Exploring young children's geometrical strategies

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This study explores young children's strategies while transforming polygons, through the use of geometrical models. Data were collected from 291 children ranging from 4 to 8 years of age in Cyprus. Children were asked to draw a stairway of specific polygons, with each shape being bigger or smaller than its preceding one. Relationships between children's responses in the transformation tasks, their ability to recognize geometric shapes and their IQ level were investigated. Results showed that children used three alternative strategies in the transformation tasks. Children's IQ score was directly associated with their transformation strategies, while only a low recognition ability was associated with the use of a defective strategy.

In this study, two dominant lines of inquiry, based on the theory of van Hiele and the use of geometric models, are taken into account. In van Hiele's theory, geometric thinking is developed through hierarchically ordered levels. Based on the notion of geometric models, an investigation of children's dynamic intuition in transformation tasks and the ideas they develop about geometric figures is possible (Gagatsis & Patronis, 1990). Previous foundational research on children's geometric conceptions (e.g. Clements et al.,1999; Hasegawa, 1997; Mesquita, 1998; van Hiele, 1999; Warren & English, 1995) has informed our line of inquiry. We add to the current understanding of children's geometric thinking, by integrating previous directions of research and by exploring young children's strategies in geometric transformation tasks, their relationship with the ability to recognize shapes and IQ level.

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Theoretical framework

The development of children's geometric thinking

Van Hiele's theory maintains that children are not initially competent in recognizing components and properties of familiar shapes (Hannibal, 1999). Therefore, young children who conceive a shape as a whole and not as a sum of its parts and identify shapes according to their appearance, recognizing them as visual gestalts by using visual prototypes, are at the visual level (Hannibal, 1999; Patronis, 2001). Children at the visual level are not in a position to identify many common shapes or distinguish among figures in the same class or, for example, include the concept of square into the concept of rectangle (Gagatsis & Patronis, 1990). At the second level, the descriptive one, a figure is no longer judged by its appearance, but rather through certain properties. At this level language is important for describing shapes. At level 3, informal deduction, properties are logically ordered. Therefore, they are deduced from one another. For example, children are able to formulate definitions for squares and rectangles and use them to justify relationships, such as explaining why all squares are rectangles.

Clements et al. (1999) conducted a research that investigated the criteria preschool children use to distinguish members of a class of geometric shapes from other figures. Findings showed that a prerecognitive level exists before the visual level. At this level children perceive shapes, but are not able to identify or distinguish them among others (Clements & Sarama, 2000; Clements et al. 1999). The van Hiele's visual level was reconceptualized by Clements et al. (1999) as ”syncretic” representing a synthesis of verbal declarative and imagistic knowledge, each interacting with the other. Children at the syncretic level develop strong imagistic prototypes and gradually gain verbal declarative knowledge. As for the descriptive level, the particular study concurs with van Hiele's theory, that children recognize and can characterize shapes by their properties.

Geometrical models, polygonal shapes and dynamic intuition

Several authors have examined the meaning and value of models in mathematics education. Fischbein (1972) considered the ”generative” function of models used in teaching as the most important one for intellectual development, which allows the construction or representation of an unlimited number of situations using a limited number of elements and rules of combinations. Furthermore, Fischbein (1972) emphasized the heuristic value of such a model for pedagogical as well as for scientific use. Examples of ”generative” models are the tree-like diagrams used in combinatorics, Euler-Venn diagrams and geometrical models. Gagatsis and Patronis (1990) give an intuitive definition of a geometrical model. They argue that a geometrical object of Σ is a collection S of points, lines or other figures in n-dimensional Euclidean space, representing a system Σ of objects or a situation or process, if the intrinsic geometric properties of the elements of S are all relevant in this representation, i.e., they correspond to properties of the system Σ. If this condition is satisfied only for the topological properties of lines or figures in S, then we shall speak of a geometrical model in the wide (or topological) sense.

A "polygon", according to Gagatsis and Patronis (1990), is a convex polygon, i.e., the convex hull of a finite set of points in the plane. In general, a "polytope" is the convex hull of any finite set of points in n-dimensional Euclidean space E^n. As for the dimensions of polytopes, 0-polytopes are points; 1-polytopes are line segments; and 2-polytopes are (convex) polygons. In this study, we were not interested in a particular polytope as a specific set of points in space, but rather in all polytopes similar to it. Similarity is an equivalence relation in the set of polytopes. In other words, the "shape" of a polytope P is its similarity class, i.e., the set of all polytopes similar to P.

Robertson (1984) states that in the space of all similarity classes of n-polytopes, given any two shapes with n>0, there is always a path which joins the two shapes. This means that any shape of polytopes can be continuously deformed into any other with the exception of 0-polytope and that a sequence of polytopes converges to a polytope of a lower dimension.

Such a kind of intuitive thinking, which involves a continuous variation process, is called "dynamic intuition" in contrast to the "static" situation of stable, non-varying figures and representations (Castelnuovo, 1972). Students visualize movement as they make connections between shapes (Owens, 1999). For example, dynamic visualization takes place when a student constructs a square that becomes a rectangle, as it gets thinner. Furthermore, Castelnuovo (1972) suggests that children do not easily observe figures and their shapes when they are steady, but rather when they move or vary in a continuous manner. If we overlook the magnitude of the sides of the changing isosceles triangle of Figure 1 and focus only on its continuously varying shape, then the geometrical model of a continuous variation of shape is attained. In Figure 1, we pass continuously from a vertical line segment, which is the limit case of a 1-polytope, to a horizontal line segment, which is another limit case. The triangle appears naturally in the continuous process.

