Review

Windows on Mathematical Meanings

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Noss, R. & Hoyles, C. (1996) Windows on Mathematical Meanings (Mathematics Education Library, Volume 17). Dordrecht, NL: Kluwer Academic Publishers, 275 p.

In September 1966 Scientific American had a special issue on Information. In this issue Patrick Suppes had an article (Suppes, 1966) on the uses of computers in education. His opening paragraph in the article ended like this:

> *One can predict that in a few more years millions of schoolchildren will have access to what Philip of Macedon's son Alexander enjoyed as a royal prerogative: the personal services of a tutor as well informed and responsive as Aristotle (Suppes, 1966, p. 207)*

With this optimistic opening Suppes goes on to discuss various possibilities of the use of technology in education. Looking back over the thirty years that have passed, we can say that this development has not quite happened the way Suppes predicted. We may also add that there have been many people expressing a similar optimism over the years, and still not much has happened that has had consequences for all students.

> *There were 'dreams' about computer-using students ... dreams of voice communicating, intelligent human tutors, dreams of realistic scientific simulations, dreams of young adolescent problem solvers adept at general purpose programming languages - but alongside these dreams was the truth that computers played a minimal role in real schools...*
> *(Becker, 1982. Quoted in Noss and Hoyles, p.159)*

However, towards the end of the article Suppes expresses some hopes:

> *It is to be hoped that, as systematic bodies of data become available from computer systems of instruction, we shall be able to think about these problems in a more scientific fashion and thereby learn to develop a more adequate fundamental theory than we now possess. (ibid., p. 219)*

Even though the use of information technology in education has been an active field for almost 40 years, it is not the education community that has led the development in this area. Many people have had the feeling that education has desperately tried to keep up with the development in hardware and software.

There have been several noticeable developments over the years. The theoretical framework put forward by Seymour Papert and others in the development of the Logo language has had a major influence in education, but has not influenced the education of all children, and now seems to be in the shadow of the present developments of using multimedia and the Internet. Another line of development that had a strong position for some time - not only in mathematics education, but in education in general - was programming, i. e. describing a construction of an object, in some formalised language. More recent developments in computer use have been towards action and not description.

After more than twenty years with a very strong focus on the use of computers in mathematics education, we must conclude that we have not had an integration of information technology into mathematics in schools in general.

Has the lack of a theoretical framework been an obstacle to the use of computers in education? Do we still need "a more adequate fundamental theory", to use Suppes' formulation?

The book by Celia Hoyles and Richard Noss is an important contribution in developing an "adequate fundamental theory". The scope of the book is wide: "[to] range over the epistemological, cognitive, cultural and methodological" concerning the use of computers in mathematics education.

Celia Hoyles and Richard Noss have worked together on developing Logo environments for many years ("part of the Logo story" as they formulate it in the book), and in this book they draw on the experiences they have had in this area. The influence of Seymour Papert and the group at MIT (Massachusetts Institute of Technology) is acknowledged and very visible throughout the book. The scope of the book is theoretical, and the task they set themselves do is to lay the theoretical foundations of the relationship between mathematics education and the use of computers.

The book consists of ten chapters:

1. Visions of the mathematical
2. Laying the foundations
3. Tools and technologies

4. Ratio world
5. Webs and situated abstractions
6. Beyond the individual learner
7. Cultures and change
8. A window on teachers
9. A window on schools
10. Re-visioning mathematical meaning.

In the first chapter the authors present their view of learning mathematics, its role in society, and the computer. There are many metaphors in the book, and one important metaphor is presented in the first chapter which is also in the title of the book - «*Windows*»:

> *Windows are for looking through, not looking at. It is true that windows mediate what we see and how we see it. Equally, windows can, at times, be objects for design and study. But in the end, what counts is whether we can see clearly beyond the window itself onto the view beyond. (p.10).*

Chapter 2 is both an overview on views of mathematics and a presentation of the authors' own view. One of their objectives in this chapter is to focus on meaning in mathematics and mathematics education. They draw upon many sources, e. g. psychology (Piaget and Vygotsky), ethnomathematics (street mathematics), and their own work with Logo.

The emerging discussion concentrates of context and abstraction. The authors disagree with the view expressed by Anna Sfard and others, that "mathematics is hierarchically organised into a spiral of abstractions, which are recursively reproduced – the stage of reification, in Sfard's terms, being the starting point of the next cycle." (p. 20-21). One concern they have with this view is that the social dimension of learning is ignored. Discussion of the social dimension in learning mathematics is present throughout the book, and is a major aspect of their attempts to build a theory of learning mathematics.

The computer is not mentioned directly as part of their argument in this chapter - except for in examples - but the authors are indirectly making a case for giving it a role in their model. They conclude the chapter by presenting their view on abstraction. They see problems with "mathematical learning ... as the acquisition of context independent knowledge within a hierarchical framework" (p.48). Hoyles and Noss view "abstraction as a process of connection rather than ascension". At the end of the chapter the computer enters the presentation: Setting acts back on thinking and tools can open new representational windows. This view is elaborated quite extensively in Chapter 3.

It is to be expected, with the authors' experience with Logo, that programming would enter their discussion. Programming underlies much of the presentation in this chapter. A strong case is made for programming (newer versions of Logo and other types of programming) and microworlds - as windows on mathematical meaning. The presentation of programming and microworlds is also an interesting historical outline of the development where they trace the origins and developments of programming concepts.

