Shared cognitive intimacy and self-defence:

two socio-emotional processes in problem solving

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This paper is an exploration into the social dimension of emotions. It is based on protocols of student problem solving sessions and comments from student interviews that are part of a three-year longitudinal ethnographic study of one classroom. Focusing on the role of emotions in social coordination of problem solving behaviour, this research extends into an area that has not received enough attention in previous research. Specifically, it will provide some insight into shared cognitive intimacy, which was occasionally experienced by students who constructed a solution to a problem together. Shared cognitive intimacy is an example of a situation where students can simultaneously fulfil both cognitive and social needs. Another phenomenon that was observed was how some students used defensive strategies to hide their lack of competency. These defensive strategies are often counterproductive in achieving the cognitive goals but they are functional in the service of social goals.

Introduction

Mathematical thinking is often regarded as the ultimate rational act where emotions are non-significant. Much of the research on problem solving has concentrated on the individual cognitive processes (e.g., Schoenfeld, 1985; Silver, 1987; Presmeg, 1999). The researchers have acknowledged that problem solving is not solely rational. The non-rational aspects of problem solving behaviour have been explained with beliefs that the solver has. A recent turn in brain studies, however, indicates that emotions are an essential part of rationality (Damasio, 1994).

Within mathematical problem solving the role of emotions as an important aspect of problem solving has been elaborated especially

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by Goldin and DeBellis (DeBellis and Goldin, 1997; DeBellis, 1998; Goldin, 2000; 2002). According to Goldin and DeBellis, affects are not merely 'noise' of human behaviour in problem solving, but a representational system parallel to, and crucial for, cognitive processing. This affective representational system is divided into four facets of affective states, which interact on the individual level: emotional states, attitudes, beliefs, and values/morals/ethics. Interactions with the socio-cultural context and the affective climate of the environment are also included in this model.

Emotions also have another function that becomes important in cooperative problem solving. Social processes are to large extent mediated through emotional messages (facial expressions, tone of voice, postures), which are often expressed and decoded unconsciously. The reform movement in mathematics education has emphasized problem solving and cooperative learning (NCTM, 1989; 2000). However, several studies indicate clearly that the implementation of cooperative problem solving instruction is non-trivial (e.g., Webb, 1991; Edwards, 1999). Those who study social interactions in mathematical thinking and learning have not usually considered the role of emotions. For example, in the Vygotskian research tradition the role of emotions has not received much attention.

Theoretical Background

There is no final agreement upon what emotions are, but there is large agreement on certain aspects. First, emotions are seen in connection to personal goals. Emotions are also known to involve a physiological reaction, as a distinction from non-emotional cognition. Thirdly, emotions are also seen to be functional, i.e., they have an important role in human coping and adaptation (e.g., Buck, 1999; Lazarus, 1991; Power and Dalgleish, 1997; Mandler, 1989).

To understand the different roles that emotions play on different levels of theorising, we use the typology of emotions by Buck (1999) (table 1). Buck distinguishes three 'readout targets' that are the different ways in which emotions are 'coded' or 'represented' in humans: an autonomic/endocrine/immune system responding (I), expressive behaviour (II), and subjective experience (III). Different readout targets relate to different functions of different emotions and to different ways of 'learning'. For example, the primary function of anger is to adapt the body for fight (I), but at the same time, it also has a characteristic expressive behaviour that allows social coordination (II). If anger proves repeatedly successful, it will lead to physiological adaptation (higher levels of testosterone) and a social development (becoming more dominant). Interest, on the other hand, primarily regulates cognitive development. Its primary read-out function is self-regulation (III) and it does not have clear expressive behaviour (II) or bodily reaction (I).

