a calculator. If their solutions and strategies were not clear, they were asked to explain their thinking further. The students were also told that if they could not understand the question they could ask for clarification, or if they could not solve a problem they could ask to proceed to the next question.

Data analysis

An individual's response to each problem was examined using field notes, tape recordings, and any written work produced by the students. The interviews were not transcribed, but used as reference if field notes were not clear. The reasoning strategy used in each problem was categorized even if it did not lead to a correct numerical answer. All student responses fit into one of six classifications (see table 2), which formed a hierarchy of reasoning sophistication. The strategy that the student used to give his or her final answer was coded. When students explained their solution using two different strategies, their response was recorded as belonging to the more sophisticated strategy.

After classifying the strategy used in each student response according to table 2, the results were organized, first by contextual structure and then by number structure. Also classified were the number of correct answers, first by contextual structure and then by number structure. A similar table was also created by gender.

Results

Students' strategies and contextual structure

The study implies that there are three contextual structures from which the students constructed different understandings of missing value proportional problems (see table 3). First, the well-chunked (W–C) and associated sets (A–S) problems call for similar interpretation from students and the pattern of strategy use is very similar. On the other hand, the part-part-whole (P–P–W) problems and the symbolic problems (S–P) indicate a somewhat different pattern of strategy use.

In the well-chunk and associated sets problems, students used fewer multiplicative and ratio differences strategies and more build-up and combined strategies than in the other two contextual structures. Combined strategies appeared in problems that had an integer-non-integer ratio relationship (i.e. $\frac{8}{12} = \frac{x}{42}$). Close to fifty percent of the well-chunk and associated sets problems were solved with multiplicative strategy compared to fifty-five and sixty-five percent of part-part-whole and symbolic problems, respectively (see table 3). This implies that a problem's contextual structure does influence the student's choice of strategies.

Table 3. *Total percentage of strategies used by contextual structures*

Strategies	Well chunked	Part-part-whole	Associated sets	Symbolic problem
No conceptualization	10	4	7	7
Ratio difference	6	20	4	10
Qualitative	0	1	0	2
Build-up	8	4	8	0
Combined	31	16	30	13
Multiplicative	46	55	51	67

Note. n=212 (n=number of problem solved in each semantic type). Column does not add up to 100 due to rounding error.

Moreover, the overall frequency of using the build-up strategy was small, which is interesting. A study in the 5^{th} grade in Iceland suggested that build-up strategies were the most common strategy used by 5^{th} graders when solving missing value proportion problems (Steinthorsdottir & Sriraman, 2007b).

For the part-part-whole problems, students tended to rely on a ratio differences strategy if they could not successfully use a multiplicative strategy. Fifty-five percent of the problems were solved with multiplicative strategies whereas twenty percent were solved with ratio differences strategies. Moreover, the students used very few build-up strategies for solving the P–P–W problems, with just four percent. P–P–W problems accounted for the fewest cases of misconception.

The symbolic problems had the highest use of multiplicative strategies with sixty-seven percent. Similar to the part-part-whole problems, the students used ratio differences strategies when multiplicative strategies failed, accounting for ten percent of the solutions. No student used build-up strategies for the symbolic problems, which also elicited the lowest rate of combined strategies with only thirteen percent of the solutions.

The findings of this study indicate that students used different combinations of strategies between contextual structures. Combined strategies appeared in problems that had an integer-non-integer ratio relationship between contextual structures (i.e. $\frac{8}{12} = \frac{x}{42}$). In thirty-five percent of the cases of combined strategies in both A–S and W–C problems, students adjusted for the noninteger multiplier using the build-up strategy (see figure 1).

In all these cases, the remainder was half of the known number in the first ratio (i.e.). It appeared to be easy for students to see the half relationship and add that number to their target number. In twenty percent of the cases, the students unitized the remainder and added that number to reach the target number. In about twenty-three percent of the cases, students used the relationship of the numbers in the ratios to estimate

Problem	Strategy
Gudrun and Thor are planning to backpack in Iceland this summer. They estimate that in 8 hours they can cover 12 km. If they walk at the same rate, how many hours will it take them if the trek is 42 km long? ($\frac{8}{12} = \frac{x}{42}$)	*I know that* $12 \cdot 3 = 36$ *and* $8 \cdot 3 = 24$. $36 + 6 = 42$; 6 *is half of* 12 $24 + 4 = 28$ *because* 4 *is half of* 8 *so I need to add that to* 24 *so the answer is* 28

Figure 1. *Student's strategy (combination of multiplication and build-up)*

Problem	Strategy
It is lunchtime at the Humane Society. The staff members have found that 6 cats can eat 8 large cans of cat food. How many large cans of cat food would the staff members need to feed 36 cats? Explain how you found your answer. ($\frac{6}{8} = \frac{x}{36}$)	8 – 16 – 24 – 32 32 + 4 = 36 6 – 12 – 18 – 24 24 + 4 = 28 *I: Why did you add 4?* *S: Because I needed 4 to get to 36 so I had to add 4 to 24 to get the answer.*

Figure 2. *Student's strategy (combination of build-up and ratio difference)*

the remainder. But only in about eleven percent of the cases did students use ratio differences to deal with the remainder (see figure 2).