A continuous variation process is a result of transformational reasoning. Markopoulos and Potari (2001) examined students' behaviour

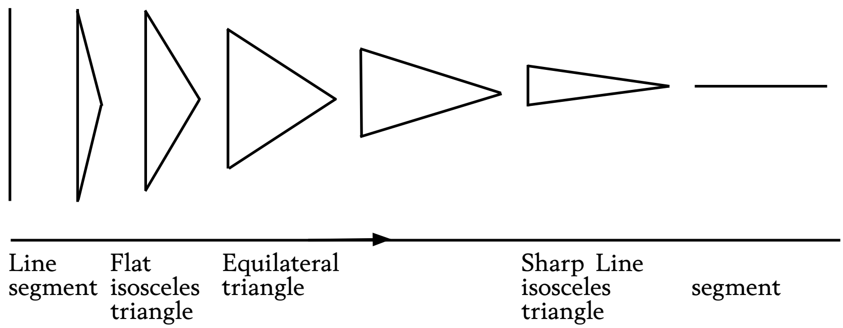

Figure 1. *A continuous variation of a triangle*

towards transformation tasks in the case of geometrical solids. In fact, they considered the dynamic transformation of geometrical solids, (which, in our opinion, can be adopted for plane figures as well), as a process where the solid (or plane figure) changes its form through the variation of some of its elements and the conservation of others. Two types of transformation have been distinguished among others: similarity and "parallel" transformations. In the first type of transformation, students kept the form of the figures constant; figures were transformed by similarity. This kind of transformation, which acted as an enlargement, or a reduction of the size of the figure, is the only one possible at the monadic stage in the development of the concept formation of n-gons, corresponding to van Hiele's visual level (Hasegawa, 1997). In the second type of transformation, students conserved all components of the figures parallel, but they did not preserve the ratio of the lengths of their sides.

Previously, Gagatsis and Patronis (1990) investigated how geometrical models could be used in learning and teaching mathematics in connection with the development of reflective thinking. More recently Elia and Gagatsis (2003) explored children's ability to transform polygonal shapes and focused on two models of action. The first model was a constant path of figures, which corresponded to invariant geometrical forms under similarity transformations. The second model represented a continuous path, which involved a topological deformation of geometrical figures, by conserving the parallels, but not the exact form of the figure. The first model of action corresponded to a primary intuitive case of mental operations in the sense that awareness of data was based on one's senses and primary conceptions and intuitions, while the second model was closer to reflective functioning (Gagatsis & Patronis, 1990).

Intelligence and mathematics achievement

IQ is a single number which defines intelligence and represents "a cohort specific index comparing the performance of a group of individuals of the same age on a battery of sub-tests designed to assess different intellectual skills" (Brody, 1999, p.19). Measuring IQ before starting formal education is predictive of the acquisition of knowledge in school (Cronbach & Snow, 1977). General intelligence is not easily changed and affects the way in which an individual responds to the environmental events. Thus, intelligence influences educational achievement, in the sense that children who differ in intelligence vary in their academic performance (Brody, 1999). Jensen (1998) maintains that IQ is the best single predictor of academic performance and many other outcomes. In other investigations, Dark and Benbow (1990) showed that high IQ adolescents with mathematical talent had a developed ability to use numerical and spatial information in working memory, in relation to their average-IQ peers with a verbal talent. On the other hand, Hoard, Geary and Hamson (1999) demonstrated that low-IQ first graders exhibited specific features of arithmetic ability that discriminated them from their average-IQ peers. In particular, low-IQ children showed particular deficits in number naming, writing, comparisons and greater difficulties in their strategies for solving simple addition problems than their average-IQ peers. Many of these differences could be explained in terms of working-memory discrepancies.

In the present study, we offer a new and powerful perspective on investigating children's construction of geometric and spatial ideas. Our work integrates the main concerns of the aforementioned research studies in this area and extends their findings. We theorize that a "good understanding" has two stages: the passive (and easier) one, such as classifying, identifying, and the active (and more difficult) one, such as doing something (Markovits, Eylon & Bruckheimer, 1986). Therefore, the present study does not examine the processes by which children identify specific shapes (polygons) (passive stage), (e.g., Clements et al., 1999) work, or how children construct shapes and use geometric transformations (active stage) (e.g., Elia and Gagatsis, 2003; Gagatsis and Patronis, 1990). It explores the relationship between the passive stage in the understanding of geometric shapes, i.e., recognition of shapes among others, and the active stage in geometric understanding, i.e., dynamic transformation of shapes. In addition, the present study investigates how these stages are associated with children's IQ, given that IQ has been studied extensively in relation to number processing, arithmetic abilities and counting methods (e.g., Hoard et al., 1999; Dark & Benbow, 1990), but has not been explored equivalently in relation to geometric understanding, strategies and skills.

Purpose

The aims of the study are the following: (a) to investigate the extent to which children would conserve the shape (constant path) of a polygon or implicitly use some model of variation of shape (continuous path) when asked to transform triangles, rectangles and squares by drawing an increasing or a decreasing stairway for each of these figures; (b) to examine how the above processes would vary in regard with children's ability to recognize triangles, rectangles and squares, IQ level and age; and (c) to identify implications of findings for theoretical descriptions of children's geometric thinking. The following research questions were formed accordingly:

1. What strategies do children use to transform polygonal shapes and to what extent? How consistent are these strategies at the transformation tasks of different shapes?
2. What is the relationship between children's transformation strategies and their ability to recognize certain shapes among others?
3. How children's transformation strategies vary in regard with their age and their IQ level?
4. Which are the possible connections of this study's findings with children's geometric or cognitive development on the basis of different theoretical perspectives?

On the basis of these research questions, considering children's geometric transformation strategies, we expect more advanced strategies parallel to the increase of children's recognition ability, IQ and age. In other words, children with a developed recognition ability or higher IQ, as well as older children are expected to be more proficient in using high level or reflective transformation strategies compared to children of a less developed recognition ability or lower IQ, or younger age, respectively.

Method

Participants

The sample of the study consisted of 291 children ranging from 4 to 8 years of age. These children were: 104 of 4 to 6 years of age (pre-primary), 105 of 6 to 7 years (Grade 1) and 82 of 7 to 8 years (Grade 2). The mean age of pre-primary children was 4.8 years, of first grade 6.4 years and of second grade 7.6 years. In particular, 49 boys and 55 girls were included in the pre-primary group; 46 boys and 59 girls in Grade 1; and finally 44 boys and 38 girls in Grade 2.

The tasks – instrument design and variables

Three measures of inquiry were considered necessary for the fulfilment of the aims of the present study: 1) an IQ test, 2) a series of recognition tasks in which children were asked to recognize rectangles, squares and triangles among other figures and 3) a series of transformation tasks in which children were asked to draw an increasing and afterwards a decreasing series of rectangles, squares and triangles. Each one of the three measures were administered separately, as group tests, and are described thoroughly below.

