The mathematics textbook

From artefact to instrument

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This paper describes the mathematics textbook from two sides, the textbook as an artefact and the textbook as an instrument. It includes results from analysis of textbooks from different parts of the world as well as studies of the actual use of textbooks. It is not an attempt to give complete coverage of previous research in the area; it is rather an attempt to highlight some important issues.

The textbook as an artefact

On terminology

The ability to collect experiences and use them for our own purpose is significant for our culture and distinguishes us from the animals. *Artefacts*, intellectual (e.g. ideas, values, knowledge) as well as physical tools, are developed, refined and changed in a continuous process in the interactions between human beings and their environments (Säljö, 2000). Wartofsky (1979) defines artefacts as "anything which human beings create by the transformation of nature and of themselves: thus, also language, forms of social organization and interaction, techniques of production, skills" (p. xiii).

Artefacts can be categorized as *primary*, *secondary* and *tertiary* (Wartofsky, 1979). 'Primary' artefacts are those directly used in production, for instance axes and clubs. 'Secondary' artifacts are externally embodied representations, created for the purpose of preserving and transmitting skills in the production and use of primary artefacts. Textbooks and other curriculum materials that are used in classrooms are examples

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of secondary artefacts. 'Tertiary' artefacts are "a class of artifacts [1] which can come to constitute a relatively autonomous 'world', in which the rules, conventions and outcomes no longer appear directly practical" (Wartofsky, 1979, p. 208).

Further examples of artefacts, which are used in mathematics classrooms, are hand-held calculators and computers. Whereas computer-aided design (CAD) programs are 'primary' artefacts, the learning-software and the computer game are 'secondary' and 'tertiary' artefacts, respectively (Strässer, 2004).

A glimpse into the history of mathematics textbooks

When discussing textbooks and their role in mathematics education, at the present time, we might forget about their importance for the development of mathematics. However, as secondary artefacts, textbooks contribute to the field of mathematics by preserving and transmitting skills and knowledge. We can notice their importance by considering the mathematicians that have a prominent position in the history of mathematics – mainly because they have written books that become frequently used in education. I can give two examples: Euclid of Alexandria (born ca. 325 BC) and Robert Recorde (1510-1558) from England.

The most famous work of Euclid is the treatise on mathematics, *The Elements*, which is a "compilation of knowledge that became the centre of mathematical teaching for 2000 years." (O'Connor & Robertson, 2002). B L van der Waerden considers the importance of the Elements in the Biography in *Encyclopaedia Britannica* (cited in O'Connor & Robertson, 2002):

> Almost from the time of its writing and lasting almost to the present, the Elements has exerted a continuous and major influence on human affairs. It was the primary source of geometric reasoning, theorems, and methods at least until the advent of non-Euclidian geometry in the 19th century. It is sometimes said that, next to the Bible, the 'Elements' may be the most translated, published, and studied of all the books produced in the Western world.

The other example of famous mathematicians (and textbooks authors) is Robert Recorde who lived in England in the 16th century, almost eighteen hundred years after Euclid. Their work and contribution to the field is not comparable to Euclid's work. Nevertheless, Recorde wrote many elementary textbooks that differ from the previous ones in that they were written in English, which was extremely uncommon at that time. In doing so, he made them available to a larger group of students, not only the few who could read Latin or Greek. Another special feature of his

books is that basically all of them are written in the form of a dialogue between a master and scholar, for instance *The Grounde of Artes* that was a very successful commercial arithmetic book and *The Whetstone of Witte* on algebra published in 1543 and 1557 respectively. The only book of Recorde that is not written in the form of a dialogue between a master and a scholar is the *Pathwaie to Knowledge* (1551), which sometimes is considered as an abridged version of Euclid's *Elements*. Recorde had a clear idea of which order the mathematical topics should be taught and wrote his books in that order with the intention to offer a complete course of mathematical instruction. In addition, he tried to make the texts easy to understand by using clear and simple expressions (O'Connor & Robertson, 2002).

> Anyone who reads Recorde's works will be led to believe that, although no evidence survives, he must have taught for some time since he has a deep understanding of how to teach. Ideas are developed clearly step by step, with difficult points being left until the student has gained enough experience to understand them. (O'Connor & Robertson, 2002).

Textbooks (in general) were also important, as basic tools, for the progress of organized education in the western part of the world in the 17 th and the 18 th century (Selander, 1988).