Table 1. *Typology of emotions (Buck, 1999)*

	Readout target	Readout function	Accessibility	Learning
I	Autonomic/endocrine/ immune system responding	Adaptation/ homeostasis	Not accessible	Physiological adaptation
II	Expressive behaviour	Communication/ social coordination	Accessible to others (and self)	Social development
III	Subjective experience	Self-regulation	Accessible to self	Cognitive development

If we now look at the possible roles that emotions may have in collaborative problem solving, we can distinguish three different types of social emotions:

1) Students in a social setting will have interpersonal relationship needs and goals (Boekaerts, 1999; Lemos, 1999). Meeting or failing to meet these needs will induce emotions. For example, involvement in group activity is usually a source of positive emotion and being excluded is a source of negative emotion.
2) Students in the group also have emotions related to their individual learning goals (e.g., frustration or joy). When they attribute the cause of failure or success to peers, these emotions extend to the social level. Students may feel angry or grateful towards peers, based on their interpretation of causes of outcomes.
3) Some emotions are related to the social coordination of individual goals. When students have different ideas of how (or whether) to proceed with the task, emotions will play an important role in the social coordination of actions. Likewise, when students share similar ideas, emotions will also play a role in coordinating actions.

These three types are not mutually exclusive, because an emotion may have more than one relevant social aspect. For example, a person who cannot contribute to a solution process may feel bad because of being excluded from the collaborative activity. This may lead to anger towards peers who excluded him/her. Furthermore, expression of this anger might induce guilt in peers, who might then make an effort to include the outsider.

Emotions related to social goals and emotions attributed to peers are rather straightforward and uninteresting. However, the role of emotions as a coordinator of collaborative behaviour is interesting. In this coordination, emotions can be expressed and interpreted unconsciously, when the participants themselves may be unaware of the role of emotions. Emotions may also be used consciously in power games or as means of solving communication problems. Furthermore, emotions may be interpreted consciously, when they may become subject to reflection and re-evaluation.

Methodology

This study is part of a longitudinal ethnographic study of one mathematics classroom, which will be only roughly outlined. The three-year-long study was done in a Finnish lower secondary school (grades 7 to 9). For two years, the researcher taught mathematics to one class, and in the third year, the researcher observed and/or video recorded several of the mathematics lessons. The students were interviewed twice each year, and several informal discussions provided further information. Furthermore, parents and primary school teachers were interviewed. A research assistant observed several lessons during the second year of the study and shared his views of the students in the class. Altogether, the study provided rich data about the students and deep tacit knowledge of them. The content of the tacit knowledge is hard to put into words (that's why it is called tacit), but it includes such elements as knowing (feeling?) when students are anxious and when relaxed. This tacit knowledge has guided interpretations of the data.

Furthermore, use of multiple frameworks to analyse students' beliefs and attitudes has enriched my understanding of these students (e.g., Hannula, 1998; 2002a). Especially relevant for the present article are the analyses of the goal structures of some of the students (Hannula, 2001; 2002b), which I will use to characterise the motivational traits of students.

This paper is based mainly on one part of the first interview done during the first year of the study. In this interview, a total of 22 students (then aged 13 to 14) were interviewed in groups of three or four. The interview was semi-structured, with topics ranging from previous experiences in mathematics to what they like or dislike in mathematics and their future career aspirations. At the end of the interview, I gave them three problems to work on, one at a time (Appendix 1).

The first problem was to define the number of colours needed to colour a figure. When students had received an answer for this specific figure, they were challenged to consider the more general case of colouring any planar figure (the four-colour-problem). The second problem was to estimate the number of letters in a novel which was handed out to pupils. Furthermore, a time limit of five minutes was set for the task. The third problem for students was to calculate the product of a specifically defined operation [$a \oplus b = (a + b) \cdot (a - b)$] with three pairs of numbers. As an extension, they were asked whether this operation is commutative.

The interviews were transcribed using a discussion analytic coding system (Appendix 2), which allows keeping track on interruptions and pauses.

Data and Interpretations

The first impression from the problem solving sessions indicated that the social interactions and the emotions varied from group to group and task to task. From the case study of Rita (Hannula, 1998; 2000; 2002a) and the first three analysed problem solving sessions I recognised the following phenomena that I decided to concentrate on in further analyses in order to see how typical they are. The two phenomena that were observed as part of the case of Rita were a shared cognitive intimacy between students when they were generating understanding together, and the defence strategies used by students who did not understand what the other students are doing.