In chapter 4 we find a description and discussion of a microworld - "Ratioworld" - for working with ratio and proportion. The microworld is based on using Logo elements. For the reviewer this part is especially interesting because of its connection with diagnostic teaching. Several of their examples give inspiration to the use of computer programs as diagnostic tools. The activities also invite the use of other types of software, e.g. spreadsheets, as microworlds, but this is not discussed explicitly.

The authors return to the theoretical foundations in chapter 5. They point to the fact that "... all technologies inevitably alter how knowledge is constructed and what it means to any individual" (p.106). They build on and adapt several of Vygotsky's ideas and concepts to the situation of having the computer as a tool. The notion of a support system (scaffolding) is developed using the idea of a web. Webbing is presented as an extension of scaffolding in several ways, e.g. the learner gaining control of the support structures. They also present several analogies with the Web (www - world wide web). After some examples to illustrate the concept of webbing the authors return to their main concern, i. e. abstractions.

Starting with the notion of situated cognition, i. e. that " mathematical knowledge becomes bound into a setting" (p.121), they arrive at different versions of mathematics: LogoMathematics, PencilMathematics etc. One major point is to discuss these notions and to develop a coherent theoretical view of mathematical meaning. This attempt is perhaps the most important part of the book. For some of us, the new mathematics we experienced through working with Logo was at one level very different from traditional mathematics (geometry), perhaps to be called RulerAndCompass-Mathematics using the language of the book. Here the authors give a convincing argument, using the notions of situated abstractions and webs, that there is a theoretical framework which is possible to encompass several types of mathematics.

In the beginning of the book, Hoyles and Noss point out the importance of the social element of education. In chapter 6 this is discussed further, based on their theoretical considerations. They describe the role of the computer in a large number of settings and issues, e.g. software as a medium through which shared mathematics can be constructed and critically observed.

The next three chapters concentrate on the issues of innovation and change, which of course are very relevant to the study of the introduction of computers in education. In chapter 7 a very interesting overview of the development and fate of Logo is given - not only in the UK but also in some other countries.

The focus on Logo is both a strong and a weak point in the book. It is a strong point because it is a good example of the historical development of computers in education, and it is interesting for those of us who at one time were fascinated by, and working in, Logo. It is a weak point because the authors are so deeply involved in the Logo culture, and it is questionable how much this development will interest readers not familiar with Logo language.

From a "Logo point of view" the discussion of the criticism from Roy Pea and Midian Kurland is very interesting. Hoyles and Noss do not excuse the fact that this argument is comes ten years late, and try to elevate their presentation to a more general level. In my opinion they have not entirely succeeded with this task, since the discussion focuses most of the time directly on issues raised by Pea and Kurland. On the other hand, the issues they raise are important and must be interesting for those who followed the controversy that Pea and Kurland's articles raised. Moreover - as the authors point out - the articles by Pea and Kurland are still a source of criticism of Logo and related issues.

In the two following chapters Hoyles, and Noss focus on two central elements relating to innovation and change; the teacher (chapter 8) and the school culture (chapter 9). They present their main points through examples - case studies of teachers and schools, and how changes and innovations affected kinds of teachers and schools. Here the authors provide valuable insight into innovations in schools. The description of the difficulties and problems in a school is probably representative even though they only consider some particular schools in the UK.

The relationship between Logo and mathematics is discussed with reference to the students' and teachers' view. Some could not see any connection between Logo and mathematics, whereas others could see a close relationship. The authors point out that such conflicting views are not easy to resolve, but they are somewhat "cautiously optimistic" that their theory - or perhaps their ways of looking at the situation - might lead to change.

The conclusion in the last chapter starts with a review of the elements of the theory, and they go on to consider some examples of what they call "epistmological revisions of mathematical activity":

Logo microworld to explore non-Euclidean geometry
Programming to reconnect with mathematics
New geometry with new tools (Cabri geometry)
Revisioning number patterns
Convivial mathematics *) (The rugby problem)
Connecting mathematics with practice (working with economic models)

*) The term convivial is taken from Ivan Illich. The following definition is quoted from Illich (1973)

> *Convivial tools are those which give each person who uses them the greatest opportunity to enrich the environment with the fruits of his or her vision.*

In conclusion they state an aim for their work:

> *Our aim is to change school mathematics and perhaps, in time, play some modest role in changing mathematics itself; not in the sense of rejecting the cultural and scientific heritage that underpins mathematical thought, but in the sense of broadening it, finding ways for it to re-enter the intellectual and cultural lives of individual and groups. (p.257)*

How can we then view the book in the light of this very ambitious aim?

In the fifty years (or so) that computers have been available for mathematicians to use in their work, there has been no coherent view of the place they have within mathematics. The same is even more true for their use in mathematics education, where "different" mathematics have been constructed, e.g. LogoMathematics. Hoyles and Noss have presented a system or theory of how we can integrate the different kinds of mathematics into a unified frame.

For many it is obvious that mathematics, and hence mathematics education, will change in the years to come because of the new technology. The book can be seen as a first important step in this development. The strength of the presentation is the discussion and problematization of the issues involved. Hoyles and Noss raise a number of issues that will have to be addressed. However, the book indirectly raises many questions which it leaves unanswered.

The book is set within a mathematical and epistemological framework. We might ask about the relationships to the world outside mathematics - how applications in other areas and disciplines influence the development of mathematics and education. Constructing a mathematics curriculum for schools depends on many choices and priorities; which choices do we have? which priorities should we make?

In conclusion, I would say that the book is very readable, and contains a number of interesting details as well as the construction of a theory. It is one important step in developing a theoretical framework for the use of technology in mathematics education. It is also a reminder that we should look back on past experiences, e. g. with Logo, with programming and the use of other tools, which we tend to forget in this world of multimedia and the Internet.

References

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