The authority and the authors of textbooks

Textbooks are special kinds of books since they are intended to be used in education, "designed to provide an authoritative pedagogic version of an area of knowledge" (Stray, 1994, p. 2). Though 'normal' books might be used in teaching situations, "the pedagogic use is marginal to the intentions of the book's producers" (ibid.). Moreover, textbooks hold a unique and significant social function in relation to other texts available for a reader since they "represent to each generation of students an officially sanctioned, authorized version of human knowledge and culture" (de Castell, Luke & Luke, 1989, p. vii). An additional view of textbooks is offered by Johnsen (1993):

> A textbook is neither just subject content, nor pedagogy, nor literature, nor information, nor morals nor politics. It is the freebooter of public information, operating in the gray zone between community and home, science and propaganda, special subject and general education, adult and child (p. 330).

For whom is the textbook written? Well, students are in general regarded as the main readers of a textbook. The role of a teacher is to mediate the text and encourage students to read the book. Since the authors cannot

intervene directly in the communication between teacher and students they usually write the textbook from the teacher's position (Kang & Kilpatrick, 1992).

The authority of the textbook comes "in virtue of its having been *authorized* by an administrative source" (Luke, de Castell, Fraser & Luke, 1989, p. 254). It could be the teacher or whoever decides which textbook to use, whose authority in turn is institutionally bound. The authorization involves the content as well as the physical object. Textbooks are, in many countries, school property that are cared for and venerated. The teachers often admonish students to be cautious, sometimes making the link between school property and private ownership with comments like: 'your parents pay tax dollars for these books!' Covering books with protective jackets is also an example of an annual ritual to show 'respect' for the text. Irrespective of that, textbooks are also disused by the students, scribbled in and dashed against walls or lockers, sometimes as a private act of criticism. But in the communicational system of the classroom, the students have a passive and *non-authoritative status* in relation to both text and teacher (Luke et al., 1989).

The authority of the textbook is more or less unrelated to the author. This is especially true for students. For them, the authors of the textbook are often anonymous and textbooks are in general identified with courses and teachers. Teachers unconsciously reinforce this when they refer to textbooks as 'the math book' or 'the blue book' rather than to their actual authors (Luke et al., 1989). This irrelevance of authorship, Luke et al. says, detach the text from the author and makes the school text "more closely associated with a corpus of 'indisputable' disciplinary or lesson content than with a potentially fallible author" (p. 258).

But the textbook, as mentioned earlier, is an artefact. And it is human-made in a specific sense; there exists an author (or a group of authors) and a producer of the textbook. One can assume that the purpose of textbook authors and publishers is to offer a well-made, carefully prepared pedagogical version of a school topic. On the other hand, publishing is an industry in most countries, so "gaining a large share of the market is perhaps the most important goal that drives the design and production of textbooks." (Chávez, 2003, p. 1). Hence, in countries where the market is free and teachers can select their own textbook, it is more likely that the economical interests have more influence than the pedagogical. However, our knowledge about textbooks authors and publishers are limited. Portraits of textbooks authors are rare (great textbooks authors in former days e.g. Euclid and Recorde excluded) and publishers are even more anonymous (Johnsen, 1993). As an unspecified group, they are however criticized from time to time. For example in a text from the beginning

of the 80's where Howson, Keitel and Kilpatrick (1981) considered the business in the USA in the following way:

> Commercial textbook publishers in the USA have set up elaborate networks for the promotion and distribution of their products. Teachers in the USA depend heavily on textbooks. This dependence has roots that go back to the introduction of free public education into a frontier society: the textbook was used to compensate for a shortage of well-educated teachers. As a result, textbooks are marketed much like automobiles: publishers' representatives find out what the consumer wants and then the publishers compete to offer it to him in the most attractive package possible. Novelty is at a premium; yet the changes offered are frequently cosmetic and rarely radical. The greatest changes in US textbooks in the last three decades have come from outside the commercial arena – many of them in response to the stimulus provided by federal-funded projects – but in each case the forces of the marketplace acted quickly to dampen the change. (p. 62).

A conclusion must be that an author's contribution to a textbook is somewhat limited. But the interests of the publisher are only one of many influential factors. Even the authors' attitudes and norms, which mark a 'normal' book, are constrained since the author himself is a product of attitudes in a given society at a given time, certainly conscious of the formal requirements regarding the given school subject. Johnsen (1993) highlights this issue and says that "It is possible to reduce the significance of an individual authors' contribution to textbooks for reasons which may in themselves provide some explanation for the low status of these writers." (p. 244).