Emotions as coordinator of productive cooperative behaviour

In this episode, we shall see productive cooperation by Tina, Elisa, Jaana, and Lilli. These students represented different levels of achievement (according to my evaluation as their teacher): Jaana had problems with mathematics, Lilli was very good at it, and Tina and Elisa were doing all right. Jaana, Lilli, and Elisa belonged to the art 'faction' of the class and they knew each other well. However, both Lilli and Elisa as well as Tina did freely interact with all students in the class. I would describe these students' motivational traits in mathematics as follows: Jaana tended to focus on avoidance in mathematics class, Tina tended to focus on performance, and Elisa and Lilli tended to focus on learning mathematics. In the following vignette the group works on the second problem: estimating the number of letters in a book that was handed over to them (see Appendix 2 for the transcription codes).

Jaana?: [three<] 367 {pages}
Elisa: And how much is here {on one page?} now these roughly.
Tina: {If you look} roughly, so<
Elisa: Roughly (--)
?: [(--)]
Tina: [(Hey-) if] you count half a page, and how many are there, and multiply by two and then times the [number of pages]
Elisa: {Tone of strong dislike in her voice} [Lett]ers. Really lot of lettershhh. Guite a lot. Umhh.
?: Okay. One two, three,
Several students: {laugh}
?: Don't [have time]
Elisa: $[Don't have time] to do it like that$. But see [here (-)]<
Tina: [If you take] one fourth?
Lilli: How many in one line?
Lilli?: One, two, three, four, five, six, [(--)]=
Tina: [That's quite good!]
Lilli?: =nine, ten, eleven, twelve, *(--)* (:05)
Tina: $Ho(h)w many lines$ are there then?
Lilli: About fifty letters in one line.
Jaana: How many lines are here? One, two {etc.} ...

Then the group finishes the task by multiplying the number of letters on one line with the number of lines on one page and multiplying the result by the number of pages in the book, which is a valid strategy.

The confused period in the beginning was long enough to indicate that no one had a routine solution for this task. One of the girls checked one piece of information (number of pages) that was easily available. Tina presented her idea for the group. Elisa gave an immediate response, where the emotion (tone of voice) conveyed half of the meaning. At this point, there was tension within the group regarding the proposed strategy. Despite the negative appraisal of proposed method, someone began counting the letters. It may have been intended as a joke or not, but it was taken as one anyway. Here, laughter served as a means to release tension as the proposed strategy was evaluated as too time consuming. Tina refined her proposal, and Lilli came up with a new strategy, and started counting the letters of one line. Tina accepted the new idea with appreciation.

I see this episode as an ideal example of a collaborative action where emotions have a significant role in the social coordination of the group behaviour. Firstly, all group members were involved in the process. Secondly, the group was effective in their process, although it was not straightforward. Thirdly, they also enjoyed doing this task, as they indicated later in the interview. Fourthly, in this episode, emotions had a significant role; even negative emotions mediated the construction of a common understanding.

Shared cognitive intimacy and a third wheel

In the following episodes, you will see how Maria and Lisa went into intensive interaction while solving two tasks. Unfortunately, the third member of the group, Rita, was left as an outsider. Maria and Lisa belong to the 'art faction' of the class, and they mostly interact in the class with other arts students. They both are also very good at mathematics. Rita, on the other hand, is a socially active 'non art'-student who has achieved below average in mathematics. The dominating goal for Maria in the classroom is learning, and performance is an important subgoal (Hannula, 2001; 2002b). The other students' goals are not so obvious.

When Maria, Lisa, and Rita start working on task 1 (see Appendix 1), Maria and Lisa soon find each other, and they go into an intensive interaction. They address each other, and Rita is left as an outsider. After a while, Rita tries to participate the discussion, but she is ignored at first. Becoming part of the interaction requires some persistence.