Twenty-four students turned to combined strategies to solve the part-part-whole problems, or sixteen percent of the problems. The patterns of the combined strategies also varied. Only four percent of the students used build-up strategies to work with the remainder, which had the multiplicative relationship of half compared with thirty-five percent in the well-chunk and associated sets problems. In twenty-eight percent of the cases, students unitized the remainder and added that number to their target number (see figure 3). On the other hand, in about forty percent of the cases, students used estimation by considering the relationship between the numbers in the ratios. Twenty-four percent used ratio differences to adjust for the noninteger multiplier.

The combination of strategies for symbolic problems was also different from those used for other contextual structures. In twenty-one percent of the items, build-up strategies were used to work with the half relationship. No one unitized the remainder. Twenty-one percent of the cases used estimation, but fifty percent of the cases accounted for ratio differences to work with the remainder.

Student's strategies and numbers structure

A clear pattern occurred in the decreased usage of multiplicative strategies and the increased usage of ratio differences as the number structure became more difficult (see table 4). For the integer-integer-integer (i.e. $\frac{3}{9} = \frac{x}{18}$) and integer-non integer-integer tasks (i.e. $\frac{2}{4} = \frac{x}{22}$), nearly all strategies were multiplicative (eighty-two percent of the I–I–I solution and seventy-two percent of the I–N–I / N–I–I solutions). The more complex number structures (N–N–I and N–N–N) generated less sophisticated strategies with only twenty-five percent of problems solved with

Problem	Strategy
In a designing program at the Hill Green School there were 2 boys for every 4 girls. How many boys participated in the game if there were 22 girls? Explain how you found your answer. ($\frac{2}{4} = \frac{x}{22}$)	*boys:* 2 – 4 – 8 – 16 – 1 – 17 *girls:* 4 – 8 – 16 – 20 – 2 – 22 *S: I figured out that there are 2 girls for every 1 boy so I had to add 2 girls to get to 22 and then I had to add 1 boy and got 17 boys.*

Figure 3. *Student's strategy (combination of build-up and unitizing)*

multiplicative strategies. Students partly relied on combined strategies to reach an answer for the more complex structures even though they had used multiplicative strategies on nearly all other problems. In these tasks, most students used multiplication to attempt to reach a target number.

The use of combined strategies increased as the numbers got more complex. In solving the I–N–I or N–I–I problems, only ten percent were combined strategies, but fifty percent of all the N–N–N problems were solved with combined strategies. There was also a decreased use of build-up strategies for more complex number structures, with the most used in the I–I–I problems (nine percent of solutions), while only one student used a build-up strategy on the N–N–N problems, one percent of the

Table 4. *Total percentage of strategies used by number structure*

Strategies	Number structure			
	I–I–I	I–N–I or N–I–I	N–N–I	N–N–N
No conceptualization	4	7	11	8
Ratio difference	5	7	10	16
Qualitative	0	0	3	1
Build-up	9	4	6	1
Combined	0	10	34	50
Multiplicative	82	72	36	25

Note. *n*=212 (*n*=number of problem solved in each semantic type).
I–I–I = Integer-Integer ratio relationship with an Integer unknown
I–N–I or N–I–I = Integer-Noninteger ratio relationship with an Integer unknown
N–N–I = Noninteger-Noninteger ratio relationship with an Integer unknown
N–N–N = Noninteger-Noninteger ratio relationship with a Non-integer unknown
Columns don't add up to 100 due to rounding error.

Problem	Strategy
Olaf has invited his friends for pizza. He estimated that he would need 2 large pizzas for 4 people. How many pizzas does he need to buy if 26 of his friends are coming? Explain how you found your answer. ($\frac{2}{4} = \frac{x}{26}$)	*S: It's 13.* *I: How did you know?* *S: There is half more friends then people so it has to be 13.* *I: Why does it have to be 13?* *S: Because 13 is half of 26.*

Figure 4. *Student's multiplicative strategy*

time. There was also an increased use of ratio differences. Combined strategies also differed among the number structures.

The reason why combined strategies accounted for no more than ten percent of the solutions to I–N–I or N–I–I problems most likely had to do with half relationships, as the remainders were half of the whole numbers in the known ratio (i.e. $\frac{2}{4} = \frac{x}{26}$; $4 \cdot 6\frac{1}{2} = 26$). The students' continual reliance on multiplicative strategies suggests that the fraction is so common to them that it might present less difficulty. Perhaps students operate with this fraction nearly as easily as they do with natural numbers (see figure 4). Ten percent of the students used ratio differences for the remainder (see figure 2) and fourteen percent dealt with the remainder by unitizing it and then adding it to the target number (see figure 3).