Perhaps as a consequence of their anonymity, textbooks authors are seldom categorized as a separate occupational group. There are however trade unions for this category of writers in Norway and Sweden, which can be a sign of professional awareness. In 1987, the Association of Norwegian Non-Fiction Writers [2] made use of their list of members and sent out a questionnaire to those who write general non-fiction books and/or textbooks in order to learn more about the authors. Most of the 731 who replied had full-time jobs in other fields. For example, thirteen percent of the authors were primary and secondary school teachers, three of four authors where men and their average age were between forty and fifty-five (Johnsen, 1993). It is hard to find documentations about the situation in Sweden, but it seems that most of the authors of mathematics textbooks are, or have been, teachers.

About mathematics textbooks

The following section contains some results from previous research on textbooks. These studies show that textbooks around the world have some common features. However, they also conclude that there exists no 'universal' book. But to begin with, I will bring forward some arguments why content analysis of textbooks is important.

Why content analysis?

The presence of textbooks in classrooms is not a clear indicator of how instruction is done since textbooks, as such, give no information on how they are used or how they influence what takes place in the classroom. So, why make an effort to find out about the content of a textbook? Is there any meaning of doing content analysis? I believe it is. Previous research suggests that:

(a) Mathematical topics in textbooks are most likely presented by the teachers (Freeman and Porter, 1989; Reys, Reys, Lapan, Holliday and Wasman, 2003);

(b) Mathematical topics not included in textbooks are most likely not presented by the teachers (Freeman and Porter, 1989; Reys et al., 2003);

(c) Teachers' pedagogical strategies are often influenced by the instructional approach of the material (Reys et al., 2003);

(d) Teachers' sequence of instruction are often parallels to that of the textbook (Freeman and Porter, 1989).

(e) Teachers report that textbooks are a primary information source in deciding how to present content (Schmidt et al., 2001).

In textbooks we can find various methods to present the same mathematical topic. A topic can be emphasized in one book and almost overlooked in another. Connections between topics can be illustrated in different ways and the topics can be organized differently. In fact, textbooks reveal underlying beliefs of what mathematics is and how it can be learned. An analysis of textbooks can identify (national or international) similarities and differences between textbooks. The findings can, subsequently, serve as one possible explanation of dissimilar learning experiences among students from different classrooms or countries (Valverde, Bianchi, Wolfe, Schmidt and Houang, 2002). Curriculum developers, teachers and others that are involved in mathematics education can hopefully learn from these studies. In addition, the results from these studies can also be helpful for the development and improvement of textbooks.

Finally, textbooks analysis can also be a record of past learning experiences and reveal historical changes in mathematics education. However, we must keep in mind that:

> Texts are often all that remain as records of past classrooms. It is tempting to make inferences about the actual curriculum and the pedagogic intentions of past authors on the basis of the texts alone. However, at the very least, the principles which governed the selection of material are clearly unavailable (Love and Pimm, 1996, p. 373).

Nevertheless, since the content and the structure of a textbook is influenced by the educational culture of a specific country (Pepin & Haggarty, 2001), a look into the historical development of textbooks can reveal interesting trends. One can for instance examine how textbooks changes with regards to mathematics content, text and physical features, from one time to another. The focus in such studies can be more or less narrow. For example, it can be the presence or absence of the proof of the general rule for finding the derivative in textbooks for upper secondary schools in Sweden, which was one of the focuses of Bremler (2003), or how well a specific textbook series is adjusted to changes in the national curriculum (Johansson, 2003).

Some content of textbooks

The presence or absence of mathematical topics and characteristics of the text and tasks are relevant issues in a content analysis of mathematics textbooks. Questions concerning the topics are for example: What are the topics in textbooks for grade eight? In what grade are linear equations introduced to students? These types of questions are maybe time-consuming but comparatively easy to find answers to. The text and the tasks in a textbook is however not that uncomplicated to characterize. First of all, there are several ways to break up a textbook into units. A unit to analyze can be a textbook (or a book-series) intended for a whole school year or a single exercise (or sentence) and everything in between. Second, it is necessary to use or develop some type of taxonomy or classification schemata.

Mathematics textbooks can also be analyzed regards to a specific mathematical topic. The focus in the study of Harries and Sutherland (1999) is for example how textbooks from five different countries [3] introduce multiplication and division to students at the age of 6 to 8. One of their findings is that the English textbook first presents multiplication as repeated addition of equivalent sets without using the standard notation of the 'x' sign. The teacher's guide implicitly suggests that pupils should first understand an idea before the mathematical notation can be

introduced. In the Hungarian and the French textbook, multiplication and division are introduced a year later than in England (age 7-8) and the students are very quickly introduced to the formal multiplication notation. Harries and Sutherland also notice that the strength of the textbooks from Singapore, Hungary and France "is the use and consistency of use of appropriate representations." (Harries & Sutherland, 1999, p. 63). In that respect, they are rather critical to the English texts.