IQ test. All participants were firstly given the Colored Progressive Matrices, for detecting their IQ (spatial) score (Raven, 1962). The test was administered for about 15 minutes, and consisted of three sets (A, AB, B) of 12 similar exercises each. In particular, in all exercises children were given a figure with a missing part and had to choose the right from 6 pieces in order to complete it. Responses were coded with 1 (correct) or 0 (incorrect) and the average score of success at the test for each child was calculated, taking into account all 36 questions. For analytic purposes, four groups were created based on the scores: 9 children of low score [0-0.4); 133 children of below average score [0.4-0.6); 122 children of above average score [0.6-0.8); and 25 children of high score [0.8-1]. The four IQ level scores were codified as **IQ1**, **IQ2**, **IQ3** and **IQ4**, respectively. It must be noted that the analysis of variance showed statistically significant differences between the three age groups regarding the IQ scores [$F(2,290)=10.125$; $p<0.01$]. In particular, a post hoc analysis (Bonferroni) revealed that these differences were due to the significantly lower IQ score of the preprimary children ($\overline{X}_1=0.46$) relative to the first grade ($\overline{X}_2=0.53$) and second grade children ($\overline{X}_3=0.55$).

Recognition tasks. Children were given three tests, based on the tests that were used in the research study by Clements et al. (1999). These tests asked children to identify and color the squares, rectangles and triangles, respectively, among other figures. The three tests were administered together and children were given 20 minutes to fill them. Correct responses were assigned the score of 1, while incorrect responses were assigned the score of -1. The total score of success at each test signified children's abilities to recognize squares, rectangles and triangles, which were codified as **recSq**, **reqRe** and **recTr** respectively. Children's mean scores of success at the three recognition tests, according to age, are given in table 1.

Transformation tasks. On the basis of the theoretical mathematical model of "polytopes" and their dynamic transformation, we developed and

Table 1. *Mean success scores at the recognition tests out of 100*

	Mean Triangle Recognition	Mean Square Recognition	Mean Rectangle Recognition
4-6 year-olds	58.27	85.96	61.54
6-7 year-olds	53.52	75.81	63.05
7-8 year-olds	55.37	87.32	60.73

administered a task asking children to ”Draw a stairway of triangles, with each one being bigger than the preceding one” and to repeat the same procedure with squares and rectangles. After a week, an analogous task was given for the same figures, but at this time children were asked to ”draw a stairway for each shape, with each one being smaller than the preceding one”. These tasks were open ended. The transformation strategies used by children were codified as follows:

T is used to represent conservation of shape by increasing or decreasing (analogous to the demands of the task) both dimensions of the figure.

O stands for children's attempt to differentiate mainly one dimension (possibly producing rectangles in the series of ”squares” or a square in the series of rectangles or an isosceles triangle in the series of ”equilateral triangles”).

N is used to show that children produced a defective series (i.e., very irregular figures non-increasing or non-decreasing at all in a regular way).

Eighteen different variables occurred representing the strategy children used when they attempted to construct the six different stairways. These are coded as follows:

(a) **SLTrt**, **SLSqt** and **SLRet**: a series of similar triangles, squares and rectangles, respectively, of continuously increasing dimensions [1].

(b) **LSTrt**, **LSSqt** and **LSRet**: a series of similar triangles, squares and rectangles, respectively, of continuously decreasing dimensions [2].

(c) **SLTro**, **SLSqo** and **SLReo**: a series of triangles, squares and rectangles, respectively, by increasing mainly one dimension of the figures.

(d) **LSTro**, **LSSqo** and **LSReo**: a series of triangles, squares and rectangles, respectively, by decreasing mainly one dimension of the figures.

(e) **NSLTr**, **NSLSq** and **NSLRe**: a defective series of irregular triangles, squares and rectangles, respectively, in the task requesting a construction of an increasing series of figures.

(f) **NLSTr**, **NLSSq** and **NLSRe**: a defective series of irregular triangles, squares and rectangles, respectively, in the task requesting a construction of a decreasing series of figures.

In (a)-(d) the last letter **t** or **o** stands for the above mentioned categories of changes in two respectively one dimension, while in (e) and (f) the first letter **N** denotes that the strategy used was defective.

Data analysis

Two separate analyses were conducted. We used a research data analysis, which enables the distribution and classification of variables, as well as the implicative identification among the variables or variable categories. This method of analysis, using the statistical computer software CHIC (Bodin, Coutourier & Gras, 2000), generated a similarity diagram (Lerman, 1981) and an implicative diagram (Gras, Peter, Briand & Philippé, 1997) of children's responses at the items of the three measures. The similarity diagram, which is analogous to the results of the more common method of cluster analysis, allows the arrangement of the tasks into groups according to the homogeneity by which they were handled by the children. The implicative diagram, which is derived by the application of Gras's statistical implicative method, contains implicative relations that indicate whether success to a specific task implies success to another task related to the former one. We also employed the notion of ”supplementary variables”, which enabled us to identify which objects were ”responsible” for the formation of particular clusters of variables. In our study, children's age was set as a supplementary variable. Consequently, we were able to know which age group of children contributed the most to the formation of each cluster.It is worth noting that the particular method of analysis derived by CHIC has been widely used by several studies in the field of mathematics education (e.g., Elia & Gagatsis, 2003; Gagatsis & Shiakalli, 2004; Gras & Totohasina, 1995), or is integrated with other statistical techniques such as Structural Equation Modeling (Gagatsis & Elia, 2004; Modestou & Gagatsis, 2004). Chi-squared tests (criterion Cramer's V and Eta) were also applied in order to ascertain the existence of possible differences concerning children's strategies at the transformation tasks with respect to their age (Cramer's V), IQ score (Cramer's V) as well as their ability to recognize squares, rectangles and triangles (Eta) among other figures.

Results

Three strategies (models of action) were observed in children's responses at the transformation tasks:

(a) T-strategy

The main feature of this strategy was the shape conservation by increasing both of its dimensions at the same time, thus producing a series of similar figures of continuously increasing dimensions at SL series (Figure 2). At a LS series a simultaneous decrease of both figure's dimensions was observed, leading to the production of a series of similar figures of continuously decreasing dimensions (Figure 3).

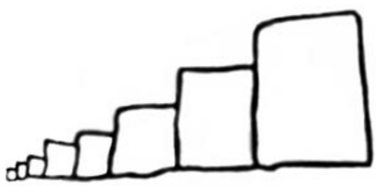

Figure 2. *Increasing both dimensions (T-strategy).*

Figure 3. *Decreasing both dimensions (T-strategy).*

(b) O-strategy

This strategy involved the differentiation of mainly one dimension of the figures: In the case of triangles this dimension was the altitude to the base; that is why children sometimes produced isosceles triangles although their paths started with an equilateral one (most common case) (Figure 4).