In the N–N–I problems, sixty-five percent of the problems solved with a combined strategy had the half relationship, which students added to their target number. Eleven percent of the students used estimation, working with the relationship of the numbers in the ratios (see figure 5, strategy a). Thirteen percent of the students used ratio differences and nine percent chose to ignore the fact that there was a remainder. In the N–N–N problems, forty-three percent of the problems that were solved with combined strategies used a multiplicative strategy and estimation. Twenty-seven percent of the students chose to fall back to ratio differences to find the remainder (see figure 2). Twenty-seven percent of the students also chose to unitize the remainder and come up with a number to add to their target number. (see figure 5, strategy b).

Correct and incorrect answers

Number structure most clearly determines the difficulty level of missing value proportion problems (Abramowitz, 1975; Tourniaire & Pulos, 1985). The results in this study support this finding. The number structure of the problems in this study clearly affected the students' abilities to respond

Problem	Strategy a
There is a feeding time at the fox farm. The farmer has found that 4 kg of meat can feed 5 foxes. How many kg does the farmer need if he has 22 foxes? Explain how you found your answer. ($\frac{4}{5} = \frac{x}{22}$)	*S: I know* $5 \cdot 4 = 20$ *and* $4 \cdot 4 = 16$ *and I need 2 more to get to 22 and I know that I can not also add 2 to 16 so it will be something like bigger than 17 but smaller than 18.* *I: How do you know?* *S: Because 4 is less then 5 but almost 5 so it has to be a little bit less than 2.*
	Strategy b
	$5 \cdot 4 = 20$ *and* $4 \cdot 4 = 16$ 1 *fox gets* $\frac{4}{5}$ *kg and* 2 *foxes get* $1\frac{3}{5}$ 22 *foxes need* $17\frac{3}{5}$ *kg*

Figure 5. *Student's strategy when solving N–N–N problem*

with correct answers more so than did semantic type (see table 5). Students did better on the associated sets problems than any of the other types, answering correctly seventy-three percent of the time.

The other contextual structures were all quite similar. On the other hand, if we look at the correct answers by number structure, integer-integer-integer problems were clearly the easiest, with ninety-two percent answered correctly. Noninteger-noninteger-noninteger problems were the most difficult ones, with only thirty-two percent solved correctly (see table 6).

Gender differences in strategy use

Correct and incorrect answers

No overall differences were identified in the rate of correct answers between boys and girls (χ^2=0.19, df = 1, p>0.05). Females solved sixty-six percent of the problems correctly and males solved sixty-eight percent of the problems correctly. Contextual structure had a bigger impact on the girls' success rate (table 5) than on the boys' success rate; that is, the success rate for females was not as similar across contextual structures as it was for males. It is also interesting to note that girls did better in the symbolic problems than did boys, while the boys did better on part-part-whole problems. The number structure did not indicate any differences by gender. Both boys and girls showed a similar pattern of a decreasing number of correct answers as the complexity of the number structure increased (see table 6).

Table 5. *Percentage of correct responses by semantic type*

	Gender		
Contextual structures	**Female**	**Male**	**Total**
Weel chunked	66	70	68
Part-part-whole	57	71	64
Associated sets	74	71	73
Symbolic problem	68	63	65

Note. Total *n*=212, Female *n*=108. Male *n*=104. (*n*= number of problem solved in each category).

Strategies

Boys tended to use more multiplicative strategies, while girls tended to use more build-up strategies. Boys had higher rates of no conception and qualitative strategies. Gender differences were not identified in the pattern of strategy use based on number structure (see table 7). Among those of both gender, the more complex the number structure became, the less sophisticated the strategy became.

Boys tended to use more multiplicative strategies than did girls in all contextual structures except for the symbolic problems (see table 8). The higher rate of use of multiplicative strategies on symbolic problems by girls reflects that the girls succeeded in solving these problems more often than did the boys. well-chunk and associated sets problems show similar patterns in strategy use by boy and girls and part-part-whole and

Table 6. *Percentage of correct responses by number structure*

	Gender		
Number structur	**Female**	**Male**	**Total**
I–I–I	92	91	92
I–N–I or N–I–I	82	88	85
N–N–I	60	59	60
N–N–N	31	34	32

Note. Total *n*=212, Female *n*=108. Male *n*=104. (*n*= number of problem solved in each category)
I–I–I = Integer-Integer ratio relationship with an Integer unknown
I–N–I or N–I–I = Integer-Noninteger ratio relationship with an Integer unknown
N–N–I = Noninteger-Noninteger ratio relationship with an Integer unknown
N–N–N = Noninteger-Noninteger ratio relationship with a Non-integer unknown

Table 7. *Percentile of strategies used by gender–number structure*

Strategies	Number Structure							
	I–I–I		I–N–I N–I–I		N–N–I		N–N–N	
	F	M	F	M	F	M	F	M
No Conceptualization	2	6	5	9	9	14	7	9
Ratio Difference	6	3	11	3	15	5	20	11
Qualitative	0	0	0	0	0	5	1	2
Build-Up	13	6	6	2	7	4	1	1
Combined	0	0	10	11	35	33	51	48
Multiplicative	79	85	68	76	35	38	20	30

