Textbooks as instruments

Three teachers' way to organize their mathematics lessons

Monica Johansson

This paper reports a study of three teachers' way to organize their lessons and how textbooks are incorporated in their work[1]. Despite the differences between the teachers, it is noticeable that in these three classrooms, the textbooks, to a large degree, guide the teaching. The textbooks are present: (a) in the students' individual work, (b) in many of the examples presented on the board, (c) as a source for background and motivational discussions, (d) in how mathematics is presented, and (e) for homework.

From previous research, we find that textbooks influence *what* to teach. If the textbooks present a certain mathematical topic, it is very likely that the teacher introduces it in the classroom. On the other hand, if a topic is not in the textbook, it is most likely not presented by the teacher (e.g. Freeman & Porter, 1989; Reys, Reys, Lapan, Holliday, & Wasman, 2003). Textbooks are also a primary information source in deciding *how* to present the content (e.g. Schmidt et al., 2001). The instructional approach of the material can even influence the teacher's pedagogical strategies (e.g. Reys et al., 2003). For better or for worse, the textbooks provide an interpretation of mathematics to teachers, students and their parents but also an interpretation of how students learn mathematics. One could for example recognize the ideas of behaviorism in a book that focuses on getting the right answers on well-defined questions. From a constructivist and socio-cultural perspective, it would be more important to start from the students own experiences and create problems that nurture discussions and cooperation (Selander & Skjelbred, 2004).

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In Sweden, there is an ongoing discussion concerning too much reliance on textbooks in the teaching of mathematics in schools. An example of a contribution in this debate comes from a current evaluation of schools. The inspectors found that the teaching of mathematics, more than any other school subject, relies on the use of textbooks.

> The evaluation shows the surprisingly dominant role of the textbooks in teaching ... especially from year 4-5 and onwards ... Content, as well as arrangement of teaching, is to a high degree directed by the textbook. Mathematics is, for both students and teachers, simply what is written in the textbook (Lindqvist, Emanuelsson, Lindström, & Rönnberg, 2003, p. 39, author's translation).

This study is an attempt to further analyze the influence of textbooks in mathematics classrooms, especially how it can be defined and discussed from a Swedish perspective.

The study focuses on teachers use of textbooks [2]. Under the assumption that a teacher uses a textbook because he or she has a more or less conscious idea that the book is important (cf. B. Englund, 1999), this paper discusses the *influence* of textbooks in classrooms. The research questions are:

- How do the teachers organize their teaching in terms of type of classroom interaction, organization of students, and content activity?
- When and how, direct or indirect, are the textbooks used in the different types of organization of teaching in the three classrooms?
- In what respect do the textbooks influence (or not influence) the mathematical work and how do the teachers highlight key ideas?

The three issues will be discussed in separate sections of this paper.

The study is guided by a theoretical perspectives that is based on what Englund (1997) describes as the third stage of the frame factor theory [3]. This means that the choice of educational content and contextualization of teaching is emphasized. A fundamental assumption is that students are offered different possibilities to create and construct meaning depending on, for example, what content is chosen and what context the textbook offers. In other words, different choices can be made, more or less consciously, which have crucial implications for teaching and learning.

The main purpose of the study is to reveal teachers' practices and relations to textbooks in order to contribute to the ongoing discussion concerning teachers' dependence on textbooks

Methodology

A study of Swedish classrooms, the CULT-project[4], forms the empirical background for this paper. Three mathematics teachers were identified for their locally defined 'teaching competence' and for their situation in demographically diverse government schools in major urban settings. Observation data is, in short, gathered using a three-camera-approach, with complementing wireless microphones, focusing teacher, class, and a group of students. Video-recorded classroom data were collected for at least ten consecutive mathematics lessons. Further data involves post-lesson video-stimulated interviews with the teachers and the students.

For this study, parts of the following types of the CULT-data are used: video-recorded lessons, teacher interviews, and teacher questionnaires. From the three teachers, Mr. Andersson, Mr. Svensson, and Mr. Larsson[5] (labeled as SW1, SW2, and SW3 in the CULT-data), a total of thirteen lessons (679 minutes) are analyzed. The selected lessons have two common features, they are consecutive (with one exception: SW1-L09 was used for a diagnostic test), and the sequences start when the teachers introduce a new chapter in the textbooks. Five video-recorded lessons are chosen from Mr. Andersson's classroom (SW1: L06-L011), four lessons from Mr. Svensson's classroom (SW2: L04-L07), and four from Mr. Larsson's classroom (SW3: L02-L05). The lessons varied in length between 38 and 70 minutes. Practical reasons, such as a lack of time, are behind the decision not to include all lessons in this study. However, in general four lessons seem sufficient to expose variations and highlight interesting phenomena in each of the three teachers' classroom practices.

The participants

Mr. Andersson and his students (SW1)

Mr. Andersson is a bit more than thirty years old and has seven years of teaching experience, which he has gained at the school where the study is conducted. His subjects are Mathematics and Science and he has been teaching grade eight students for four years. The class, which consists of 26 grade eight students, is a mixed-ability group. The students are mainly working on an individual basis but every second or third week, he organizes them to work in groups.

The mathematical content, which is treated during the period of video recording for the CULT-project, concerns mathematical relationships (i.e. coordinate system, proportionality, and linear equations). It is, in most parts, a new subject area for the students. The teacher practice is *individual pace learning* (cf. Löwing, 2004). This means that students are working with tasks in the textbook[6] in their own pace. Mr. Anderson's idea of individualized teaching is indicated in an interview. When one of his students works faster and reaches further than the other, he offers her another textbook and asks her to work with the more demanding tasks in that book.

Mr. Andersson does not assign homework on a regular basis. He thinks it is difficult to go through the homework in a whole class setting, when the students are working at different speeds.

