STRATEGIC COMPETENCE:
Issues of task-specific strategies in arithmetic

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This article focuses on the term strategy and on strategy use in elementary arithmetic. The central theoretical viewpoint in the article includes strategy variability as a fundamental characteristic of mathematical cognition. The theoretical and research base for the acqusition and development of strategic competence is reviewed, and the characteristics of children are discussed from a developmental perspective. Across all areas, the primary focus of this review is to provide a framework for studying the differences, if any, between mathematically normal and mathematically disabled children with regard to the pattern of development that unfolds as the children move up through primary school.

Introduction

Learning has been a central topic within psychology from the field's earliest days as a science. During the first half of this century, many researchers investigated stimulus-response explanations of learning. The grand theories of Thorndike, Gutherie, Skinner, and Hull established learning as the central topic in psychology in the 1930s, 1940s, and 1950s. Nevertheless, the standard unit of analysis was stimulus-response connections and not strategies.

The beginnings of interest in children's strategies can be traced to the neo-behaviourists. Some theorists suggested that learning could be better explained in terms of intervening mediators, covert stimuli and responses that operated between observable stimuli and responses. However, they did not refer to covert mediators such as strategies. The uncommon idea of strategies was brought in with the cognitive revolution of the late 1950s. Gradually, a paradigm shift occurred within education research. Learning began to be looked at as the result of an active process from the student's point of view and not as a result of passive acceptance of knowledge of the outside world or the environment. Researchers expanded their field of study from laboratories to the places where training was happening in more

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natural circumstances, that is, in kindergartens and schools. This expansion resulted in the training units that the researchers then had to be concerned with becoming a lot more complicated. Strategies became therefore more realistic units of measurement, not only as training units but also as standard units of measurement for researchers' analysis of learning (e.g., Bruner, Goodnow & Austin, 1956).

Developing out of the cognitive psychology of the late 1960s and early 1970s were questions concerning the nature of knowledge representation. For example, researchers postulated that the manner in which information was represented in memory affected the way the information was processed, and proposed that developmental differences in children's knowledge base significantly influences cognitive task performance, including the use of strategies (e.g., Chi, 1978). Gradually, the expression *cognitive arithmetic* developed and was established as a common expression in the literature of the field as the spotlight was more and more directed to the question, "How do people do arithmetic in their heads?" (Ashcraft, 1992, p. 77).

During the last 25 years, research designed to identify strategies used by children at different ages or at different levels of skill acquisition has been central to the study of cognitive development. Concern with strategies used to perform arithmetic tasks is generally agreed to have begun with the influential work of Groen and Parkman (1972), in which they presented subjects with a series of addition problems and analyzed response times with respect to several different solution algorithms (more details below).

The purpose of this article is to discuss the notion of strategies as it applies to research on elementary arithmetic. Accompanying theories are presented to highlight the various arithmetic strategies that have been investigated. Following this, I give a review of research with the goal to show that strategy use in mathematics is a product of a complex cognitive activity and is not a simple unitary act. Moreover, I discuss the characteristics of strategy development by mathematically normal children. Across all areas, the primary focus of this review is: (1) to provide a framework for studying the potential cognitive mechanisms that might contribute to mathematics difficulties, and (2) to offer suggestions for future strategy research involving children with mathematics difficulties.

Strategy definition

The term *strategy* derives from the Greek work *strategos*, a general in the military. One dictionary definition is "the science of planning and directing large scale military operations, and specifically ... of maneuvering forces into the most advantageous position prior to actual engagement with the enemy" *(Webster's New Twentieth Century Dictionary,* 1978, p. 1979). Another is a "procedure to attain a goal" *(Universitetsforlagets bokmålsordbok*, 1986, p. 503).

The issue of strategy definition is somewhat controversial, and definitions of strategy vary widely in the research literature (Schneider & Weinert, 1990). In the early 1970s the term had a fairly fixed connotation. It referred to a procedure for rehearsal of the items to be learned in a memory experiment; most commonly they were specific rehearsal procedures (e.g., Ashcraft & Kellas, 1974). Nowadays, the term strategy used in the area of arithmetic performance has a considerably broader connotation, loosely meaning "how some arithmetic task is performed mentally" (Ashcraft, 1990; Bjorklund & Harnishfeger, 1990; Siegler & Jenkins, 1989).