Note. Total *n*=212 (*n*=number of problems solved in each category)
F = Females, M = Males
I–I–I = Integer-Integer ratio relationship with an Integer unknown
I–N–I or N–I–I = Integer-Noninteger ratio relationship with an Integer unknown
N–N–I = Noninteger-Noninteger ratio relationship with an Integer unknown
N–N–N = Noninteger-Noninteger ratio relationship with a Non-integer unknown
Columns don't add up to 100 due to rounding error

symbolic problems have different patterns. In W–C and A–S problems, girls tended to use more build-up strategies than did boys, and in P–P–W and symbolic problems they tended to use ratio differences more often than did boys. Differences in solution patterns between contextual structures are more extreme with girls than with boys, indicating that the contextual structure influences girls more than it does boys. Finally, looking at the combined strategy, there seem to be no differences between girls and boys as both genders have similar patterns.

Discussion and conclusions

The purpose of this study was to identify whether the contextual or number structures of missing-value proportional problems have a greater influence on a student's choice of solution strategy. A second purpose was to investigate gender differences in strategy use within both contextual and number structures. The major results of this study are the following:

- The number structure influenced students' use of strategies for solving missing value proportional problems more than did contextual structures.

- The number structure most clearly determined the level of difficulty of the problem.
- There were no overall gender differences in the number of correct solutions.
- There are indications that the contextual structures more strongly influenced the girls' use of strategies for solving missing-value proportional problems than it did the boys' use of a strategies.

The influence of contextual structures

The results indicate that number structure, more than contextual structure, influenced the students' use of strategies for solving missing value proportional problems. In addition, number structure most clearly determined the level of difficulty of the problem. Consequently, the number structure of the problem appeared to affect students' ability to respond correctly. The influence of the contextual structures should not be overlooked.

In examining the contextual structures, the well-chunked and associated sets problems show similar patterns in strategy use. Some aspects of these problems call for a very similar interpretation from students. The context of these problem types is familiar to the students; students deal with speed, price, and amount of food in their daily lives. The familiarity of the context is what well-chunked and associated sets have in common. These types also show the fewest number of 'pure' multiplicative strategies (table 3). We call it 'pure' because the combined

Table 8. *Percentile of strategies used by gender–semantic type*

Strategies	**Contextual structures**							
	Well chunked		Part-part-whole		Associated sets		Symbolic problems	
	F	M	F	M	F	M	F	M
No Conceptualization	9	11	1	6	5	10	6	9
Qualitative	0	0	1	0	0	1	0	5
Ratio Difference	7	5	30	10	6	1	13	7
Additive	12	3	5	4	10	5	0	1
Combined	33	28	12	19	33	26	11	15
Multiplicative	38	54	51	60	45	58	71	63

Note. Total n=159 (n=number of problems solved in each category)
F = Females, M = Males
Columns don't add up to 100 due to rounding error

strategies were usually a combination of multiplicative and other strategies. If combined strategies are taken into account, then these two contextual structures do in fact elicit the most frequent use of multiplicative strategies. This is consistent with the number of correct solutions (table 5), since well-chunked and associated sets have the highest number of correct solutions. It can therefore be argued that students use their most sophisticated strategies in problems set in contexts with which they are familiar and understand.

For students, the part-part-whole problems called for a different interpretation. In the part-part-whole problems, the two elements in a given ratio are subsets of one whole. By looking at the number of multiplicative strategies and combined strategies it can be said that part-part-whole items elicited the fewest solutions with multiplicative strategies. It is interesting that build-up strategies were not a practicable way for students to solve these problems, but ratio difference strategies were.

For symbolic problems, students relied on multiplicative strategies. Failing that, they used ratio differences. It is interesting to note that no attempts were made to use build-up strategies as most students treated these problems as fractions, not ratios. The students were familiar with how to find equivalent fractions and most knew that they were supposed to multiply. Use of build-up strategies was not considered, since they had nothing to build up and no elements made sense to them. Also, students probably remembered from their textbooks that adding on was not something you did when making equivalent fractions.

It is difficult to talk about symbolic problems without taking into account the number structure. Since the students treated the proportion as two fractions, it was difficult for them to see the answer as a mixed number; moreover, they were not familiar with complex fractions. A very common answer when dealing with an N–N–N problem was, "This is not possible. The answer is not a whole number." When faced with this difficulty, many students tried something else to attain a whole number answer. Also interesting is that the students who combined strategies used multiplicative strategies to reach the target number and then used ratio differences for the remainder. We believe that this is connected with students' understanding of fractions, which is limited when dealing with complex fractions.