Mr. Svensson and his students (SW2)
Mr. Svensson is about sixty years old. He has been working in the current school almost all the time of his thirty-three years of employment as a teacher. The school practice is tuition in ability groups and the twenty-five ninth-grade students in his class are regarded as high-achievers. According to Mr. Svensson, the students in this class intend to be prepared for the Natural Science program, which they attend after finishing compulsory school.

The mathematical content that is treated during the period of video recording for the CULT-project is, according to the teacher, partly new and partly repetition. It is about equations. About ten to fifteen lessons are what the teacher usually plans for each topic. Homework is assigned on a regular basis; in order to practice new skills, to establish a topic, and ending work that was started during the lesson, the teacher says.

The teacher has mixed feelings about the textbook, which is a book from one of the most common textbook series for grade seven to nine in the compulsory school in Sweden. Some tasks are quite good, he says, but the word problems are too unrealistic.

> Well, the equations as such, the ready-made, they are okay. They review algebraic knowledge and some other understanding also. But then, the word problems, some of them you can be without. If you are supposed to count the three consecutive even numbers (refers to a task in the textbook) ... they are a bit 'non-realistic', I think (translation to English by the author).

The students do not solve all tasks in the textbook. Some problems, for example the A-tasks [7], are left out and some new are jointly constructed in the classroom.

Mr. Larsson and his students (SW3)
Mr. Larsson is about sixty years old. He has a long-standing experience of teaching and a long period of employment, more than thirty years of teaching, at the school where the study is conducted. Besides mathematics, he teaches Physics and Technology in grade eight and nine.

The textbook[8], which is used in this particular class, is a book from the same textbook series as Mr. Svensson's. The teacher seems to adhere very closely to the textbook in the private as well as the public part of the lesson, even if he from time to time brings up examples from outside the book. In one of the interviews, he confirms the strong reliance on the book. He was asked why he uses concrete numbers to show the students how to simplify an expression. The teacher answered:

> It is generally so that I just follow the usual way to do it ... this is normally how it is done in all books and I have not wondered about it so much, I think it is a system that works (translation to English by the author).

The class consists of 22 grade eight students. The school practice is tuition in ability groups and the students in this class are identified as high achievers. According to Mr. Larsson, they are quite homogeneous. He thinks of them as a hard working group that concentrates on mathematics. It is a good group, he says, nice and positive, sometimes a bit too chatty, but ambitious. When the teacher decides to give the students homework, which is not on regular basis, it is because he needs it for the grading or to gather the students for the next chapter in the textbook. As regards to how the students work in the textbook, he seems to think it is important to keep them together. He also uses the thematic tasks in the textbook to gather them.

The mathematical content that is treated during the period of video recording for the CULT-project is, according to the teacher, partly new and partly repetition. The chapter in the textbook has the title *Negative numbers, variables and expressions*.

The coding procedure

A coding procedure is used in order to capture sequences of the lessons that are of special interest for this study. These sequences are analyzed from three different perspectives: (1) the type of classroom interaction, organization of students, and content activity; (2) the use of textbooks, when and how, indirect or direct; and (3) the role of textbooks in different types of teacher activity. Two types of codes are used, *coverage codes* and *occurrence codes* (see next paragraphs). The coding procedure is guided by the coding manual of the TIMSS Video Study (Jacobs et al., 2003) – a study that has the following research objectives: a) to develop objective, observational measures of classroom instruction, b) to compare teaching practices among countries and, c) to describe patterns of teaching practices within each country.

Coverage codes

The coverage codes are used to code a lesson, or a defined period of a lesson, in its entirety. The codes have three mutually exclusive and exhaustive options. Thus, only one of these options is applied to each defined period in the lesson. The graphs in Figure 1 show the coverage codes and their subcategories.

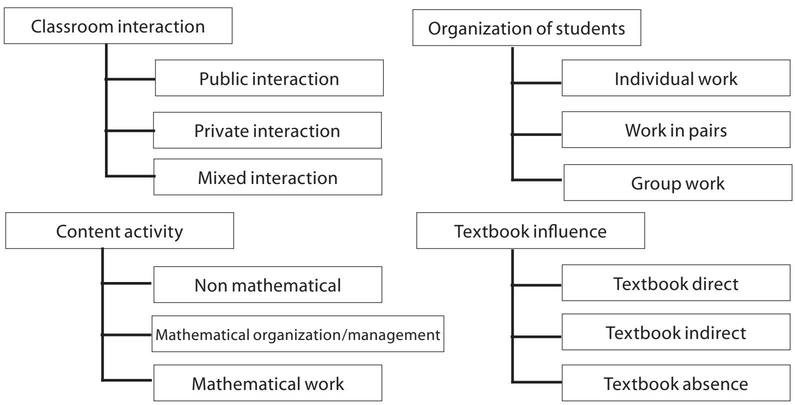

Figure 1. *Coverage codes*

Classroom interaction and *Content activity* are coverage codes for all parts of the lessons. This means that all points in the lessons are coded as one of the three, mutually exclusive subcategories for each code. *Organization of students* is a coverage code for all parts of the lesson that are coded as periods of *Private interaction* or *Mixed interaction*. How the students are working, individually, in pairs or in groups, is categorized in order to describe student organization and cooperation.