As indicated above, strategy can be defined very narrowly or very broadly. In some studies and reviews, strategy refers to any procedure used to accomplish a task. In the simplest and most straightforward definitions, strategies are organized problem-solving behaviours that are directed toward a goal (e.g., Willatts, 1990). In other cases, strategies are viewed as relatively grand entities that encompass a variety of means toward an end, including "all" processes involved in execution of some tasks. Thus, some researchers view strategies as necessarily being invoked in a "flexible, goal-directed manner...that influences the selection and implementation of subsequent procedures" (Bisanz & LeFevre, 1990, p. 236). For example, Ashcraft (1990) defines strategy as "any mental process or procedure in the stream of information-processing activities that serves a goal-related purpose" (p. 207). Procedures that create new procedures or alter old ones in flexible ways are also considered strategic (Bisanz & LeFevre, 1990). Others emphasize the potentially conscious and controllable nature of strategies, as well as the dynamic interaction of strategies (Pressley, Forrest-Pressley, Elliot-Faust & Miller, 1985). In the last case it becomes a generic term, no different from any of a number of synonyms for thought, such as cognitive operation or mental processing (Bjorklund, Muir-Broaddus & Schneider, 1990).

In yet other cases, strategy is not defined at all, as if the meaning were self-evident (see, e.g., Bisanz & LeFevre, 1990).

Goldman (1989) makes a distinction between *task-specific strategies* and *general strategies.* According to Goldman, task-specific strategies focus on the cognitive operations necessary for carrying out a particular kind of problem (e.g., a sequence of operations that is specific to adding two numbers together.) "These tell a person what to do in face of the particular number task" (p. 44). The task-specific strategies are not a homogeneous group. Research shows that they can be of different forms and complexities (Pellegrino & Goldman, 1987). General strategies, which could be called metacognitive strategies, are defined very broadly, involving cognitive operations for orienting, organizing, and evaluating problem solutions. "Strategies of this sort provide youngsters with a general framework for approaching mathematics tasks and obtaining information about the progress of the solution effort" (Goldman, 1989, p. 44).

For children to behave strategically in solving problems in mathematics, they must first realize that their actions influence their progress toward a goal and must keep that goal in mind as they are solving the problem (English, 1991). Examination of past research on task-specific strategies in arithmetic reveals that *the goal-directed nature of behaviour* is the cornerstone of strategies. But once we try to get beyond this most rudimentary of definitions, the consensus fades. There are at least two fundamentally different ways of defining strategies: (1) as planned, goal-oriented activities (e.g., Flavell, 1970; Siegler & Jenkins, 1989), or (2) as planned, goal oriented activities that also include the process prior to the selection that results in the decision of a particular procedure for solving the problem (e.g., Bisanz & LeFevre, 1990). One of the most notable advantages of the first of these alternatives is that it limits the range of cognitive operations to which the term can be applied. For example, Siegler & Jenkins (1989) make a distinction between procedures and strategies. Procedures, unlike strategies, may represent the only way to achieve a goal. When a driver shifts from first gear to second gear, the activity is directed towards a definite goal. But the procedure that the driver uses is obligatory (required). There is only one relevant procedure that is applicable. Therefore the procedure in this case is not a strategy. When, on the other hand, a pupil solves the problem $4 + 3 = \square$, the activity is directed towards a definite goal. There are a series of different procedures to choose from in order to arrive at the right answer. The procedures that the student makes use of is goal directed and nonobligatory. Therefore the procedures in this example are, according to Siegler and Jenkins, strategies. In view of these considerations, they reserve the term strategies for procedures that are chosen among others. Thus, they define "a strategy as any procedure that is nonobligatory and goal directed" (p.11), and include the nonobligatory feature to distinguish strategies from procedures in general.