The influence of number structure

The number structure was carefully manipulated within planned parameters of complexity. The number complexity formed a parallel hierarchy among the contextual structures. Consequently, the more complex the number structure, the less sophisticated the strategies used by students. The frequent use of multiplicative strategy in the I–N–I problems might support the idea that students chose the 'easier' multiplicative relationship to work with such problems. Two of the problems had an integer multiplier between ratios and two had one within ratios. But when looking at the problems, it can be seen that there was an integer relation within the ratio and, a relationship between ratios. That is, the multiplier between ratios was an integer plus a half. Students' reliance on multiplicative strategies might also suggest that the fraction is so common to them that it might have presented fewer difficulties. It is well documented that students operate with this fraction nearly as easily as they do with natural numbers (Noelting, 1980a, 1980b). On the other hand, with the N–N–I problems, which all had a $\frac{1}{2}$ relationship between ratios (within the measurement space), there was a considerable drop in use of 'pure' multiplicative strategies. The difference between these two number structures is that in three of the N–N–I problems, the denominator in the known ratio was larger than ten, with the known quantity in the second ratio larger than 30. Some research has suggested that problems with numbers larger than 30 influence students' proportional reasoning. The results corroborate the literature.

The last number structure, N–N–N, showed the fewest cases of correct solutions as well as the least usage of multiplicative strategies. Students had to rely partly on other strategies to reach an answer even though they used multiplicative strategies on nearly all the other problems. Such use of fallback strategies has been found in previous research on proportional reasoning (Tourniaire & Pulos, 1985; Karplus et al., 1983; Lamon, 1993b; Kaput & West, 1994).

In these tasks, many students used multiplication to attempt to reach a target number. When this number could not be reached with an integer multiplier, they used the nearest multiple and applied a less sophisticated strategy to the remainder. It was this remainder or the adjustment for the noninteger multiplier that caused difficulty on the N–N–N tasks for many of the students as they had to give fractional answers of thirds, fourths, fifths and sixths. This would suggest that these fraction families were less understood by the students interviewed. It also suggests students' unfamiliarity with complex fractions and mixed number answers within the contextual structures of the problems.

An alternative way to look at the N–N–N problems is to combine the number of multiplicative and combined strategies. By doing this, one can see that approximately seventy-five percent of the problems were solved by a strategy that was multiplicative in nature, since the combined strategies were a combination of multiplicative and one other strategy. In the method the students used to reach the target number, as described above, it was not their lack of proportional reasoning that hindered them in determining the correct answer but their lack of computational skills. On the basis of our results, it is not possible to determine whether students had difficulties with proportional reasoning in N–N–N problems. One could argue that a student who had a preference for using a multiplicative strategy understood some aspects of proportion, while also understanding that the same relationship applies in more complex number structures. He or she might not have used computational knowledge to solve a complex calculation, or might not have seen the need to come up with an exact number in some cases. Thus, it might not make any sense for the student to go through a complex manual calculation when a close estimation can be obtained

Gender differences in strategy use

The data indicated no overall gender differences in the number of correct solutions. Girls did better than boys in solving the symbolic problems and associated sets problems and girls tended to use less complex strategies, except in the symbolic problems.

An important factor to look at is that the symbolic problems were very familiar to the students. These problems were common in the textbooks and the students most likely had already solved this kind of problem, even though their recognition was related to fractions and not proportion. For girls, it seems important *to consider what they had learned before*. It might be that girls had paid attention to what was taught and were good learners. There are studies that support the idea that girls do better than boys on content that has been covered in the classroom (Kimball, 1989). Another study implies that contextual structures create a greater effect on females than on males (Pulos, Karplus & Stage, 1981). A noticeable difference exists in this data in the variation of strategy used by males and females, as well as the difference in the success rate (see table 5 and 7). The associated sets included problems such as how much food was needed for a certain number of animals or people. Such a situation is most likely very common for children in their daily life. Meanwhile, well-chunked problems involve problems related to speed and price. These are also common circumstances in daily life. Moreover, the symbolic problems might not have been connected to students' daily life but it is quite likely that the students had already learned a procedure that helped them to solve this type of problem. On the other hand, the part-part-whole problems are not as much a part of the students' daily life activities. For students to wonder about the number of people in a group and how much linear expansion there would be if some particular part of the group grew would certainly be less likely to be a part of the students' daily practice.

In conclusion girls were more successful than boys in handling associated sets and symbolic problems, and boys were more successful than girls in part-part-whole problems. Moreover, the data suggest that the contextual structures influence females' choice of strategy more than that of males. Follow up studies that call for the use of proportional reasoning on real world problems can help further illuminate factors related to gender and strategy use. It would be of interest to the Nordic community to conduct similarly designed studies to examine whether such differences are present elsewhere.

References

Abramowitz, S. (1975). *Adolescent understanding of proportionality: the effects of task characteristics* (Study undertaken as a doctoral dissertation). Palo Alto, CA: Stanford University.

Beaton, A., Mullis, I., Martin, M., Gonzalez, E., Kelly, D. et al. (1996). *Mathematics achievement in the middle school years: IEA's third International mathematics and science study (TIMSS)*. Chestnut Hill, MA: Boston College, TIMSS International Study Center.

Behr, M., Harel, G., Post, T. & Lesh, R. (1992). Rational number, ratio, and proportion. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 296–333). New York: Macmillan.

Behr, M., Harel, G., Post, T. & Lesh, R. (1994). Units of quantity: a conceptual basis common to additive and multiplicative structures. In G. Harel & J. Confrey (Eds.), *The development of multiplicative reasoning in the learning of mathematics*, (pp. 121–176). Albany, NY: State University of New York Press.