Textbook influence is a coverage code for all parts of the lessons that are coded as periods of *Mathematics organization/management* and *Mathematical work*. This means that all points in these segments of the lessons are coded as one of the three, mutually exclusive categories:

a) Textbook direct. There is an open and explicit use of the textbook: the students are working individually or in groups with exercises from the textbook; the teacher makes, explicitly, comments about a text, a problem, or a picture in the textbook; or the teacher reads directly from the textbook.

b) Textbook indirect. The teacher makes explicit verbal or written statements that are parallel and comparable with the text in the textbook without referring to it. Examples: the teacher explains a graph without declaring that there is a similar graph in the book; the teacher shows a worked example on the board that is similar or exactly as an example in the textbook; the teacher talks about mathematical aspects (generalization/statements) in the same way as the textbook.

c) Textbook absence. There is a clear difference between how the teacher introduces, explains, draws, or comments on a mathematical subject and how it is presented in the textbook. Examples: the teacher makes connection to other mathematical areas but the textbook does not show this link; the teacher makes connections to every day life or applications, which are not in the textbook.

Occurrence codes – teacher activity

Occurrence codes are used in order to highlight *how many times* and *where* a specific event occurs within a particular lesson. Since the focus is on the teacher it is the activity of the teacher that counts. An occurrence code can be applied several times within a lesson but it can also be the case that there is no event to apply to. *Teacher activity* includes eight occurrence codes for all parts of the lessons that are coded as *Mathematical work*. These categories are marked each time they occur. Figure 2 shows the occurrence codes.

If the code *Problems and tasks* is marked, it means that there is an interaction, public or private, between the teacher and the students. The discussion concerns a mathematical problem, for example a task in the textbook.

There are two conditions that must be evident for an occurrence to be coded as *Mathematical generalizations*. There must be generalized mathematical information and there must be an explicit attempt to point out

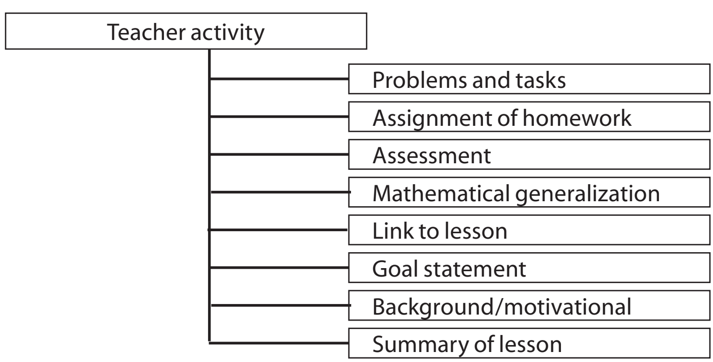

Figure 2. *Occurrence codes*

the generality (for further details see Jacobs et al., 2003). Mathematical generalizations are marked for each explicit statement, verbally or written. An example is when the teacher says that "the angles of a square always add up to 360 degrees". Exception: if the statement is made during an individual guidance as a support in a problem-solving sequence and the generality is a part of the solution strategy.

The code *Link to lesson* is marked whenever the teacher explicitly refers to particular mathematical ideas discussed or worked on within the current or a different lesson. The link should help students organize related information. For example, "now we are working with expressions and formulae and since you learn about sound in Physics this fall, we shall look at the formula of the sound-wave".

Goal statements are explicit verbal or written statements made by the teacher about the mathematical topic, which will be covered in the specific lesson.

The code *Background/motivational* is marked when the teacher connects the mathematical content to its historical background (e.g., Pythagoras as the originator of a mathematical theorem) or to its practical use in or outside the school context.

Methodological discussion

Since this study is based on the data of the CULT-project, there are some issues that have to be clarified. First of all, I am not a member of the research team and I had no part in the planning of the project or the data gathering. Following the guiding principle for the project, for example protecting the anonymity of the participants, I had permission to use the data for this study. However, the responsibility for the methodological approach (the coding procedure), the results from the analysis, and the conclusions presented in this paper are mine.

Coding reliability is measured by percentage agreement[9]. A re-coding of one of the thirteen lessons was made one year after it was coded for the first time. In both cases the coding was made by the author. The reliability score is calculated by dividing the number of agreements by the number of agreements plus disagreements. All codes included, the score is 95.8 %.

In this paper, most of the transcripts, and all translations from Swedish to English of transcripts, are made by the author. For all possible errors, the author takes full responsibility. One remark, however, is in place: The text should be seen as a report of the conversation in the classroom. Thus, making an allowance for readability, it is not a word for word description of what the teacher and the students say in the classroom.

The organization of the lessons

In this section of the paper, the teachers' way to organize their teaching in terms of type of classroom interaction, organization of students and content activity will be discussed. In the analysis of the thirteen video-recorded lessons, one can observe that there are in principle two types of interaction in the classrooms, private and public. A lesson normally starts with a public part. The teacher stands in front of the class. He writes on the board, presents problems, poses questions and verifies or disproves answers.

In other parts of a lesson, the students, at least most of them, are engaged in 'practice'. This means that they are working, mainly on an individual basis, with tasks in the textbook. The teacher walks around the classroom, he observes and interacts with the students (*Private interaction*).

A third type of interaction, *Mixed interaction*, is observed in Mr. Svensson's lessons. These are occasions when a student stands in front of the class, writing a solution on the board, and any other student participation is optional.

In the three teachers' classrooms, private interaction is more common than public interaction [10]. Figure 3 displays the distribution in all thirteen lessons and shows that the teachers' way to interact with the students varies. Mr. Andersson's lessons SW1-L10 and SW1-L11, for example, show notable differences. About eight percent of SW1-L10 is devoted to public interaction in comparison to about fifty-five percent of SW1-L11.