Some characteristics of textbooks

The characteristics of mathematics textbooks are reported in several, mainly comparative, studies from all over the world. Sometimes the focus is very broad and sometimes it is rather narrow. Probably the most comprehensive study of textbooks and curriculum material is conducted as a part of the *Third International Mathematics and Science Study*, TIMSS. Textbooks are for this study regarded as one of the representatives for the intended curriculum that could *explain* resulting differences (Robitaille et al., 1993). A considerable amount of material was collected from about 50 participating countries[4]. Three parameters are used to characterize each section of every textbook, syllabus and curriculum guide: subject matter content; performance expectations; and perspectives or context.

The data from the textbooks' analysis part of TIMSS curriculum study shows that textbooks around the world differ greatly in size, length and other structural features. There are also differences regarding the types of chapters and units they contain as well as lay out. In addition, a deeper examination reveals notable differences in sequencing and complexity (Valverde et al., 2002, p. 21).

One of the most obvious physical characteristics of textbooks is the number of pages. In the TIMSS's sample of textbooks[5], the average number of pages is about 125 pages for mathematics textbooks intended for fourth grade. The textbooks for eight grade have 225 pages on average. Extreme numbers of pages are rare. Only about 10 percent of mathematics textbooks have less than one hundred pages. Germany, the Russian Federation and Hong Kong and nine other countries had this type of textbooks. About 10 percent of mathematics textbooks consist of more than five hundred pages and US is certainly an exceptional case. With only one exception, a textbooks intended for nine-year-olds with 484 pages, all of their textbooks have more than 500 pages. To cover all of the content, the amount of time for doing this would be severely limited. But if not all content is covered, there must be some basis for choosing what to be enclosed and what to be exposed (Valverde et al., 2002).

Except pure text, many textbooks contain pictures with different functions. There are for example images for primarily illustrative purposes or photographs that suggest a context. But there are also images to be worked on mathematically, for instance measure length or angle (cf. Grevholm, Nilsson & Bratt, 1988; Love & Pimm, 1996). The sample of textbooks in TIMSS vary substantially in their use of visual aids such as photographs, pictures, tables, and graphs (Valverde et al., 2002).

As regards the physical features of the textbooks, Valverde et al. (2002) note that some textbooks correspond to the description:

– emphasizes exercises and problem solving to a high degree;
– have a small number of strands;
– are of moderate size,

Other textbooks can be described in the following way:

– have many pages;
– cover a moderate array of topics with less focus;
– are fragmented;
– are mostly made up of exercises (Valverde et al., 2002).

The majority of the mathematics textbooks in the TIMSS curriculum study correspond to these two descriptions (43,5% to the first and 47,3% to the second one). In general, the countries have textbooks with different structures but some country patterns are revealed. For instance, all Cypriot mathematics textbooks in the TIMSS study correspond to the first of these two descriptions and almost all US mathematics textbooks correspond to the second (Valverde et al., 2002).

The text in a textbook is often organized in an 'exposition–examples–exercises' model (Love & Pimm, 1996). In the exposition part, one can find sequences of tasks that are intended to support students' concept formation through a kind of 'guided discovery'. However, the text is usually moving to a particular destination and "students are often impatient with the exposition and skip to the 'essential' results" (p. 387). The exposition part is usually followed by examples, which offer a prototype for students to copy for the next part, the exercises. The exercise sets are often 'graded' and progresses from easier to more difficult or consist of parallel sets with different levels of difficulty. Such features have inbuilt assumptions about ability and learning. For example that progressing in small, well-defined steps is the best way to achieve learning (Love & Pimm, 1996).

In Swedish mathematics textbooks it is extremely common that exercise sets are graded according to their level of difficulty. Which level each individual student should work with is usually determined by a diagnosis

in the textbook. For better or for worse, this facilitates individual work by the students but is also a way to differentiate them. How this is done is examined by Brändström (in preparation) through a content analysis with a special focus, inhomogeneity of learners (open differentiation).

An exception from the 'exposition–examples–exercises' model is the 'activities–cours–exercises' model, which can be found in textbooks from France (Pepin & Haggarty, 2001). The 'activities' part intends to introduce a notion to the students through small investigations. The 'cours' describes what needs to be taught, both in words and in worked examples. According to Pepin and Haggarty, this model seems to fit with Piaget's notions of constructivism.