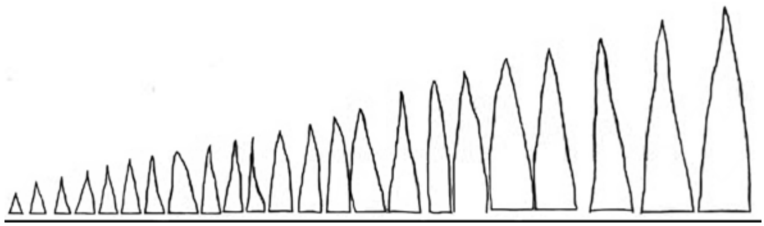

Figure 4. *Increasing only one dimension (O-strategy).*

In the case of rectangles it was usually the longer side (Figure 5); sometimes a square occurred among the rectangles in a very natural way (Figure 5).

In the case of squares children produced rectangles (Figure 6).

Figure 5. *Squares in the series of rectangles.*

Figure 6. *Rectangles in the series of squares.*

(c) N-strategy

The application of N-strategy produced a defective series of irregular figures.

Next, descriptive results are presented, followed by the outcomes derived by CHIC, which are enhanced by the results of the chi-squared tests.

Descriptive results: children's transformation strategies by age

Figures 7, 8 and 9 illustrate the percentages of children of every age (4-6, 6-7 and 7-8 years old), based on the strategy they used to carry out the tasks for each figure (triangles, squares and rectangles).

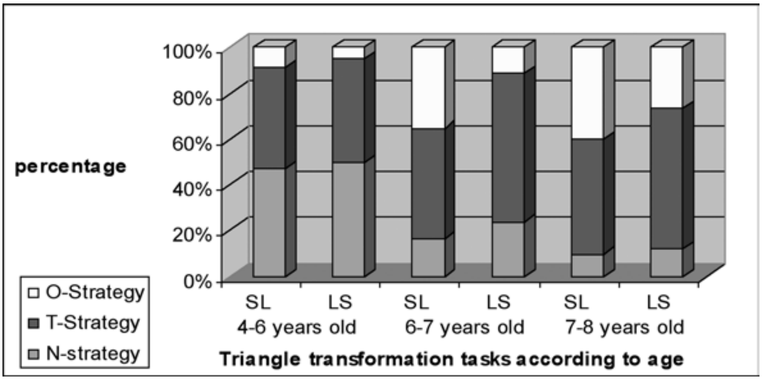

Figure 7. *Percentages for strategies O, T and N at tasks SL and LS for triangles.*

Note. SL = an increasing series of triangles at the transformation tasks. LS = a decreasing series of triangles at the transformation tasks. O-strategy = differentiation of one dimension of the triangles. T-strategy = differentiation of both dimensions of the triangles. N-strategy = construction of a defective series of irregular figures.

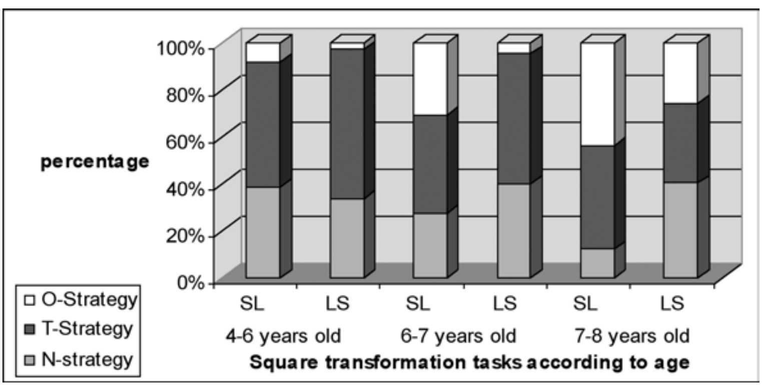

Figure 8. *Percentages for strategies O, T and N at tasks SL and LS for squares.*

Note. SL = an increasing series of squares at the transformation tasks. LS = a decreasing series of squares at the transformation tasks. O-strategy = differentiation of one dimension of the squares. T-strategy = differentiation of both dimensions of the squares. N-strategy = construction of a defective series of irregular figures

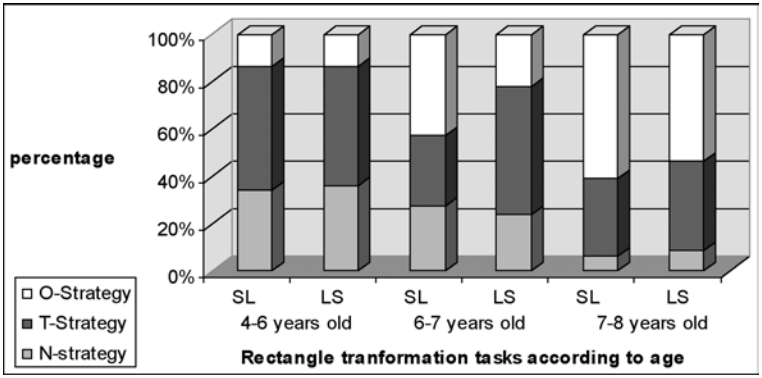

Figure 9. *Percentages for strategies* O, *T and N at tasks SL and LS for rectangles.*

Note. SL = an increasing series of rectangles at the transformation tasks. LS = a decreasing series of rectangles at the transformation tasks. O-strategy = differentiation of one dimension of the rectangles. T-strategy = differentiation of both dimensions of the rectangles. N-strategy = construction of a defective series of irregular figures.

A significant observation that can be derived from the comparisons of the above outcomes is the fact that in the case of 7-8 year-old children the strategy of differentiating one special dimension of the figures appears in a frequency which is higher compared to the cases of the younger children for all figures. The chi-squared test results reinforce this observation as a statistically significant relation ($p<0.05$) exists between children's age and the use or not of the O-strategy (Cramer's V SLtro=0.313, Cramer's V SLsqo=0.336, Cramer's V SLreo=0.399, Cramer's V LStro=0.260, Cramer's V LSsqo=0.328, Cramer's V LSreo=0.370). In particular, as children get older they tended to use the O-strategy more frequently, with the age group of 7-8 year-olds differentiating the most from the younger age groups.