Maria: And so this could be like blue, red. If this was blues, so these could be both red (--). This one? And that could be again blue, blue, blue, red, [red, red]
Lisa: [Because< so if that's] always four, like (-) four here=
Maria: =yes [(-) can't]=
Lisa: [four corners]
Maria: =there can be two [colours]
Lisa: [there] can be like this
Maria: Mm
Lisa: but then (:02)
Maria: (for example [--])
Lisa: [But look.] But if this then is (--)
Maria: Uhm
Lisa: And so, those then (-)
Maria: Aren't they blue, blue, red, red=
Lisa: =But look, it (may not except) at corner
Maria?: Ahem.
Rita: I don't like this t(h)ask at all.
Maria: Wait, it there is so that (.) (-) this would be blue. And that would be red, and that'd be red. That one could be blue again. And thiis could be< Could this one be blue? That is=

Lisa: =[yeah]
Maria: =[a little] like in the corner [(-- connecting)]
Lisa: [(-- three)]
Maria: But if that one were. Krhm. Yellow. Then this could be blue again (--)
Lisa: (- too)
Maria: What could this be? Krhm. That could be red and blue, so this would be ye[llow]
Lisa: [ellow]
Maria: And then=
Lisa: =But (-)
Maria: This could be red again. (0:02) Couldn't it?
Rita
& Lisa: Yellow. Uhm. Yes {2.51}
Maria: [(-this then<)]
Rita: [Is this then] yellow, 'coz that (-). {2.53}
Maria: How about this one? This could be bl[ue. Because it does not touch]
Lisa: [(ue. Because (-) touches] at corner
Maria: Blue. Then this one. Umm, yes
Rita: [Hey! How come it's blue then?] {3:10}
Lisa: {to Maria} [Yes, probably it would go] with three colours.
Maria: {to Lisa} Yes, with three colours.
Rita: Because that one is yellow. (3.0) Hum?
Maria: Where?
Rita: Why you put that blue?
Lisa: Which one?
Maria: Well, may not touch [except with a corner.]
Lisa: [Ah, this piece] It may not be red, because these are red. And it could be yellow, of course.

When they work on the third problem, applying a new operation ⊕, a fairly similar thing happens. Maria is the first one to grasp the idea and Lisa picks up her explanation easily. They start working in intensive interaction, and when Rita tries to participate she is not listened to.

Lisa: No+but. But there's two minuses. Or is [it like]
Maria: [minus 1] times minus 6
Rita: That will be [minus (-)]
Lisa: [(-) THEN] {talks over Rita}
Maria: minus 1 times *minus 6*
Lisa: Did we count this (somehow) wrong? Will it be plus 6?

Rita {to Markku}: Will it be plus 6?
Maria: I think it won't be, there's =

In these episodes, Lisa and Maria were intensively interacting, while Rita remained an outsider. Originally, I started the analysis from the exclusion of Rita. However, when trying to find the mechanism of exclusion, I realised that the intensity of the interaction is the dominating phenomenon. The intensive interaction between Lisa and Maria was characterized by how one student frequently continued or completed what the other had said. At times they even spoke in unison. There was very little need for explanations, as understanding was almost immediate. As an outsider, it was often difficult to see how the understanding was communicated.

This phenomenon seems to relate to the notion of addressivity (Gordon Calvert, 1999). In the group two of the students are talking to each other, and they are listening to what the other is saying. To explain it further, we will use the term 'mathematical intimacy'.

Mathematical intimacy is the feeling of closeness with the task that a person may experience while engaged in mathematical problem solving. It is intrinsically rewarding to the individual, and may lead to such focusing that one is temporarily unaware of external noises (DeBellis, 1998).

In the given example, there is, in fact, a dual intimacy. On one hand, there is the mathematical intimacy with the task, and on the other hand, there is the social intimacy with the fellow problem solver. Although in these examples the dual intimacy is around mathematics task, I do believe that it is characteristic to all cognitive activities. Therefore, I have labelled the phenomenon **shared cognitive intimacy**.

When I had recognised this phenomenon, I looked for it in all groups. I began with a holistic view of episodes, but gradually recognised the following characteristics of shared cognitive intimacy: pleasant atmosphere during the problem solving process, frequent overlapping of utterances and occasional speaking in unison, and positive emotional evaluation of the process afterwards. Shared cognitive intimacy seemed to be a rather common phenomenon. It was observable in every group, and most students got into it at least for short times. Shared cognitive intimacy was intrinsically rewarding for those experiencing it.