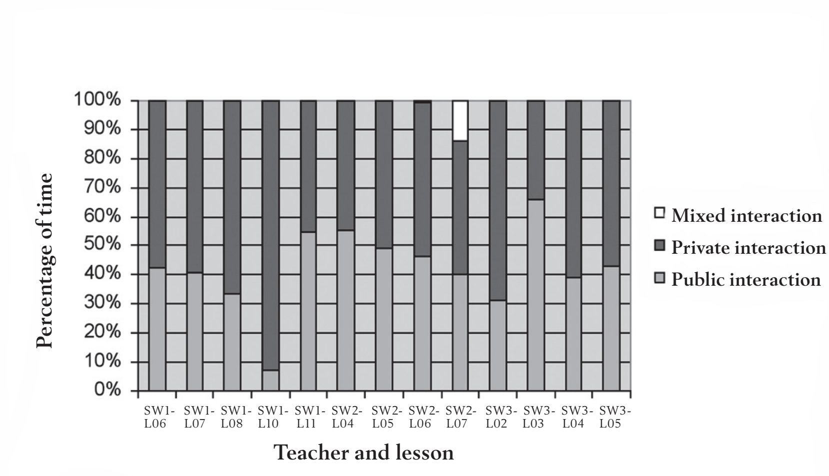

Figure 3. *Classroom interaction in all thirteen lessons.*

In all three classrooms, most of the students are working by themselves during *Private interaction* time. There were no occasions of group work throughout the thirteen lessons in this study. Some of the students seem, however, to work in pairs from time to time, but this is optional and not incited by the teacher.

With regards to *Content activity*, Table 1 displays the percentage of time devoted to mathematical work, mathematical organization, and non-mathematical work for each lesson. From the picture, we can read that the three teachers spend most of their lesson time on mathematical work and that there are no big differences between them.

Table 1. *Content activity, percentage of total time of lessons*

Content activity	Mr. Andersson (SW1)	Mr. Svensson (SW2)	Mr. Larsson (SW3)
Non mathematical	17,93 %	10,77 %	9,62 %
Mathematical organization	5,44 %	2,29 %	4,39 %
Mathematical work	76,62 %	86,94 %	85,99 %

Mr. Andersson seems to use more time on non-mathematical work than the other two teachers. From the video-recordings of his lessons, one finds that he more often is involved in disciplinary discussions with students but also conversations that can be regarded as 'from one friend to another'.

The use of textbooks

In all lessons, there is an extensive use of textbooks, especially if one looks at the *Private interaction* part of the lessons. The textbooks are in direct use about sixty percent of the time. For Mr. Andersson and Mr. Svensson, the textbook is definitely the main resource. There are very few occasions when the textbook influence seems to be 'absent' [11]. For Mr. Larsson, the picture is slightly different. He uses other sources than the textbook in about one fifth (18.40 %) of the time. Table 2 shows the average distribution of *Textbook influence* for the three teachers, measured during lesson periods of *Mathematical work* and *Mathematical organizations*.

In all three classrooms, there were few occasions of non-textbook tasks and the mathematical content is to a large extent introduced and

Table 2. *Textbook influence, percentage of total time of lessons* *

Textbook influence	Mr. Andersson (SW1)	Mr. Svensson (SW2)	Mr. Larsson (SW3)
Textbook direct	57,55 %	61,67 %	57,54 %
Textbook indirect	23,92 %	24,41 %	14,44 %
Textbook absence	0,60 %	3,15 %	18,40 %

* *Note*. Non-mathematical work is excluded

elaborated via the textbooks. But what kind of mathematical activity is offered when the teacher teaches without the textbook? The following example is from a lesson of Mr. Larsson (SW3-L03).

Example of a non-textbook task

The teacher presents an information sheet from the municipalities. It shows the price of water and drainage supply and the formula $A \cdot x + B \cdot \sqrt{x}$ describes the charge.

On the board, the teacher gives three examples of costs for water and drainage supply for three different types of consumers, a lonely person that consumes little amount of water, a lonely person that consume a larger amount of water, and a family. The teacher suggests that they could compare the costs. First the teacher asks how many times bigger the consumption is for the person that consumes a hundred cubic meters of water in comparison to the person that uses twenty-five. One of the students says it will be four times bigger. Thereafter, the teacher asks about how many times bigger the fee will be. Three times, another student says. "What should the charge be, if it would be really fair?", the teacher asks and one of the students suggests that it should have been 2,000 kr instead of 1,500 kr.

In this part of the lesson, which is coded as *Public interaction*, *Mathematical work* and *Textbook absence*, the teacher uses a resource that is regarded as a real world object. It is a task derived from an information sheet, which the teacher solves together with the students. The episode entails a repetition of some known concepts, square root and cubic meter, but also a concrete example of how constants and variables work in an equation and affect the results. There is a task in the textbook (see Figure 4), which in some sense concerns the same topic. The differences between these two

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The cost for water supply in a private house per year can be calculated with the formula

$K = 295 + 16x + 6.5y$

K = the cost in kronor

x = the amount of consumed warm water in cubic meter (m^3)

y = the amount of consumed cold water in cubic meter

a) How much is the cost for a family in one year when the family consumes 60 m^3 warm water and 200 m^3 cold water?

b) What do the numbers 16 and 6.5 stand for in the formula?

Figure 4. *A textbook task (Undvall et al., 2002, p.219, translation by the author)*.

tasks are left to the reader to think about. I could offer one inference, and that is that the textbook task is less related to the use of mathematics in the 'real world'.

The role of the textbooks

The activity of the teachers is indicated by means of occurrence codes. Thus, the data from the coding procedure describes number of occasions when a certain activity is noted. The role of the textbooks, in the activity of the teachers, will be discussed in this part of the paper. First, how the teachers arrange for mathematical work, and thereafter how they highlight key-ideas.

To arrange for mathematical work

In this study, as well as in the TIMSS Video Study (Hiebert et al., 2003), teachers and students spend a considerable part of each mathematics lesson solving problems or tasks. But mathematical work can also involve assessment and homework. This part of the paper reports how the teachers arrange for mathematical work and the way textbooks are incorporated.