The strategy of differentiating both dimensions of the figures appeared more frequently in the case of 4-6 year-old children. The younger children used at a statistically significant level ($p<0.05$) more often the T-strategy compared to older children, especially in the case of squares and rectangles (Cramer's V LSsqt=0.237, Cramer's V SLret=0.226).

In addition, as expected, the percentages of 4-6 year-old children producing a defective series were considerably higher than the corresponding percentages of the older children for almost all the figures. This difference is statistically significant ($p<0.05$) as children's age appears to relate to the production of the defective series (Cramer's V NSLtr=0.367, Cramer's V NSLsq=0.227, Cramer's V NSLre=0.272, Cramer's V NLStr=0.334, Cramer's V NLSre=0.252). Therefore, the older children were the ones producing the smallest amount of defective series.

Similarity and implicative relations

Figure 10 illustrates the similarity diagram that occurred from the use of the statistical tool, namely, CHIC. It shows how children's responses at the tasks are grouped, according to the homogeneity.

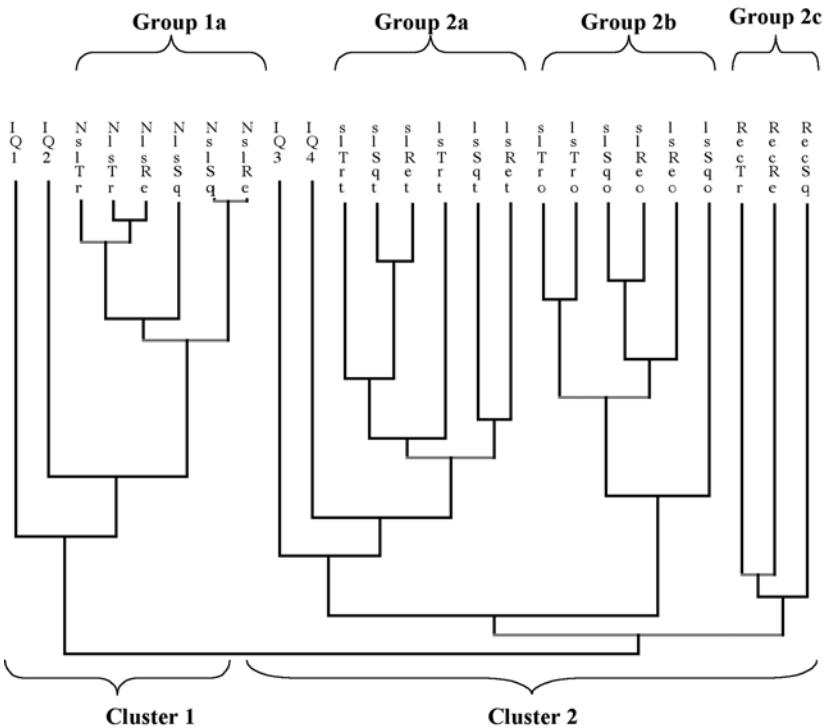

Figure 10. *Similarity diagram of the variables concerning children's responses at the tasks.*

Note. Similarities presented with bold lines are important at a level of 99%.

Two clusters can be identified in the similarity diagram. The first cluster concerns the N-strategy and the second cluster refers to T- and O-strategies as well as the recognition responses. In both clusters children's responses in the transformation tasks can be classified with respect to the strategy they applied. The group of variables (Group 1a) within Cluster 1, which has the greatest similarity, consists of NslTr, NlsTr, NlsRe, NlsSq, NslSq, NslRe and concerns the N-strategy on both types of transformation tasks, i.e., decreasing or increasing series of figures. The first similarity group of variables within Cluster 2 (Group 2a) consists of slTrt, slSqt, slRet, lsTrt, lsSqt and lsRet, which represent the application of T-strategy on both types of tasks. The second similarity group of variables within Cluster 2 (Group 2b) consists slTro, lsTro, slSqo, slReo, lsReo, lsSqo and refers to O-strategy anboth types of tasks, as well. The formation of these groups of variables reveals that children tended to approach the variations of all three kinds of figures in a similar and consistent way; that is, each child, was more likely to apply the same strategy, rather than a different one, at the diverse stairways of figures of the two types of tasks (increasing or decreasing series of figures).

The variables representing children's outcomes in the recognition tasks form another group within Cluster 2 (Group 2c), and are separated from the other groups of variables. This observation implies that children's ability to identify triangles, squares and rectangles is not closely related to the process of transformation of the corresponding shapes. However, the recognition group is more directly associated with the transformation groups of T- and O-strategy compared to the transformation group of the N-strategy. This indicates that children who constructed a series of irregular figures have not yet developed the ability to identify geometrical shapes of these figures. The above observation was reinforced by the Eta values [3] given from the chi-squared tests between children's ability to recognize rectangles and the strategies used when constructing the increasing or decreasing series of figures. In particular, the results of the analysis ($p<0.05$) indicate that children who failed to recognize rectangles among other figures were the ones most probable to construct defective series, independently of the figure. This behaviour was also repeated in the case of recognition of triangles and squares. Specifically, most children that did not recognize squares, were the ones constructing a defective series of rectangles or squares, while in the case of triangles low recognition ability leaded to a defective series of triangles. Further analysis on the relation between children's recognition abilities and the N-strategy used at the transformation tasks, revealed that this relation was stronger for pre-primary children compared to the older ones.

Children's IQ scores had a complementary role in the two groups of variables and more specifically at the group of N-strategy (Group 1a) within Cluster 1 and at the group of T-strategy (Group 2a) within Cluster 2. In particular, the variables of low and below average IQ scores are connected to the N-strategy group, while the variables of above average and high IQ scores are associated with the T-strategy group. These similarity relations indicate that children who had a below average IQ score applied mostly the N-strategy in the transformation tasks of polygons, while children achieving above average IQ score used mainly the T-strategy in the aforementioned tasks.