As a backside of this intimacy, it seems to easily lead to ignoring other peers. When Rita wished to contribute, she was ignored (task 1) or even quite aggressively denied access to the interaction (task 3). I do not see evidence that it was done on purpose. More likely, Maria and Lisa were so deep in their intimacy and so focused on their own process, that they did not observe Rita's effort to be included.

For some students, intimacy may also be a threat. In this classroom Maria, Tina, and Laura clearly expressed their preference for solitary work to group work. Tina and Laura were also the clearest examples of students whose primary motivation is to perform in class, and for Maria it was an important secondary goal. It may be that performance goals are more difficult to accomplish in small group settings. Performance-motivated behaviour may be counterproductive in group work. Tina effectively disengaged her group from problem solving (analysed later). Sometimes performance-oriented students may also threaten the emotional climate of the group. For example, Laura made sarcastic comments to Airi during problem solving: ”Well, you tell now, you are so intelligent!”

Defence strategies

When emotions function as means of social coordination, they do not only have a positive role. Emotions are also used as weapons and camouflage in power struggle. Some students, when they lost their trust in finding the solution, seemed to change the game they played. If possible, they often gave up the task altogether. However, if other students continued with success, more sophisticated defence strategies were applied:

1) stop really trying and just answer something (express lack of interest);
2) show disinterest or contempt towards the task;
3) give an answer, and say that the answer is wrong (express lack of interest); and
4) laugh at one’s own mistakes (expression of relief).

The first sub-strategy is functional because it changes the attribution of failure from talent to effort. The second sub-strategy works in a similar way, but also lowers the value of the task. The third sub-strategy may be used to prevent a complete failure. After all, either the answer or the evaluation of that answer is going to be correct. Furthermore, such an approach indicates low commitment to one’s own answer, which will lower the significance of possible failure. Finally, the fourth sub-strategy imitates a spontaneous laughter of students who recognise and correct a ’silly’ error they have made.

These strategies are not used often, but students who use them seem to apply several different strategies. Based on this small sample it seems that the students who use them are usually less concerned with learning than they are with showing their ability (or hiding the lack of it). Thus, these defensive strategies could also be used as diagnostic indicators for finding students whose primary motivation is not learning. Naturally, more empirical work is needed to refine the picture. The self-defensive strategies are often counterproductive for the individual student's learning goals, for the group's learning goals, and for the group's emotional climate. Therefore, it would be useful to diagnose these students in the classroom and concentrate on their cooperative skills while engaging the class in co-operative tasks. It might also be useful to assign them such roles in the group that would focus their attention on process instead of product.

Discussion

This paper has described examples that indicate the relevance of socio-emotional phenomena in cooperative problem solving. One example illustrated the potential power of emotional communication in the coordination of collaborative problem solving behaviour. Shared cognitive intimacy was recognised as a phenomenon that takes place quite frequently in problem solving. In shared cognitive intimacy, students enter an intimate interaction with each other and with a task. This intimacy is indicated by frequent continuing or completing the other student's utterances and occasional speaking in unison. It is an example of a situation where students can achieve their cognitive and social goals simultaneously. This dual intimacy with peers and mathematics is rewarding for the students and, furthermore, it can be an extremely useful tool for enhancing the classroom climate. A problem with this kind of intimacy is that sometimes it may exclude other students.

Such pleasurable, shared intimacy with a mathematical task is also described by Williams (2002). Similar to the examples presented here, the students' interaction in her example was characterised by frequent completing or extending of other student's utterances. Students were also very focused and ignored such distracters as a buzzer (denoting the end of collaboration time), the teacher, and the third group member. Williams uses the concept of 'flow' (Csikszentmihalyi and Csikszentmihalyi, 1992) to explain the experience of positive affect when task demand is optimal to the skill level of the solver.