In consequence of their view of strategies, Siegler and Jenkins qualify retrieval as a strategy, "as long as it is directed at a goal and is used in a context in which other strategies can meet the same goal" (p.12). Retrieval is one among several task-specific strategies that can be used to attain a goal. Accordingly, they distinguish between two main groups of strategies: that is, *retrieval* and *backup strategies*. These expressions are the consequence of theories in which scientists think of students' mathematical knowledge as a reserve of knowledge units (Anderson, 1983; Ashcraft, 1992; Campbell & Clark, 1989).

Consider a 7-year-old child confronted with the problem 5 + 3 =☐. In observing the child's actions, you might conclude that the youngster would use a backup strategy to solve the problem. A backup strategy used for the addition problem 5 + 3 = , could, for example, undergo the following steps: the student first counts five fingers "1-2-3-4-5" and continues on his or her other hand "1-2-3". Then he or she goes back and counts all eight fingers "1-2-3-4-5-6-7-8", repeating "8" and writes the answer 8 (Ostad, 1991). A few years later, the child's solution process changes. Presumably, the child might get the answer to the same problem with no obvious external cues or actions in much the same way as recalling his or her middle name (Bisanz & LeFevre, 1990). According to Siegler, children use retrieval strategies, and answers for problems such as 5 + 3 =☐ are retrieved directly from long-term memory: "Children simply state the answer following presentation of the problem" (Siegler & Jenkins, 1989, p. 27). A backup strategy is defined as "any strategy other than retrieval" (Siegler & Jenkins, 1989, p. 27). The preceding examples illustrate relative simple strategies. Most strategies associated with test questions in mathematics at the high school or junior high school level, in contrast, are often much more complex. A division problem with multi-digit numbers, for example, presupposes that the pupil can master several single strategies (addition strategies, subtraction strategies, and multiplication strategies).

To sum up, although there are diverse opinions on what constitutes a strategy (Bisanz & LeFevre, 1990), there is some agreement on its key features. Thus, it seems unquestionable that strategies are goal directed and nonobligatory actions; that is, strategies are not the only way that the strategy user can achieve her or his goal (e.g., Siegler & Jenkins, 1989). Furthermore, although it is usually accepted that strategies are goal directed operations (e.g., Harnishfeger & Bjorklund, 1990), action that "is goal directed but disorganized is not strategic because it lacks a definite pattern in its attempt to solve the problem" (Willats, 1990, p. 24). Moreover, strategies are frequently seen as domain-specific actions employed to facilitate both knowledge acquisition and utilization (e.g., Pressly, Borkowski & Schneider, 1987).

With the aim of studying developmental differences between mathematically disabled children and their mathematically normal peers, as these pupils move up through primary school, I define *task-specific strategies as organized, domain-specific, nonobligatory patterns of decisions activated when confronted with mathematical (arithmetical) problems, and goal directed to attain the solution of the problems.*

In agreement with Siegler and Jenkins, I include retrieval as a strategy, emphasizing that "retrieval is not a simple unitary act. Instead, like other strategies, it is itself the product of complex cognitive activity" (Siegler & Jenkins, 1989, p. 13).

Models of mental arithmetic

How is a person's mathematics knowledge organized in memory, and what are the processes by which this knowledge is accessed as applied in various settings? I now turn to three well-known models of simple arithmetic processing. These models illustrate the complexity of processes.

Groen and Parkman's counting model

Groen and Parkman's (1972) model of mental arithmetic is a *chronometric model.* It is based on two fundamental assumptions. These are that: (1) cognitive tasks (e.g., solving problems in arithmetic) take a finite and measurable length of time to execute, and (2) the length of time needed to perform a particular mental task is proportional to the amount of information processed, so complex tasks require more time to complete than less complex tasks. Groen and Parkman suggested that children are equipped with an "internal counter" that is activated while solving problems. If counting plays a major role when it comes to problem solving, would the problem size (size of numbers) influence the time needed. Based on this suggestion, they postulated a *problem size effect,* which is concerned with basic facts in arithmetic. The investigators suggested that, for series number fact problems, response time would be a linear function of the number of increments required to perform each answer. More precisely, the reaction times would be longer for the larger facts; that is, that problems with larger addends or multipliers, and hence with larger answers, would in some fashion be more difficult to solve than those with smaller numbers and answers. These arguments are the basis of Groen and Parkman's (1972) most widely reported research study. They presented children (first graders) with a series of addition problems of the form "X + Y =□ " and analyzed response times with respect to three possible solution algorithms. These were:

Counting-all: The counter is set to 0, incremented X times followed by a further Y times. (Total increments = X + Y.)