The textbook as a teacher

Some mathematics textbooks contain only problems and exercises, other consist of two separate parts with theory in one part and problems and exercises in the other. Both kinds require support from a teacher who will play a central role in mediating the text to the students (Love & Pimm, 1996). There are also textbooks that have theoretical notes (remarks, clarifications, generalizations, etc.) interspersed with problem, exercises and other assignment. Van Dormolen (1986) notices that a book like this seems to be a teacher in itself. He asks if the author had wanted to write a teacher-proof text and if the teacher just sits back and let students work with the book. Van Dormolen continues his discussion questioning whether it is really possible to write teacher-proof texts. Books of such kind must, on the one hand, teach new concepts that might imply lot of texts, and on the other hand serve as a reference book that demands a short, concise text.

Another relevant question to ask in this respect is if mathematical problem solving is bookable [6]. In other words, is it possible to transform the dynamic process of problem solving into a linear, static text in a textbook? If the text contains a solution to the problem, one obvious dilemma is how to hinder the students from looking at it. Other difficulties concerns how to 'book' blind alleys, incorrect solutions, reformulations of the problem, hints, etcetera (Kang & Kilpatrick, 1992).

A more global question is if textbooks, themselves, can contribute to mathematics learning. The issue is especially relevant to Sweden where students and teachers seem to be extraordinarily dependent on textbooks. In an evaluation of schools in 40 municipalities (out of 290) it is reported that from year 4-5 and onwards, the teaching of mathematics is in principal based on the use of textbooks. Content as well as preparation and organization of the lesson is very much dictated by textbooks.

Hence, they seem to define 'school mathematics' as well as the 'learning path' for the majority of students, at least in lower and upper secondary school (Skolverket, 2003).

Some textbooks are even designed with the idea that students can learn mathematics directly from the text. Since student readers have to be able to work through the texts without help from a teacher, there are some special demands for such books. To eliminate the risk of causing difficulty to an isolated student, thus prevent him or her for asking (relevant) questions, the mathematical demands of a task need to be on a lower level than if a teacher is supposed to be around (Love & Pimm, 1996).

But if student-readers should handle texts with little or no assistance from a teacher, it is important that the text contains *clear* instructions. On the one hand, the instruction should guide the student so s/he can decide what to do. On the other hand, the decision should correspond with what the author or the teacher has in mind. Van Dormolen (1986) shows an example when the task is unclear for the students:

Compute

6 x -3 =

5 x -3 =

4 x -3 =

3 x -3 =

2 x -3 =

1 x -3 =

0 x -3 =

-1 x -3 =

-2 x -3 =

What do you notice?

> It was the teacher's purpose that the students should discover that the product of two negative numbers is a positive number. Everybody thought, however, that they should recognise the pattern, a task that they did not find difficult at all. This exampel shows how it may seem clear to students what they have to do, but the actual activity generated is not what the author had in mind. One can find many similar examples in mathematics books. (van Dormolen, 1986, p. 159).

If we assume that textbooks can contribute to mathematics learning by themselves, is one type of textbook better than the others? Several studies are trying to find answers to that question. Many of them stem from the USA, probably because of their development of new materials that intends to match up to the *Standards*. However, to answer these questions without taking into account variables such as quality of teaching is not sufficient. Nevertheless, there are indications that students who used a certain (reform-based) material perform better on achievement tests than their counterparts who used traditional textbooks (see for instance Reys et al., 2003) and many researcher are convinced that the choice of textbook matters. "It is clear that in these cases, the nature of the mathematical tasks presented to students, their richness and quality, does depend on the textbook used by the teacher." (Chávez, 2003, p. 159).

The textbook as an instrument

On *terminology*

A user of a textbook, whether it is a teacher or a student, develops an individual system to read the book, an individual *utilization schema*. But the emergent schemes are results from a collective process that both users and designers of textbooks contribute to so they have a 'social' dimension as well. Rabardel and Samurçay (2001) describe the process of learning with artefacts as an *instrumental genesis process.*

The subject's instrument is not a 'given', but is elaborated by the subject during the instrumental genesis process. This process concerns both the artifact and social utilization schemes (p. 15, section 2).

The *instrumental genesis process* has two dimensions:

> *Instrumentalization* – the emergence and evolution of the components (e.g. selection of functionally pertinent parts of the artefact)
>
> *Instrumentation* – the appropriation of social utilization schemes, the emergence and development of private schemes (Rabardel & Samurçay, 2001).

Figure 1 illustrates these two dimensions with a bidirectional arrow. The *instrumentalization* is the arrow from *artefact* to *utilizations schemes*. The *instrumentation* is illustrated by an arrow from *utilizations schemes* to *artefact* and demonstrates how a subject (for instance a person) develops a private schema for the specific artefact. For example, a scissor, which is

mainly constructed for the purpose to cut paper, can be most useful for taking out thumbtacks from a bulletin board.