There is clearly need for further research on the conditions that make such shared cognitive intimacy possible. Does it require working at the limit of one's competencies as the concept of flow suggests? How important is the prior relationship between the collaborators? Does it only occur between two students at a time, or could a larger group also reach

such intimacy? Anyway, we researchers should keep our eyes open for this phenomenon.
Another phenomenon was related to mistakes and giving up efforts when success does not seem likely. Some students use different strategies to create a more positive image of their competence in these situations. The used strategies include (1) attributing failure to effort instead of talent, (2) devaluing the relevance of the task, (3) evaluating one's own answer as incorrect, and (4) strategic laughter at one's own mistakes.

In the review of literature, it became apparent that affect is an important regulator of problem solving behaviour. The examples presented in this paper indicate that affect can also play an important role in the social regulation of problem solving behaviour of a group. Emotions may be a pitfall of collaborative work, or an important element in and motivator for it.

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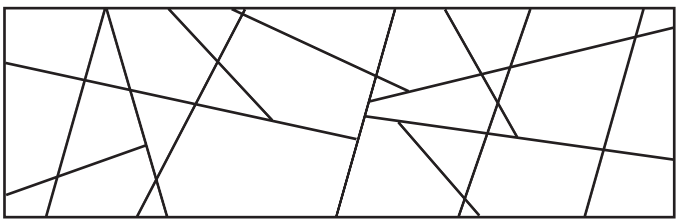
Appendix 1

Task 1.

Salla is working on an abstract painting. She has divided an area with straight lines into parts. She wants to paint the picture with as few colours as possible. Parts that are side by side may not be of same colour, but those touching only in corners may. How many colours will Salla need?



Task 2.

Estimate in five minutes how many letters did Aleksis Kivi write to his novel, "The Seven Brothers". (The book was handed out to the pupils. It had 367 pages, 33 lines on a page, and approximately 50 letters per line.)

Task 3.

Addition, subtraction, multiplication and division are operations. Let's define a new operation $\oplus$ in a following way:

When a and b are numbers, then $a \oplus b = (a + b) \cdot (a - b)$

An example: $2 \oplus 3 = (2 + 3) \cdot (2 - 3) = 5 \cdot (-1) = -5$

a) Do the following calculations:

$2 \oplus (-3) =$

$(-2) \oplus 3 =$

$(-2) \oplus (-3) =$

b) Addition is commutative. For example, $2 + 3 = 3 + 2$.

Is the previously defined operation $\oplus$ commutative?

Appendix 2

Code-key for the transcriptions:

(x.y)	a pause, x.y seconds
(.)	a pause, less than 0.5 seconds
(-)	unidentified word
(--)	several unidentified words
(*text*)	unclearly heard words
text	text spoken quietly
<*text*>	text spoken slowly
$*text*$	'smiling' voice
wo(h)rd	word has been spoken while laughing
[*text1*]; [*text2*]	texts 1 and 2 spoken simultaneously
word+word	words 'glued' together
wo-	interrupted word
text<	interrupted flow of speech
=	talking continues immediately after the other speaker (cut/latching)
{text}	about context or tone of voice
{...}	text omitted
?:	unidentified speaker
name?:	uncertain identification of a speaker

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Sammanfattning

Denna artikel utforskar den sociala dimensionen av känslor. Den baserar sig på material från elevers problemlösningssessioner och intervjuer som ingår i en treårig etnografisk studie av en klass. Artikelns fokus ligger på känslornas roll vid social koordinering av problemlösandet, och härmed utvidgar den forskningen till ett område som hittills inte har fått tillräckligt med uppmärksamhet. Speciellt erbjuder den en insyn i en gemensam kognitiv intimitet, som ibland upplevdes av elever som tillsammans konstruerade en lösning till ett problem. Den gemensamma kognitiva intimiteten är ett exempel på en situation, där eleverna samtidigt kan fylla både sina kognitiva och sina sociala behov. Ett annat fenomen som observerades var hur några elever använde defensiva strategier för att dölja sina kunskapsbrister. De defensiva strategierna motverkar ofta uppnåendet av kognitiva mål, men de är funktionella med avseende på andra mål.