Problems and tasks are offered in two different settings: a) the teacher (or sometimes a student) stands at the board in front of the class (*Public interaction*), and b) the teacher walks around in the classroom and gives individual assistance (*Private interaction*). In the latter type of interaction, the problems and the tasks origin from one exclusive source: the textbook. In the *Public interaction* part of the lesson there are a few occasions, less than one percent, when the task seems to be derived from a different source (see the previous section for an example of a non-textbook task).

Considering the number of occasions that involve problems and tasks, Mr. Andersson seems to be most busy among the three teachers in this study, at least in the first lesson (SW1-L06). During individual work, when the students are solving tasks in the textbook, he assists their problem solving processes thirty-three times in a period of thirty-four minutes. Figure 5 displays, however, that this varies between lessons.

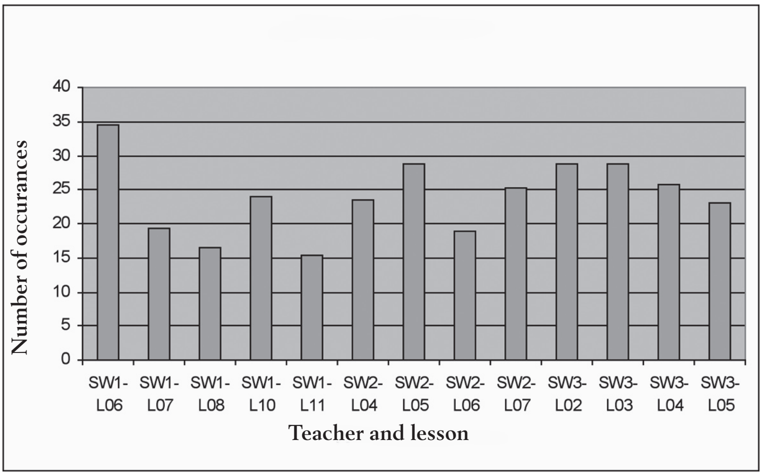

Figure 5. *Problems and tasks*[12]

The following transcript serves as an example of a common type of interaction in a *Problems and tasks* situation. In this case, the teacher, Mr. Larsson, funnels[13] the student until he reaches the solution. According to the options of the four coverage codes, the episode is coded as *Private interaction*, *Individual work*, *Mathematical work* and *Textbook direct*. The task, which the student is trying to solve, is as follows:

5033
x is an odd number, any one. Write an expression for the two consecutive numbers.

The solution is presented in the answer key: $x + 2$ and $x + 4$

Student: (...) odd numbers
Teacher: Odd numbers, yes. Any one, yes ... two consecutive. If you think of an odd number, for example eleven, what is the next odd number then?
Student: Thirteen
Teacher: Yes. How do you get eleven then ... you add ... ?
Student: Plus two
Teacher: Yes, and then the next number ... how much should you add then do you think?
Student: Two more
Teacher: Yes of course
Student: Yes, okay

In the thirteen lessons included in this study, no time is spent on assessments or diagnostic tests. This does not mean that there are no occasions of assessment in these classrooms. First of all, one has to consider the kind of indefinable assessment that could be a part of the teachers' performance in the teacher-student interaction. Secondly, Mr. Andersson uses one whole lesson, SW1-L09 in the CULT-study, for a test. For this study, however, this lesson was excluded, mainly because there was not much teacher activity.

There are two occasions when the students get homework. One occazsion occurs in a lesson of Mr. Andersson and one occasion occurs in a lesson of Mr. Larsson. Both times, the assignments concern tasks in the textbooks. Mr. Andersson tells the students that if they have not completed the tasks till page number 177, they should work with it at home. Mr. Larsson writes *Homework: 5090–5096* on the board[14].

To highlight key ideas

How can a teacher help students to identify key mathematical points in a lesson? One way to do it, is of course to offer verbal or written mathematical generalizations or statements. The teacher could also choose to explain the goals for the current lesson or connect to a different lesson, in mathematics or another school subject. Key ideas *about* mathematics could also be acknowledged. For example when the teacher talks about the historical background of a mathematical concept or the use of a specific mathematical knowledge in everyday life, the society or within school. This is emphasized in the mathematics syllabus for compulsory school.

> Mathematics is an important part of our culture and the education should give pupils an insight into the subject's historical development, its importance and role in our society (Skolverket, 2001, p. 23).

In this subsection of the paper, I will try to illustrate the three teachers' ways to highlight key ideas and how the textbook is involved, or not involved, in this.

In all thirteen lessons, 119 occasions of *Mathematical generalizations or statements* have been identified. Most of them, eighty-one, are found in the *Public interaction* part of the lessons. Only nineteen of them are recognized as different from the textbook. This implies that the textbooks, to a large extent, influence *how* mathematics is presented in terms of mathematical procedures and concepts.

Figure 6 shows how often the code is marked in each lesson. Mr. Andersson seems to offer mathematical generalizations more often than the other two teachers, especially if one considers the first lesson. One plausible reason could be that the specific chapter, which is about 'relationship' (i.e. coordinate system, diagram, and linear relationships), involves many, for the students, new concepts and terms. The subjects in the other teachers' lessons are partly new and partly repetition for the students.