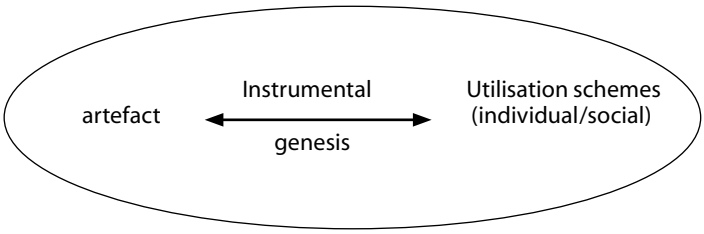

Figure 1: *Schematic representation of an 'instrument'*[7]

In educational research, the 'instrumental approach' is commonly used for studies of teaching and learning with computers or other technical devices. However, considering that textbooks are artefacts that are most important in many classrooms, especially in Sweden, it is meaningful to extend the theoretical framework to also include research on textbooks. Because when a teacher or a student uses a textbook, it becomes an instrument.

The use of textbooks by teachers

In general, textbooks should be *mediated* by teachers, not *replace* them. One reason is that a text will never be able to communicate directly to the students with strategies that a teacher can use, for instance voice tones and commentaries (Love & Pimm, 1996). Even though the book is an instrument for both teachers and students, it is the teacher who primarily influences how the students use their textbook. The discussions in this section are based on results from previous research that consider the actual use of textbooks. Under the assumption that it is the teacher who decides which textbook to use and how to use it, the focus in this part is solely on 'utilization schemes' of teachers.

The variation among teachers' use of textbooks is considerable (Sosniak & Stodolsky, 1993; Stodolsky, 1989; van Dormolen, 1986). Why? One reason is that teachers, even if they work with identical material, have a different understanding of the ideas that the material attempts to communicate (Lloyd, 1999). Another reason is that teachers' use of textbooks is related to their beliefs about mathematics and teaching (Chávez, 2003). The teachers can use their textbook rather consciously, for instance as a source of mathematical and representational ideas from which s/he can

adapt and invent tasks (Remillard, 2000). But textbooks can also be used without awareness (cf. Sosniak & Stodolsky, 1993).

Teachers use textbooks in different kind of activities. Pepin and Haggarty (2001) found that textbooks are used "for teaching in order to lay down rules and conditions; for explaining the logical processes and going through worked examples; and for the provision of exercises to practice." (p. 169). The use of textbooks for exercises is emphasized by teachers in the three countries, France, Germany and England, but the theoretical parts and the worked examples in the textbooks are conceived differently. French teachers present the rules in a different way than the book and German teachers use different worked examples than the textbook (Pepin & Haggarty, 2001).

Do teachers depend upon textbook to a large extent? Well, there is a good deal of evidence that many teacher do (Chávez, 2003; Love & Pimm, 1996). Surprisingly not since the material facilitates teachers' work; the topics are hopefully ordered in the best way and notations are most likely consistent. Moreover, they do not need to be concerned whether a student will have met the necessary pre-requisites for a new topic.

Teachers can defend their decision to follow the textbook closely by arguing that it prevents them from skipping important topics or teaching topics out of an appropriate sequence. However, even those teachers who follow the book very closely do not necessarily cover it as completely as is commonly believed (Freeman & Porter, 1989).

Freeman et al. (1983) categorize teachers' use of textbooks into four distinct styles: textbook-bound, selective omission, focus on the basics, and management by objectives. Teachers that are 'textbook-bound' progress almost page–by–page through the entire book over the course of the year. If the teacher is textbook-bound but skips some chapters, for instance geometry because s/he thinks that the topic is unimportant for students to learn, then the teacher has a 'selective omission' style of use. Certain general topics are emphasized by teachers who adopt the 'focus on basic style', for instance the four fundamental rules for arithmetic. Lessons in the textbooks that are not directly related to basic mathematical concepts and skills are therefore omitted. In some schools, teachers are required to work in a system that is designed to ensure that all students reach a minimal level of competencies in mathematics. The 'management by objective' style of use corresponds to teachers who define all textbook assignments by the system (Freeman et al., 1983).

In a study of four elementary school teachers, Freeman and Porter (1989) find that the greater the extent to which the teacher rely on the text the stronger is the relationship between topic emphasis in textbooks and topic emphasis in instruction. Nevertheless, even the teacher

who followed her textbook very closely failed to use 40 % of the lesson in her textbook.