The results of chi-squared tests [4] between children's IQ score and the strategy used when constructing the increasing or decreasing series of figures concur with the above findings as children with low IQ score, irrespectively of their age, were more probable ($p<0.05$) to construct a defective series of figures, while children with higher IQ score ($p<0.05$) tended to use the T-strategy at the respective tasks. Concerning the O-strategy, the results of the same analysis indicate that only in the case of the construction of an increasing series of rectangles, children with higher IQ score tended to use the aforementioned strategy. The variable, which contributes the most to the establishment of the groups of variables in the similarity diagram, representing the use of N-strategy (Group 1a) and T-strategy (Group 2a), respectively, is the group of 4-6 year-old children, while the most contributing variable for the establishment of the class of responses concerning O-strategy (Group 2b) is the group of 7-8 year-old children. Consequently, taking into account the IQ score combined with age, it can be inferred that IQ had a role on the application N- or T-strategy, which were used mostly and more consistently by 4-6 year-old children. Children of this age achieving low IQ scores ($p<0.05$) produced a defective series of irregular figures while children of the same age achieving high IQ scores ($p<0.05$) differentiated both dimensions of the figures. Similarly to the youngest children's group, low IQ achievers' tendency to use N-strategy was observed also in the older groups of children. However, no connection between high IQ scores and the T-strategy was revealed in these groups, indicating that the tendency of children with high IQ scores to use the T-strategy, which seemed to hold for the whole sample, was explained only by the youngest children's performance.

In the case of O-strategy, it must be noted that the group of 7-8 year-old children, that contributed the most to the establishment of this class of responses, tended to apply it at the transformation task regardless of their IQ level. The results of the chi-squared test between the oldest children's IQ levels and the use of the O-strategy provided further evidence to the above finding, as no statistically significant differences between the children with different levels of IQ, who tended to use the O-strategy, was revealed.

The implicative diagram, which presents the implicative relations between the variables concerning children's responses towards the tasks, is illustrated in Figure 11.

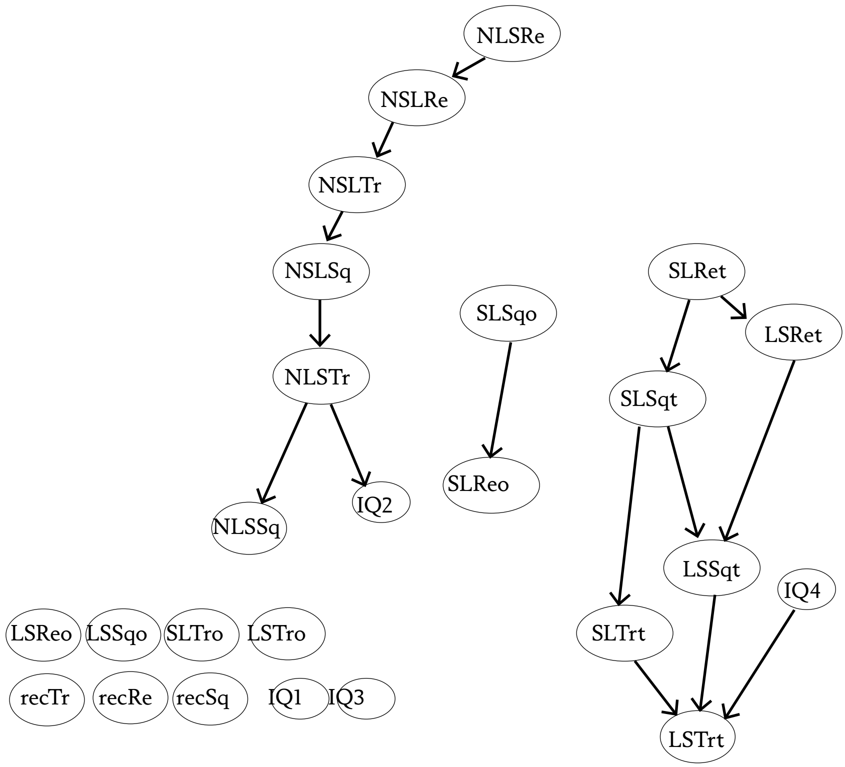

Figure 11. *Implicative diagram illustrating implicative relations among children's responses at the tasks*

In particular, an implicative chain, which connects all the variables of N-strategy, is created. The fact that this chain ends in the variable of the below average IQ score (IQ2) indicates that N-strategy was mostly applied by below average achievers in the IQ test. This finding concurs with the close similarity connection of the group of N-strategy with the below average IQ scores in the similarity diagram.

Concerning T-strategy, the implicative group of links consists of two types of relationships: first, of variables of the same kind of transformation tasks, that is a decreasing or increasing series; and second, of variables of the same figure, that is rectangles, squares, or triangles. Within the first type of implicative relationships, the relationship Ret → Sqt → Trt emerges in a consistent manner in both types of transformation tasks. Based on these implicative relationships, it can be inferred that children who differentiated both dimensions of the rectangles, followed the same strategy in the case of squares and triangles. Within the second type of implicative relationships, the relationship SL → LS occurs constantly for the tasks of all the figures. This indicates that the construction of increasing series of figures, where T-strategy was applied, led to the construction of decreasing series of the corresponding figures, using the same strategy. It seems that starting the series with a large figure which gradually gets smaller was easier for the children who used T-strategy, than starting the series with a small figure which gradually gets larger. This finding provides support to children's higher percentages in the construction of decreasing series using T-strategy, relative to the construction of increasing series using the same strategy in most of the cases (see Figures 7-9). The group of implicative relationships of T-strategy is connected with the variable of high IQ score. This relationship is in line with the corresponding connection of the similarity diagram, indicating that T-strategy was mostly used by (young) children achieving a high IQ score.

An important remark that arises from the implicative diagram concerns the lack of implicative relationships between children's responses at the recognition tasks of figures and their responses at the transformation tasks of the corresponding figures, involving the application of T-, O- or N-strategy. This observation provides further evidence to the formation of a separate group of the recognition tasks in the similarity diagram, and therefore to the assertion that the ability to identify geometrical shapes among other shapes is not closely related to the construction and dynamic transformation of shapes.

Discussion and implications

Transformation strategies and different levels of cognitive development

Two of the main concerns of the study were to examine the strategies adopted by young children when constructing an increasing and a decreasing series of plane geometric figures, and discuss possible links of their range of behaviour with the development of geometric thinking, established by relevant theoretical perspectives and previous empirical findings. In line with the findings of the studies of Gagatsis and Patronis (1990) and Elia and Gagatsis (2003), three distinct geometric transformation strategies were exhibited by the children: a) O-strategy involving the differentiation of only one dimension of the figures; b) T-strategy representing the differentiation of both dimensions of the figures, thus conserving the initial form of the figures; and c) N-strategy standing for the drawing of a defective series of irregularly increasing or decreasing figures.

The present study's findings reveal that each child tended to use a specific strategy consistently in her attempt to tackle the tasks. This kind of behaviour may be attributed to children's different levels of thought in geometry.