Bell, A., Fischbein, E. & Greer, B. (1984). Choice of operation in verbal arithmetic problems. The effects of number size, problem structure, and context. *Educational Studies in Mathematics*, 15 (2), 129–148.

Brandell, G., Nyström,P. & Sundqvist, C. (2004, July). *Mathematics – a male domain*. Paper presented at the quadrennial conference of the International Congress on Mathematical Education (ICME 10), Copenhagen.

Carpenter, T.P., Fennema, E., Franke, M.L., Levi, L. & Empson, S.B. (1999). *Children's mathematics: cognitively guided instruction*. Portsmouth, NH: Heinemann.

Carr, M. & Jessup, D. (1997). Gender differences in first grade mathematics strategy use: social, metacognitive, and attribution influences. *Journal of Educational Psychology*, 89 (2), 318–328.

Dienes, Z. P. (2000). The theory of the six stages of learning with integers. *Mathematics in School*, 29 (2), 27–33.

Dienes, Z. P. (2004). Six stages with rational numbers. *New Zealand Mathematics Magazine*, 41 (2), 44–53.
Fennema, E. (1995). Mathematics, gender, and research. In B. Grevholm & G. Hanna (Eds.), *Gender and mathematics education: an ICMI study in stiftsgården Åkersberg, Höör, Sweden 1993* (pp. 21–38). Lund University Press.
Fennema, E., Carpenter, T. P., Jacobs, V. R. & Levi, L. (1996, April). *Gender differences in mathematical thinking*. Paper presented at the annual meeting of the American Educational Research Association, New York.
Forgasz, H. (2004, July). *Computers for mathematics learning and gender stereotypes*. Paper presented at the quadrennial conference of the International Congress on Mathematical Education (ICME 10), Copenhagen.
Gallagher, A. M. & De Lisi, R. (1994). Gender differences in scholastic aptitude test-mathematics problem solving among high-ability students. *Journal of Educational Psychology*, 86 (2), 204–211.
Hart, K. M. (1981). *Children's understanding of mathematics: 11–16*. London: John Murray Ltd.
Inhelder, B. & Piaget, J. (1958). *The growth of logical thinking from childhood to adolescence*. New York: Basic Books.
Kaput, J. & West, M. (1994). Missing value proportional reasoning problems: factors affecting informal reasoning patterns. In G. Harel & J. Confrey (Eds.), *The development of multiplicative reasoning in the learning of mathematics* (pp. 235–287). Albany, NY: State University of New York Press.
Karplus, R., Pulos, S. & Stage, E. K. (1983). Proportional reasoning of early adolescents. In R. Lesh & M. Landau (Eds.), *Acquisition of mathematics concepts and processes* (pp. 45–90). Orlando, FL: Academic Press.
Kieren, T. (1993). Rational and fractional numbers: from quotient fields to recursive understanding. In T. Carpenter, E. Fennema & T. Romberg (Eds.), *Rational numbers: an integration of research* (pp. 49–84). Hillsdale, NJ: Lawrence Erlbaum.
Kimball, M. M. (1989). A new perspective on women's math achievement. *Psychological Bulletin*, 105 (2), 105–214.
Lamon, S. (1993a). Ratio and proportion: connecting content and children's thinking. *Journal for Research in Mathematics Education*, 24 (1), 41–61.
Lamon, S. (1993b). Ratio and proportion: children's cognitive and metacognitive processes. In T. Carpenter, E. Fennema & T. Romberg (Eds.), *Rational numbers: an integration of research* (pp. 131–156). Hillsdale, NJ: Lawrence Erlbaum.
Lesh, R., Post, T. & Behr, M. (1988). Proportional reasoning. In J. Hiebert & M. Behr (Eds.), *Number concepts and operations in the middle grades* (pp. 93–118). Reston, VA: National Council of the Teachers of Mathematics.

Linn, M. C. & Pulos, S. (1983). Aptitude and experience influences on proportional reasoning during adolescence: focus on male-female differences. *Journal of Research in Mathematics Education*, 14 (1), 30–46.

Lubienski, S. T. & Bowen, A. (2000). Who's counting? A survey of mathematics education research 1982–1998. *Journal for Research in Mathematics Education*, 31 (5), 626–633.

Mahlomaholo, S. & Sematly, M. (2004, July). G*ender differences and black learners' attitudes towards mathematics in selected high schools in South Africa*. Paper presented at the quadrennial conference of the International Congress on Mathematical Education (ICME 10), Copenhagen, Denmark.

Marshall, S.P. & Smith, J. D. (1987). Sex differences in learning mathematics: a longitudinal study with item and error analyses. *Journal of Educational Psychology*, 79 (4), 372–383.

Noelting, G. (1980a). The development of proportional reasoning and the ratio concept. Part 1 – Differentiation of stages. *Educational Studies in Mathematics*, 11 (2), 217–253.