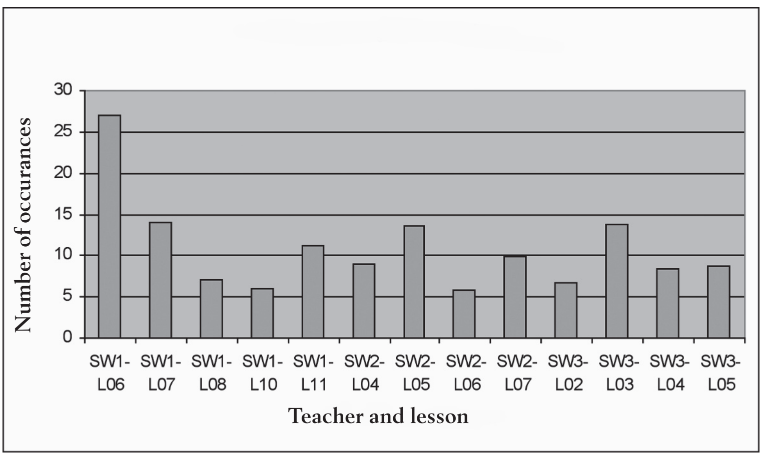

Figure 6. *Mathematical generalizations or statements, number of occasions.* [12]

Mathematics is a subject that can be related to other school subjects such as Physics, Chemistry, and Social science. Links can also be made between different topics within mathematics. *Link to lesson* is marked each time the teachers make connections to other lessons, in mathematics or other school subjects. Fourteen occasions are recognized in all thirteen lessons. Some of them are rather vague links to previous or future lessons and some of them refer to Physics lessons.

The following example, which includes a task that deviates from the text-book, illustrates a rather clear link to Physics. It is an episode in one of Mr. Larsson's lessons (SW3-L04) and it is about sound-waves.

> **Example of a link to a lesson in Physics**
>
> The teacher starts by explaining that sound is a wave movement and that there is a formula connected to this. He writes $\lambda = v/f$ on the board. After a discussion about the different parts of the formula, he holds up a tuning-fork and hits it with a pen. This particular tuning-fork has the frequency 1700 Hertz, he explains. In cooperation with the students, the teacher calculates the wavelength to twenty centimeters.

A difference between the teachers in this study concerning non-text-book work (see Table 2), which is discussed in the previous section in this paper, could be explained by the fact that Mr. Larsson can rely on his knowledge as a teacher in Physics when he presents examples. As regards to the current topic in his lessons, which is about formulae and equations, this is especially suitable. However, Mr. Larsson makes a choice that could be discussed. In this study, he always presents these examples in the *Public interaction* part of the lesson and not as tasks for the students to work with, individually or in groups. It would certainly be a different experience if the students, by themselves, work with this type of activity. This would require a move from the traditional mathematics education, "the exercise paradigm", towards an investigative approach and, hopefully, make the intentions of the students become the driving elements in the learning process (see Skovsmose, 2001).

Concerning *Goal statements*, there is also a difference between the teachers in this study. Mr. Svensson states the goal for each lesson [15], Mr. Andersson and Mr. Larsson talks about lesson goals in two (of five) and

Goal
After you have studied this chapter, you should be able to:

- draw and identify points in a coordinate system
- use proportional relationships, i.e. cost-per-unit prices
- use relationships that consists of a fixed and a flexible part
- interpret different types of linear relationship

Figure 7. *Goal statement in the textbook* (Carlsson et al., 2002, p.171, translation by the author).

one (of four) lessons, respectively. There are three types of goal statements, which emphasizes: (a) the subject: "today we are going to work with … which is about …", (b) the target: "this work will lead to …", or (c) what is most important.

Goal statements can also be found in the textbooks, often in the beginning of a chapter. They are of different kind and more or less explicit. In the textbook that Mr. Andersson uses, the text, which describes the goals, is placed in a framed textbox on the first opening of the chapter.

The example in Figure 7 shows the kind of learning goals that the authors had in mind when they made this particular chapter. There are goal statements in the other two textbooks as well, not framed in textboxes though. For example:

> In this introductory part, we repeat how to solve equations. In the next part, you practice your skills on different problems (Undvall, Olofsson, & Forsberg, 2003, p. 96, translation by the author).

What is this good for? How can we make use of this? Mathematics is a school subject that we sometimes take for granted. However, some students, for example Beata in Mr. Andersson's class, seem to need more motivation and justifications than others. In the first lesson, when Mr. Andersson introduces a new topic (the chapter in the textbook starts with an introduction to coordinate systems), Beata asks: "why do we have to know this?". "Well, first of all, coordinate system is very useful if you are going to New York", the teacher says. He explains that the streets are arranged like a coordinate system. Beata, and some other students too, seem to be a bit doubtful. "You don't need to learn a whole system for that?", she says. It seems like a reasonable comment, it really surprised me that the teacher gives such an example. However, a look into the textbook, which shows a map of New York on the first page of the chapter, reveals the likely source of inspiration.

In order to convince his students, Mr. Andersson offers more examples of usefulness. He says that they are going to draw curves and do estimates of costs. Later in the lesson, Beata calls for the attention from the teacher.

Beata:	Well, why do I have to do this when there is probably a computer somewhere and a program that can calculate this rather easy
Teacher:	Yes, but why should you do anything at all, Beata?
Beata:	But …
Teacher:	But this, I mean …
Beata:	Yes, I want to do funny stuff (laughing)

Teacher: Yes, but perhaps a computer can do that for you, too ... we don't have to do so much

Beata: No, because these ... it feels so meaningless if I don't know how I can use it

Three lessons later (SW1-L10), Beata is still not convinced about the usefulness of the current topic. She calls for the attention of the teacher again and in the three minutes that follow; Mr. Andersson tries to persuade her and makes her keep on working with the textbook tasks.

The other two teachers seem not to have the same type of motivational discussions with their students. When Mr. Svensson justifies a subject or suggests aims for learning a certain topic, he often refers to future work in the textbook. "This group of students accepts the somehow poor explanation that it will be useful for them later on", says Mr. Svensson.