Specifically, the application of O-strategy by many children at different transformation tasks (e.g., starting their paths with a square and ending in a rectangle by conserving the parallel lines but by differentiating only one dimension), signifies a global combination with partial analysis and use of the characteristics of the figures. This behaviour seems to correspond to a transitional stage beyond the van Hiele's visual level, at which individuals recognize shapes according to their overall appearance, and lower from the descriptive one, which is exemplified by the ability to identify the whole range of the properties of geometric shapes. It is possible that children who used O-strategy (mainly 7-8 year-old children) were in this intermediate phase close to the descriptive level, since they did not only attend a figure as a holistic visual prototype, but they also seemed to focus on single or a subset of properties and components that constitute a shape (e.g., parallel lines), although these features were not clearly defined in a global or uniform manner. In other words, in their construction of continuous series as a product of the O-strategy, they decomposed the figures into components, omitting some of them, thus producing series of figures without conserving their original shape.

Moreover, if we take into consideration the research perspective examining the dynamic transformations (Markopoulos & Potari, 2001), it becomes obvious that the children who applied O-strategy used the "parallel" form of transformation or the continuous variation of a shape (Gagatsis & Patronis, 1990), where all the components of the figures are conserved parallel, but the ratio of the lengths of their sides is not kept constant. Therefore, by employing the "parallel" or "continuous" transformation a rectangle can easily occur in a series of squares as well as an isosceles triangle in a series that begins with an equilateral triangle. On the other hand, the similarity transformation or the constant variation process seems to correspond to the application of the T-strategy as the figures are transformed by similarity and their form remains constant.

Children who applied T-strategy (mainly pre-primary children with high IQ scores and 6-7 year-old children) may have had the characteristics of the visual level. The differentiation of both dimensions of the figures may be due to children's attempt not to change the holistic appearance of the figure. This assumption is also supported by Hasegawa (1997) who claims that the regular enlargement, or reduction of the size of a figure, is the main characteristic of the monadic stage in the development of the concept formation of n-gons, which corresponds to van Hiele's visual level. Furthermore, it can be assumed that prototypical images of shapes have influenced children's reasoning. Therefore, children's limited views of the particular geometric shapes may have enhanced their tendency to keep the prototypical form of the initial shape constant in the series they produced, because, otherwise, different forms of shapes may have occurred, not as "good" as the first one (Mesquita, 1998). However, on the basis of the findings of this study it cannot be determined whether the children who used T-strategy tended to keep the holistic form of the figure, implying a visual level of geometric thinking, as analyzed above, or whether they combined the different features of the figure and focused on the two dimensions of it, illustrating a descriptive level performance.

Children who constructed a defective series of irregular figures (N-strategy) were mostly of pre-primary age, achieving low IQ scores. This group of children seemed to have the characteristics of the prerecognitive level (Clements & Sarama, 2000; Clements et al., 1999), as they have not yet developed the ability to identify geometric shapes among other figures. Indications for this assertion were provided by the similarity diagram, where the N-strategy similarity group was distant and completely separated from the recognition task variables, as well as, the significantly higher proportion of low recognition ability children, who applied the particular strategy relative to the high recognition ability children.

Discussing the different strategies for the transformation of geometric figures in relation to the levels of van Hiele's theory or other relevant theories on the development of geometric thinking, is based on our assumption that these strategies belong to different levels of cognitive development. An explanation of this claim may be given by Karmiloff-Smith' s (1992) model of representational redescription (RR), which seems to be associated with this study's findings, despite the differences between the context of the present study and the domains at which the particular model has been examined and validated. It is noteworthy that this connection could be uncovered mainly by the transformation tasks, involving the active stage of geometric thinking, rather than the recognition tasks, having the characteristics of the passive stage of geometric understanding.

The RR model describes the way knowledge is initially represented in an implicit form and is subsequently processed to become explicit to consciousness and for linguistic processing. Karmiloff-Smith identified at least four levels of knowledge representation from an implicit level to a conscious level, which may have a correspondence with the children' s strategies, observed in this study, for the transformation of geometric shapes. In particular, the T-strategy which is applied by children in an attempt to conserve the holistic form of the figure, without considering consciously its features, may be related to Level I (Implicit), whose representations are procedural in format, intuitive in nature and not accessible to consciousness. The O-strategy which is implemented mostly by the 7-8 year-olds, who attended only to one dimension of the geometric shapes and thus omitted some of their properties, may correspond to the Level E1 representations, which provide only general descriptions in the sense that they lack many detailed elements of the codified knowledge and are difficult to access. The application of the T-strategy, as a product of the attention to both dimensions of the figure and therefore the conscious concentration on the figure's features and overall appearance, may be explained by Level E2 representations based on knowledge which is accessible to consciousness. At Level E3, knowledge is held in various notation systems, such as the natural language. Since the use of different systems of representation for the solution of the transformation tasks was not intended to be examined in this study, the latter level cannot be related to our findings. The lower level solution method, i.e., N-strategy, which involves the production of a defective series of figures, may be close to a level that was identified in a research study by Peters, Davey, Messer and Smith (1999), whose rationale and goals were directly linked to the RR model. Children at the particular level, namely, "unsuccessful", were not in a position to respond adequately in the majority of the tasks that were asked to tackle. From Gagatsis and Patronis (1990) point of view the passage from the N-strategy to the conscious use of the T-strategy may indicate the continuous progress from intuitive to reflective thinking in the particular geometric activities.

On the basis of the RR model or Gagatsis and Patronis (1990) work on reflective thinking in mathematical activity, an optimal method for tackling the transformation tasks could be the T-strategy (i.e., differentiating the two dimensions of the figures), in the sense that children consciously focused on the two dimensions and took into consideration different attributes of the figure as well as its overall shape. Nevertheless, as already noted, our study provided no evidence of the distinction between children who used T-strategy due to their tendency to keep the prototypical form of the initial shape and children who implemented the T-strategy in an optimal level, or the connection of these strategies with age. This could be an interesting issue for future investigation, which can be studied by using a systematically designed qualitative approach, probably involving clinical interviews at the time that children with different IQ scores of the three age groups are constructing the required series of figures. In addition, our findings raise issues about validating the developmental ordering and continuity of these strategies in relation to the representational redescription model by Karmiloff-Smith, the van Hiele levels or the stages of the process of reflective thinking proposed by Gagatsis and Patronis (1990) and exploring the effect of specific learning experiences on developmental change.