Noelting, G. (1980b). The development of proportional reasoning and the ratio concept. Part 2 – Problem-structure at successive stages; Problem solving strategies and the mechanism of adaptive restructuring. *Educational Studies in Mathematics*, 11 (3), 331–363.

OECD (2004). *PISA Learning for tomorrow's world: first results from PISA 2003*. Paris: OECD.

Olafsson, R. F., Halldorsson, A. M. & Bjornsson, J. K. (2006). Gender and the urban-rural differences in mathematics and reading: an overview of PISA 2003 results in Iceland. In J. Mejding & A. Roe (Eds.), *Northern lights on PISA 2003 – a reflection from Nordic countries*, (185–198). Copenhagen: Nordic Council of Ministers.

Pourkazemi, M. H. (2004, July). *In the name of God. Gender and mathematics*. Paper presented at the quadrennial conference of the International Congress on Mathematical Education (ICME 10), Copenhagen.

Pulos, S., Karplus, R. & Stage, E. K. (1981). Generality of proportional reasoning in early adolescence: content effects and individual differences. *Journal of Early Adolescence*, 1 (3), 257–264.

Resnick, L. B. & Singer, J. A. (1993). Protoquantitative origins of ratio reasoning. In T. Carpenter, E. Fennema & T. Romberg (Eds.), *Rational numbers: an integration of research* (pp. 107–130). Hillsdale, NJ: Lawrence Erlbaum.

Soro, R. (2004, July). *Teachers' beliefs about girls and boys equity in mathematics*. Paper presented at the quadrennial conference of the International Congress on Mathematical Education (ICME 10), Copenhagen.

Sriraman, B. & English, L. (2004). Combinatorial mathematics: research into practice. Connecting research into teaching. *The Mathematics Teacher*, 98 (3), 182–191.
Sriraman, B. & Lesh, R. (2006). Modeling conceptions revisited. *Zentralblatt für Didaktik der Mathematik*, 38 (3), 248–254.
Sriraman, B. & Lesh, R. (2007). A conversation with Zoltan P. Dienes. *Mathematical Thinking and Learning*, 9 (1), 59–75.
Steinthorsdottir, O &, Sriraman, B. (2007). Iceland and rural/urban girls – PISA 2003 examined from an emancipatory viewpoint. In B. Sriraman (Ed.), *International perspectives on social justice in mathematics education. The Montana Mathematics Enthusiast* (Monograph 1), 169–178.
Steinthorsdottir,O. & Sriraman, B. (2007b). *Icelandic 5th grade girls developmental trajectories in proportional reasoning*. Paper submitted for publication.
Tourniaire. (1986). Proportion in elementary school. *Educational Studies in Mathematics*, 17 (4), 401–412.
Tourniaire, F. & Pulos, S. (1985). Proportional reasoning: a review of the literature. *Educational Studies in Mathematics*, 16 (2), 181–204.
Vergnaud, G. (1994). Multiplicative conceptual field: what and why? In G. Harel & J. Confrey (Eds.), *The development of multiplicative reasoning in the learning of mathematics* (pp. 41–59). Albany, NY: State University of New York Press.
Wiest. L. R. (2004, July). *The critical role of an informal mathematics program for girls*. Paper presented at the quadrennial conference of the International Congress on Mathematical Education (ICME 10), Copenhagen, Denmark.

Notes

1 Here we define ratio as the relationship between two quantities that have two different measure units.

Appendix A

Well Chunked Problems

Integer – integer with an integer answer (I–I–I)

In 3 hours, the students solve 9 math problems. How long will it take the students to solve 18 math problems? Let's assume that it takes the same amount to solve each problem. Explain how you found your answer. ($\frac{3}{9} = \frac{x}{18}$)

Noninteger – integer with an integer answer (N–I–I) or integer – noninteger with an integer answer (I–N–I)

My friends and I are planning to backpack in Iceland this summer. We estimate that in 5 hours we cab cover 7 km. If we walk at the same rate, how many hours will it take them if the trek is 21 km long? Explain how you found your answer. ($\frac{5}{7} = \frac{x}{21}$)

Noninteger – noninteger with an integer answer (N–N–I)

Gudrun and Thor are planning to backpack in Iceland this summer. They estimate that in 8 hours they can cover 12 km. If they walk at the same rate, how many hours will it take them if the trek is 42 km long? Explain how you found your answer. ($\frac{8}{12} = \frac{x}{42}$)

Noninteger – noninteger with a noninteger answer (N–N–N)

Johanna lived in the USA for one year. She went to the candy store and bought some candy for her party. The price of the candy was 2 pounds for 3 dollars. How many pounds of candy could she buy for 17 dollars? Explain how you found your answer. ($\frac{2}{3} = \frac{x}{17}$)

Part-part-whole problems

Integer – integer with an integer answer (I–I–I)

In the after school program there are 5 girls for every 15 boys. How many girls are in the after school program if the boys are 45? Explain how you found your answer. ($\frac{5}{15} = \frac{x}{45}$)

Noninteger – integer with an integer answer (N–I–I) or integer – noninteger with an integer answer (I–N–I)