One can consider whether individual teachers are consistent in their style of textbook use across the subjects they teach. In other words, does the subject matter? Sosniak and Stodolsky (1993) found that single teachers vary considerably in their use of textbooks from one subject to another. For example, the distribution of instructional time with textbooks, for one of the teachers in the study [8] (Carol), was 98 % in mathematics but only 17 % in social studies. Carol reports that she did read the teacher's guide for the mathematics text when the book was new to her, but after one year – she only skims it. "Where reading textbook materials serve Carol as food for thought in the course of building her own reading program, mathematics textbook materials free Carol from having to do much thinking at all about her mathematics program." (Sosniak & Stodolsky, 1993, p. 260). Teachers are rather autonomous, and how they use textbooks for instruction is, according to Sosniak and Stodolsky, not strongly affected by district policy or the nature of the subjects themselves. Sometimes the teachers make heavy use of the textbook materials with careful attention to the full nature and the theoretical perspective they are based on. Sometime they are less aware of the ideas embodied in the materials. Their degree and type of thoughtfulness and purposefulness differs – with varying consequences for the students.

That teachers vary in their use of mathematic textbooks is confirmed in an earlier study of Stodolsky (1989). Generally, topics in instruction correspond to topics in the textbooks and vice versa [9], but it was less agreement between suggested activities in the teacher's editions and actual classroom practice. Manipulative activities and suggestions for enrichment were in particular seemed as dispensable to all of the teachers.

There are several aspects of instructions that textbooks rarely determine, for instance, time allocation and standards of students' achievement. The individual teacher can often decide:

a) which textbooks to use

b) which sections of the textbook to use

c) when and where the textbook is to be used

d) what topics to teach

e) the sequencing of topics in the textbook

f) how much time to spend on each topic

g) the way in which pupils engage with the text

h) the level and type of teacher intervention between pupil and text

So, do teachers, in general, feel that textbooks control their teaching? Well, not the teachers in the study of Pepin and Haggarty (2001) at least. They asserted that they were in charge of their teaching and student learning. A similar opinion was expressed by the teachers in the study of Sosniak and Stodolsky (1993). They did not perceive textbooks to be good or bad, strong or weak, in categorical terms. For these teachers, they were regarded as resources with strengths and weaknesses that teachers themselves had to decide upon.

> Textbooks apparently served as neither a blueprint nor a source of great frustration for the teachers. Instead, texts were merely materials available – tools, props, curricular embodiments (Sosniak & Stodolsky, 1993, p. 270).

Teachers deviate from textbooks for different reasons. One reason is if the suggested teaching method in a textbook does not corresponds to the way the teacher perceives that the subject should be taught. The individual teacher, on the base of his/her own learning experience, can then decide to not use the book (Sosniak & Stodolsky, 1993). Further reasons are that the teacher may judge that the textbook is inappropriate because the language is too difficult or because the exercises are inadequate (van Dormolen, 1986). It could also be the situation that the material is unfamiliar to the teacher and not provide enough pedagogical guidance (Remillard, 2000, p. 345).

Clear pedagogical guidance may however not necessary mean that a curriculum material is used in accordance to the intentions of the authors. The teachers in Lloyd's study, who used a curriculum material that emphasizes exploration and cooperation, saw similar benefits of this way of working. They interpreted however the suggested activities in terms of those benefits differently. One of the two teachers viewed the problems in the program as 'challenging and open to student interpretation – at times *too* open'. Whereas the other teacher claimed that the problems were 'overly structured and did not permit students to solve problems in their own ways or explore concepts sufficiently' and that 'students are led through the problem' – comments that bear remarkable similarity to the other teachers' remarks about the traditional curriculum's exercises (Lloyd, 1999).

So teachers conceive textbooks and other curriculum materials differently, but can they learn from them? Can textbooks change the way teachers teach mathematics? What can we say about the 'instrumental genesis process'? In a study of teachers reading of a reform-based curriculum material, Remillard (2000) found that Catherine, one of the two teachers in the study, made a considerable shift. After only one year with the new material, she changes her view of problem solving and

increases her awareness of its importance and also of her own and her students' abilities to solve problems. But can the textbooks alone form the curriculum? Remillard doubts that they can since different teachers read texts in different ways.

> The variation in the two teachers' reading of the text [...] illustrates the powerful role that teachers play in mediating the textbook's contribution to the enacted curriculum. Consequently, it is unlikely that textbooks can shape the curriculum directly (Remillard, 2000, p. 344).

Chávez (2003) notes another interesting phenomenon; teachers who use reform-based material (or more precise – NSF-funded curricula) "seem to adhere more closely to their textbooks. They are also more likely to read and use the teacher guide" (Chávez, 2003).

Conclusions and discussion

The history of mathematics shows that textbooks are artefacts that preserve and transmit knowledge in the educational systems. But their role in this system is not easily determined; they can be regarded as both important tools *and* obstacles for the development. On the one hand, they facilitate the daily work for the teachers and serve as some kind of guarantee that the students have the necessary basic knowledge and training for the next level in the school system. On the other hand, they seem to reduce both freedom and responsibility of the teachers.