Mr. Larsson often offers motives and background implicitly when he presents examples on the board. For instance when he, at the beginning of one of his lessons (SW2-L02), writes $E = mc^2$ on the board and asks the students if they recognize the formula. According to the options of the coverage codes, the episode is coded as *Public interaction*, *Mathematical work* and *Textbook absence* (the occurrence code is *Background motivational*). However, the textbook might have inspired the teacher. On page 217 one can read, "Formulae are often used within Science. One example is Einstein's famous formula $E = mc^2$". But, since Mr. Larsson elaborates and discusses this further, this episode is not regarded as influenced by the textbook.

Another kind of support for students' recognition of key ideas in a lesson is a summary statement. This is when the teacher, near the end of the lesson, highlights points that the students have been studying. In this study, there were no occasions at all that could be regarded as lesson summaries[16].

Main results

The main purpose of the study is not to compare the three teachers' teaching methods or to make generalizations about mathematics teaching in Sweden. For this particular study, it would not even be fair to make such comparisons between the three teachers since one of them, Mr. Andersson, works under different condition than the other two. He is less experienced as a teacher and teaches a mixed ability group of students. Furthermore, it is not a matter of proposing criticism towards teachers or textbooks. The intention is rather to analyze some mathematics classroom in order to reveal teachers' practices and relations to textbooks, which hopefully will stimulate discussions about choices.

An underlying assumption in the TIMSS Video Study is that there exists a culturally-based 'lesson script' (cf. Clarke & Mesiti, 2003). In this study of three teachers' way to organize their lessons, a definite 'script' is not recognized. However, with regards to the use of textbooks, there are some observable patterns. What is noticeable is that the textbooks to a large degree, guide teaching in these three classrooms:

a) Students are exclusively working with tasks in the textbook during individual work, which on average is more than half the time of a lesson.

b) In the *Public interaction* part of the lesson, the examples and the tasks that the teachers present are mainly from the textbook. An exception is the teacher Mr. Larsson who uses his experiences as a Physics teacher in some of the examples on the board.

c) The way that mathematics, as a scientific discipline, is presented is comparable with the approach in the textbook. A hundred of totally 119 occasions of *Mathematical generalizations or statements* are coded as comparable or the same as in the textbook. In principal, this means that hardly any other definitions, conventions, or rules than the textbook offers are presented to the students. It also means that the mathematical procedures, for example how to solve an equation, and how the structural features of mathematics are portrayed, are mainly the same as in the textbook. To the students, this means that mathematics appears to be a static subject, formulated by an external authority, and not a subject to explore.

d) Two of the teachers, Mr. Andersson and Mr. Svensson, use their textbooks as the main sources for background and motivational discussions.

e) Homework is not assigned on a regular basis. However, when the teachers do give assignments, students are supposed to work with tasks from the textbooks.

Besides these results of quantitative nature, there are some aspects of the three teachers' teaching that I would like to highlight as well. It is 'uniqueness' of each teacher's teaching style.

Mr. Andersson is teaching a mixed-ability group of students, which means that the students are working at a different pace. As a consequence, after just a few lessons of a new chapter, the class is spread out in terms of the tasks in the textbook. Mr. Andersson seems to be the busiest teacher among the three teachers in this study, at least when the students are working individually. In one of his lessons, he assists students' problem solving processes thirty-three times in a period of thirty-four minutes.

Mr. Svensson sequences his lesson differently than the other two teachers; he alters between types of classroom interaction several times in his lessons. In this case, it means that each student in his class is probably working with the same task as all the other students throughout the lesson. Other differences between Mr. Svensson and the other two teachers are that he sometimes chooses to let a student write the solution on the board and that he presents the goal each lesson.

Mr. Larsson uses supplementary sources more often than the other two teachers when he presents examples on the board. Instead of taking the examples from the textbook, he often relies on his knowledge as a teacher in Physics. In this study, he always presents these examples in the public part of the lesson and not as tasks for the students to work with, individually or in groups.

Discussion

A fair question to ask in the discussion about the results from the study is: *Does it matter if the teacher (heavily) relies on the textbook when teaching mathematics*? First of all, we need to keep in mind that the textbook facilitates the daily work of the teacher. It is not rational or even realistic to just expect that the teachers' dependence on textbooks will be reduced without good reasons. Further advantageous is that, within the school system, the textbooks serve as some kind of agreement and support for the uniformity. A textbook is often organized in such a way that it covers the topics that students should encounter during a particular school year. Thus, teachers can defend their decision to follow the textbook closely by arguing that it prevents them from skipping important topics or teaching topics out of an appropriate sequence (Freeman & Porter, 1989). Moreover, in many textbooks, at least in Sweden, the tasks are graded by level of difficulty. This means that the students can work individually and hopefully be challenged at their own level (see Brändström, 2005, p. 71). If these arguments are convincing enough, we should not be worried. Thus, the answer to the question would be: *No, it does not matter that the teacher relies on the textbook when teaching mathematics*.

If we, on the other hand, find it problematic and believe that *it does matter that textbooks have such an influence on mathematics teaching*, we need to discuss some options. Considering, for example, the discussion between the student Beata and her teacher about monotonous work with tasks in the textbook, one could argue that the textbook fails to encourage the student's joy to learn (cf. Lindqvist et al., 2003). One could also contrast the textbook task about the water consumption in a household (Figure 7) with the example that Mr. Larsson brought up on the same topic using an information sheet from the municipality. This could be a starting point in a discussion of textbook tasks from the perspective of their richness and relatedness to out-of-school 'reality' (for a detailed study on this see Palm, 2002).