In a designing program at the Hill Green School there were 2 boys for every 4 girls. How many boys participated in the game if there were 22 girls? Explain how you found your answer. ($\frac{2}{4} = \frac{x}{22}$)

Noninteger – noninteger with an integer answer (N–N–I)

A constructor is building apartments building in a new neighborhood. These buildings have two and three bedroom apartments. It has been decided that for every 6 three bedroom apartments there should be 14 two bedroom apartments. How many three bedroom apartments are needed if there are going to be 35 two bedroom apartments? Explain how you found your answer. ($\frac{6}{14} = \frac{x}{35}$)

Noninteger – noninteger with a noninteger Answer (N–N–N)

Karl was mixing fruit drink for his birthday party. According to the recipe, there are 5 cans of Sprite to every 6 packs (jugs) of orange juice. For the exact same taste, how many packs of orange juice would be needed if Karl used 19 cans of Sprite? Explain how you found your answer. ($\frac{5}{6} = \frac{x}{19}$)

Associated Sets Problems

Integer – integer with an integer answer (I–I–I)

Thor is feeding his fish. The directions on the box tell Thor that 4 scoops of food is enough for 12 fish. How many scoops of food should 24 fish in Thor's aquarium? Explain how you found your answer. ($\frac{4}{12} = \frac{x}{24}$)

Noninteger – integer with an integer answer (N–I–I) or
integer – noninteger with an integer answer (I–N–I)

Olaf has invited his friends for Pizza. He estimated that he would need 2 large pizzas for 4 people. How many pizzas does he need to buy if 26 of his friends are coming? Explain how you found your answer. ($\frac{2}{4} = \frac{x}{26}$)

Noninteger – noninteger with an integer answer (N–N–I)

It is lunchtime at the Humane Society. The staff members have found that 6 cats can eat 8 large cans of cat food. How many large cans of cat food would the staff members need to feed 36 cats? Explain how you found your answer. ($\frac{6}{8} = \frac{x}{36}$)

Noninteger – noninteger with a noninteger answer (N–N–N)

There is a feeding time at the fox farm. The farmer has found that 4 kg of meet can feed 5 foxes. How many kg does the farmer need if he has 22 foxes? Explain how you found your answer. ($\frac{4}{5} = \frac{x}{22}$)

Symbolic Problems

Integer – integer with an integer answer (I–I–I)

What number does the *x* stand for? ($\frac{2}{6} = \frac{x}{12}$)

Noninteger – integer with an integer answer (N–I–I) or
integer – noninteger with an integer answer (I–N–I)

What number does the *x* stand for? ($\frac{3}{5} = \frac{x}{20}$)

Noninteger – noninteger with an integer answer (N–N–I)

What number does the *x* stand for? ($\frac{10}{16} = \frac{x}{24}$)

Noninteger – noninteger with a noninteger answer (N–N–N)

What number does the *x* stand for? ($\frac{3}{4} = \frac{x}{17}$)

Olof B. Steinthorsdottir

A former mathematics classroom teacher in her native country of Iceland, assistant professor Olof Bjorg Steinthorsdottir teaches mathematics education courses. Her scholarly interests include the teaching and learning of mathematics among students in pre-kindergarten through middle school, specifically students' understanding of mathematics and how teachers can use that knowledge to make instructional decisions. Her interests also include gender and mathematics. Steinthorsdottir has been active member within PME and has in resent years co-organized the working/ discussion group about gender and mathematics. She received her PhD in Mathematics Education from the University of Wisconsin in Madison.

Olof B. Steinthorsdottir
Assistant Professor
School of Education
University of North Carolina at Chapel Hill
CB#3500 - Peabody Hall
Chapel Hill, NC 27517
Phone: 919-962-6608
Fax: 919-961-1533
steintho@email.unc.edu

Bharath Sriraman

Bharath Sriraman is associate professor of mathematics at the University of Montana, with a wide range of eclectic research interests including mathematical cognition and gender issues. Bharath is the editor-in-chief of the Montana Mathematics Enthusiast, associate editor of ZDM – the International Journal on Mathematics Education, consulting editor of Interchange: A Quarterly Review of Education, and reviews editor of Mathematical Thinking & Learning as well as ZDM. He holds very active research ties with researchers in many countries including Iceland.

Bharath Sriraman
Associate Professor
Dept. of Mathematics
The University of Montana
Missoula, MT 59812
Phone: 406-243-6714
Fax: 406-243-2674
sriramanb@mso.umt.edu

Sammendrag

Denne artikel handler om strategier ved løsning af proportionalitetsproblemer og det undersøges hvordan opgavernes kontekstuelle og talmæssige strukturer influerer på henholdsvis drenges og pigers valg af løsningsstrategier.

Der kunne ikke påvises kønsforskelle i elevernes generelle succesrater. Pigerne var dog mere succesfulde end drengene ved A–S og S–P opgaver mens drengene klarede P–P–W opgaver bedre end pigerne. Endvidere indikerer data, at den kontekstuelle struktur i højere grad påvirker pigernes valg af strategi end drengenes.