Age, IQ and ability to recognize shapes

The study also set out to investigate the relations of children's age, IQ level and recognition ability, signifying the passive stage of geometrical thinking, with transformation strategies, representing the active stage of geometrical understanding.

Children's behaviour towards the tasks varied with respect to their age. Clearer conclusions though, can be drawn for the O- and N-strategy, rather than the T-strategy, with respect to children's age. In particular, the O-strategy was used mostly by 7-8 year-old children, while the N-strategy was used mostly by 4-6 year-old children. As children got older, they tended to apply the dynamic transformation process of O-strategy more frequently and coherently in the different figures, with the age group of 7-8 year-olds outperforming the younger children. The T-strategy was applied more often by 4-6 year-old children in certain tasks of figures and by children of 6-7 years of age in others. Nevertheless, it was revealed that 4-6 year-old children used the T-strategy in a more consistent way at the transformation tasks of the different figures relative to the 6-7 year-old children. This indicates 6-7 year-old children's tendency to move forward to a more dynamic transformation process, such as the O-strategy. The above results provide support to the findings of the study carried out by Elia and Gagatsis (2003), which revealed that older children differentiated mainly one dimension of the figures, while younger children differentiated both dimensions of the figures in transformation tasks.

Younger children tended to use more frequently and consistently the N-strategy in the different figures, leading them to the construction of defective series for the corresponding figures. Specifically, N-strategy was used mostly by 4-6 year-old children with a significant difference from the older children, who produced the smallest amount of series of irregular figures. This is consistent with Gagatsis and Patronis (1990) findings that drawing a defective series of irregular figures appeared mostly in the younger children of the study.

Nevertheless, children's age combined with the strategy that they used do not allow us to make clear conjectures for their developmental geometric stage. As illustrated and discussed above, the main reason for this is that we cannot be sure at what stage children of T- or O-strategy are. Consequently, further and more systematic research is needed to explore this issue.

In addition, the IQ score was found to be directly associated with the transformation processes of figures, applied mostly by 4-6 year-old children. Particularly, low IQ performance seemed to have a close relationship with the production of a defective series of figures for all the children of this study, while high IQ performance was found to be directly associated with the construction of a series of similar figures by increasing or decreasing both dimensions of the figures in the transformation tasks only for the youngest children. The significant role of IQ on mathematics achievement was also showed by Hoard et al. (1999), who found that low-IQ children exhibited certain deficits in arithmetic ability that distinguished them from their average-IQ peers. However, IQ score was not related to the strategy of differentiating one dimension of the figures, which was mostly used by the older children of the study. These inconsistencies characterizing the connection of IQ with the geometric strategies of children of different ages may be attributed to the varying nature or amount of children's systematic instructional experiences in geometry with respect to their age. For instance, older children's wider practice in spatial and geometric concepts may have a moderate role on the effect of the IQ level on their performance in the transformation tasks.

Children's ability to recognize triangles, squares and rectangles was not found to affect or connect directly with the use of T- and O- strategies. However, an incompetence to recognize these figures among others was often related with the construction of a defective series of figures (N-strategy). Thus, it can be asserted that recognition tasks require different types of abilities from construction and transformation tasks, and that recognition abilities of geometric figures do not automatically implicate success at geometric construction or transformation tasks. These findings are in line with Markovits et al. (1986) view that the passive stage of understanding, corresponds to the recognition tasks, is less complicated than the active stage of understanding, which corresponds to the transformation tasks.

We admit that the transformation tasks of this study were open-ended in order to give children the opportunity to reply according to their level of geometric understanding. It would be interesting and perhaps more meaningful for a future research to overcome this "limitation" by examining children's behaviour in more definite activities, such as transformation tasks that give the former two figures of an increasing or a decreasing stairway and ask children to carry on, or transformation tasks performed in a geoboard or grid, and examine its relationship with children's recognition abilities of the corresponding shapes.

A strong understanding of geometry is a necessary component of a child's mathematical foundation (Oberdorf & Taylor-Cox, 1999). Various studies seem to agree at one important point in relation to geometry instruction: teaching geometry needs to begin early, since young children's conceptions remain constant after six years of age, without necessarily being accurate (Clements & Sarama, 2000; Gagatsis & Patronis, 1990). Children's consistent behaviour towards the tasks of our study seems to provide further support to the above assertion.

Some useful implications for teaching geometry can be drawn from this study's findings. The present study raises a question that needs to be examined further: How important is the use of tasks involving recognition of geometric shapes and tasks involving transformation of geometric figures in early geometry instruction, aiming at children's geometric and mathematical development? This study's findings indicated that the ability to identify geometric shapes is not closely related to the ability to construct and transform geometric shapes. Thus, instruction which aims at children's overall geometric development needs to promote both kinds of ability, by providing not only recognition activities of geometric shapes, but also construction and transformation activities of shapes. Moreover, encouraging children's "dynamic intuition" in transformation tasks through geometrical models, may facilitate geometry instruction which emphasizes shape properties and characteristics, as well as, the interconnectivity and hierarchical commonalities and differences among shapes, such as rectangles and squares. Quantitative methods such as our use of implicative statistical analyses are a powerful way of empirically verifying and extending the current findings on young children's geometric thinking generalisable to the Nordic milieu.

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Notes

1 The SL symbol indicates a series of similar shapes beginning with a Small shape and ending with a Large one

2 The LS symbol indicates a series of similar shapes beginning with a Large shape and ending with a Small one

3 These Eta values not reported here due to space constraints but available from authors for interested readers.

4 These quantitative values are again not reported but available to interested readers.

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Sammendrag

I dette studium undersøges 4-8 årige børns strategier til transformation af trekanter, kvadrater og rektangler. Datamaterialet omfatter tegninger fra 291 cypriotiske børn, der har løst hver seks transformationsopgaver. Børnene blev bedt om at tegne serier af specifikke ligedannede polygoner af henholdsvis stigende og faldende størrelse. Sammenhænge mellem børenes transformationsstrategier, deres evne til genkendelse af de geometriske figurer og deres scorer i en IQ-test blev undersøgt statistisk. Resultaterne viser, at børnene bruger tre alternative strategier i transformationsopgaverne, at børnenes strategier er direkte forbundet med deres IQ-scorer, og at dårlig genkendelse af geometriske figurer har en sammenhæng med brugen af utilstrækkelige transformationsstrategier.