Another issue concerns individualized teaching, which is emphasized in the national curriculum in Sweden. This means that teaching should be adjusted to each student's ability and needs. In a mixed-ability group of students, such as Mr. Andersson's class, this could be more or less complicated. However, if the tasks are graded by level of difficulty, as in many Swedish textbooks, one can think of them as proper tools to accomplish individualized teaching and let the students work on their own level and pace. To some extent, this explains why much of the activity in many Swedish classrooms consists of 'silent calculation' in the textbook. But if the students are working individually and with different tasks, just as Mr. Andersson's students do, how could you nurture mathematical discussions? Furthermore, it is also important to question if: a) a certain textbook offers something for *each* student's ability [17] and needs, b) it is appropriate for all students to learn mathematics by themselves through a textbook (and with some help of the teacher of course), and c) if it is possible to fulfill the national objectives for mathematics education by means of a textbook.

A final comment may raise the question if there are any differences between the school subject Mathematics and other school subjects, i.e. *Is a study of the influence of textbooks more relevant in research in mathematics education than in educational research in general*? I believe it is, but to prove that the assumption is correct is out of the scope of this study. Nevertheless, Sosniak and Stodolsky (1993) noticed that none of the teachers in their study used textbooks in the same way when teaching different subjects. One of the teacher reports that the textbook for the reading program serve as "food for thoughts" but the mathematics textbook frees her from "having to do much thinking at all about her mathematics program" (Sosniak & Stodolsky, 1993, p. 260).

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Notes

1 This paper is a shortened and revised version of an article in my thesis (see Johansson, 2006)

2 Textbooks are in this study defined as "fairly large and printed objects, which intend to guide students' work throughout the year" (Johansson, 2003, p. 20)

3 The frame factor theory, or model, origins from work of the Swedish educationalist U. Dahllöf and his colleagues in the 1960s. In its early stage, it focus on how political decisions regarding teaching and education (e.g. time schedules, grouping, etc.) influenced the pedagogical work (Lundgren, 1998).

4 For a comprehensive description of the methodological and technological design in the CULT-study, see Häggblom (2005). Information about the study can also be found on: http://www.ped.uu.se/kult/default.asp. The fieldwork and the data collection in the CULT-project is based on the research design set out for the Learner's Perspective Study (http://extranet.edfac.unimelb.edu.au/DSME/lps/).

5 All names in this paper are fictitious.

6 The teacher, Mr. Andersson, uses the textbook Matte Direkt, år 8 (Carlsson, Hake, & Öberg, 2002).

7 The textbook, which is Matematikboken Z Röd (Undvall, Olofsson, & Forsberg, 1997), differentiate the tasks. A-tasks require the lowest demands.

8 The textbook is Matematikboken Y Röd (Undvall, Olofsson, & Forsberg, 2002). According to the authors, it is intended to be used by students who are interested and have good skills in mathematics.

9 The method was also used in the TIMSS Video Study. The minimum acceptable reliability score for an individual coder was 85 %.

10 In the TIMSS Video Study, the teachers from different countries divided their time between public or private interaction differently. However, apart from two countries, a greater percentage of lesson time was spent in public interaction. In Australia, there was no detectable difference between time on public and private interaction. In the Netherlands, fifty-five percent of lesson time was spent in private interaction (Hiebert et al., 2003).

11 Since 'absent' in this case means that one cannot notice an influence of the current textbook, influence of another textbook cannot be excluded.

12 Because of different length of the lessons, the number of occurrences is based on 45 minutes lessons.

13 The 'funnel pattern of interaction' is a well-known activity in many mathematics classrooms – an activity where the teacher provides individual guidance through a step-by-step reduction of the demands (cf. Alrø & Skovsmose, 2002; Bauersfeld, 1988).

14 In the TIMSS Video Study, the teachers in all seven countries, except Japan, assign homework in at least 57 percent of the lessons (Hiebert et al., 2003).

15 How frequently the teachers state the goal of a lesson differs between the countries in the TIMSS Video Study. The teachers from the Czech Republic, for example, do so in almost every lesson (91 % of the lessons) but the teachers from the Netherlands state the goals in only 21 % of the lessons (Hiebert et al., 2003).

16 In the TIMSS Video Study, lesson summaries, which are less common than goal statements, were found in at least twenty-one percent of the lessons in Japan, the Czech Republic, and Hong Kong SAR, and in ten percent of lessons in Australia (Hiebert et al., 2003).

17 Brändström (2005) analyzed tasks in mathematics textbooks in order to reveal their level of difficulty and how they are differentiated. She found that though textbooks tasks are offered on different levels, ”the processes and required demands are too low” (p. 75) on all levels.

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Sammandrag

Den här artikeln handlar om hur tre lärare organiserar sina lektioner och på vilket sätt läroboken är integrerad i deras arbete. Trots skillnader när det gäller lärarnas undervisningserfarenhet och elevgruppernas sammansättning visar det sig att läroboken bestämmer undervisningens innehåll i mycket stor utsträckning. Läroboken används som källa: (a) då eleverna arbetar individuellt, (b) i många exempel som läraren visar på tavlan, (c) för diskussioner om bakgrund och syfte, (d) för hur matematiken presenteras och (e) för hemläxa. Huruvida lärobokens styrande roll i matematikundervisningen ska betraktas som ett problem eller inte diskuteras. Å ena sidan är det inte rimligt att förvänta sig att lärare ska frångå boken utan goda skäl – läroboken är ett hjälpmedel som förenklar deras dagliga arbete. Dessutom kan den ses som ett stöd för progressionen och likformigheten i elevers väg genom skolsystemet. Å andra sidan kan innehållet i läroböckerna diskuteras och ifrågasättas – till exempel hur rika och verklighetsnära uppgifterna är samt hur väl de är anpassade till varje elevs behov. Viktiga frågor i detta sammanhang är om (a) läroboken erbjuder utmaningar för varje elevs förmåga och behov, (b) elever kan lära sig matematik på egen hand (med viss hjälp från läraren) och (c) om det är möjligt att uppnå styrdokumentens strävansmål när det gäller matematik via en lärobok.