The authority of textbooks is an important issue that needs further discussion. In comparison to other school subjects such as History and Social studies, it is perhaps not so important if mathematics text is regarded as a corpus of 'indisputable' lesson content (cf. Luke et al., 1999) given that it is expected that the book is accurate with regards to mathematics. It is however important to stress that the book presents a selected part of the subject area. It paints a certain picture of mathematics, its role in the society and in the history. But on what basis is this made and what are the consequences?

What appears in textbooks is influenced by several aspects of the educational culture. That is a plausible reason why textbooks from different part of the world are different. However, under the assumption that the textbook is the predominant source in the teaching and learning of mathematics, this means that students are faced with different opportunities to learn (cf. Valverde et al. 2002). But who is responsible for the content of a textbook: the author; the publisher; the educational authority; the mathematical society; the teachers; or maybe the society at large?

In many countries, a textbook needs an approval by an educational authority before it reaches the market. In Sweden, there are no restrains (except economical of course). Which textbook to choose is often a decision of a teacher or a group of teacher. Thus the demands and the expectations of the teachers will influence the development of textbooks. If the teachers' choices of textbooks are more or less unanimous the selection and diversity of textbooks on the market will be more or less limited. Hence, the market offers books that most teachers want so the teachers have to choose among them. An alternative way for the authors/publishers to reach more purchaser is to make textbooks that are "all–in–one", i.e. designed in order to adhere to different styles of teaching. The fact that many textbooks have a large number of pages, which covers more than the class can handle during a school year, can be an indication of that.

Regarding the textbooks as an instrument, this text focus solely on 'utilization schemes' of teachers. My intention was however to put equal attention on the two main persons in the classroom: teachers and students. But I had to learn that very few studies involve the students' use of textbooks. Thus, there is a special need for more knowledge – and research – on the use of textbooks by students. That teacher depend on textbooks to a large extent is one conclusion from previous studies. But at the same time the variation among teachers' use of textbook seem to be considerable. It is also evident that teacher could interpret the vision and aim of a textbook differently and deviate from it for different reasons. This is important to have in mind when we think about and discuss the role of textbooks in classrooms, and especially their position in a reform of the curriculum.

For the understanding of the processes of teaching and learning mathematics, I believe it is important to increase the awareness of textbooks and how they are used. As a predominant source in many mathematics classrooms, textbooks have a unique status. They often determine what is school mathematics and also what is mathematics, for both teachers and students (cf. Skolverket, 2003; Valverde et al., 2002). However, textbooks and their role in the teaching and the learning of mathematics should be discussed in a thoughtful way. In order to achieve this goal, we need to analyze the situation in classrooms. Not only the *content* of textbooks and *how much* textbooks are used in relation to other activities should be analyzed, but also *how* and *why* textbooks are used. Studies of the textbooks and the use of them in classrooms would also offer further knowledge about the practice of mathematics teaching. This can for example be done through a combination of: observations in classrooms studying the actual use of textbooks by students and teachers; teachers'

interviews asking for the role textbooks play in their professional life as teachers; an analysis of teachers' logbooks and content analysis of the textbooks in use.

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Notes

1 With an exception of quotations, I have chosen to use the word *artefact* in this paper, which is chiefly the British variant of *artifact*.

2 Currently *Norwegian Non-fiction Writers' and Translators' Association*

3 Harries and Sutherland compare textbooks from England, France, Hungary, Singapore and the USA.

4 The resulting sample included for instance 241 curriculum guides and 318 mathematics textbooks (Schmidt, McKnight, Valverde, Houang and Wiley, 1997).

5 The TIMSS curriculum study contain 318 mathematics textbooks from the participant countries, selected for the purpose to represent material that no less than half the students in each TIMSS population are likely to have in their classes.

6 Kilpatrick [1980] asked the question, 'Is problem solving bookable?' at the 58th annual meeting of the National Council of Teachers of Mathematics in Seattle

7 The picture originates from Strässer (2004)

8 Four teachers teaching in fourth grade participated in this study.

9 Six elementary school teachers took part in this study

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Sammanfattning

Denna artikel beskriver läroboken utifrån två aspekter, läroboken som artefakt och läroboken som instrument. Resultat från studier av läroböcker från olika delar av världen samt studier av hur läroböcker används i klassrummet behandlas. Syftet med artikeln är inte att ge en heltäckande bild av tidigare forskning inom området utan snarare att lyfta fram några viktiga frågeställningar.