Counting-on: The counter is set to the first number (X) and incremented Y times. (Total increments = Y.)

MIN Model: The counter is set to the larger of the two numbers and incremented a number of times equal to the smaller number. (Total increments = MIN (X,Y).)

Since each of these counting algorithms required a different number of increments, Groen and Parkman suggested that analysis of the students' response time would reveal which of the three models the students were using.

For example, a child who used the MIN-model to solve the problem 3 + 6 = □ would start by identifying the highest numeral (i.e., 6). Thereafter, the child would go up three counting steps. The time required to identify the highest numeral is constant and independent of the number's size. According to Groen and Parkman's hypothesis, the problems 3 + 6 = □, 6 + 3 = □, and 4 + 3 = □ require the same response time. The same applies to the three problems 2 + 3 = □ , 3 + 2 = □, and 2 + 9 = □ Excluded from this rule, however, are "tie problems" (i.e., 2 + 2, 6 + 6), which were proposed as being retrieved directly from long-term memory.

Groen and Parkman (1972) observed that the size of the smallest addend gave the best prediction for response time and that the principal source of the variation in response time for different problems was the number of counting steps upward from the larger addend that were needed to solve the problem. If the above suggestion is valid, 4 + 3, 3 + 7, and 6 + 3 would produce the same solution times because all three require 3 upward counts. Assuming that the solution time is a linear function of the number of counting steps, the researchers proposed that the solution time's length indicated which of the three models the pupils had used. On the basis of this assumption, Groen and Parkman concluded that the MIN strategy was the one that children in the first grade consistently used to solve such problems.

Initial scrutiny of their data led Groen and Parkman to propose that both children and adults were using the MIN counting algorithm, with adults performing it much more rapidly. However, closer inspection of adult response times revealed evidence of direct retrieval for approximately 95 per cent of the problems and MIN counting for the remaining 5 per cent. They concluded therefore that young children used a computational algorithm (usually MIN), while older children and adults used some direct access retrieval process, with counting used in the event of retrieval failure. Thus, a clear implication of Groen and Parkman's original results is the notion that strategy use changes across ages, from strategies that rely heavily on counting during the first years of elementary school to retrieval strategies later on.

Ashcraft's network retrieval model

Ashcraft, in a number of publications (Ashcraft, 1982, 1990 & 1992; Ashcraft & Battaglia, 1982), has been critical of Groen and Parkman's counting model. The criticism has been directed towards the fact that their model is used as a general model applicable also to older students and adults. Ashcraft's early research (Ashcraft & Battaglia, 1978) revealed strong evidence against counting-based models of adults' performance. Instead, his research indicated that adults performance on basic addition and multipication fact problems was attributed to retrieval processes operating on an organized, long-term memory network of fact knowledge. Accordingly, he argued that calculation time indicates the difficulty of accessing a stored answer from this network. In Groen and Parkman's model, the choice of strategy is dependent on factors that are tied to a structural characteristic of the problem, such as the problem size or whether or not the problem is a tie problem. Ashcraft developed a model that, to a greater degree, directs attention to knowledge structure, that is, how knowledge units are represented in memory. Furthermore, the model focuses on which connections exist between knowledge structure and the strategies used by the students.

As proposed by Ashcraft, the two most important structural aspects of the network involved the concepts of strength and relatedness among nodes. In the network, each problem-to-answer association was represented in terms of strength or degree of accessibility. Furthermore, "the network also coded the degree of relatedness among problems and answers, in that adjacent, 'near neighbor' nodes were more strongly interlinked than more distant, non-adjacent nodes" (Ashcraft, 1992, p. 85). The reference to those units of knowledge that are near and those that are far was in the first part of Ashcraft's publications linked to a model where fact knowledge was represented in table-like structures like addition tables or multiplication tables (Ashcraft & Battaglia, 1978). More precisely, Ashcraft suggested that knowledge units (such as, $2 + 3 = 5$, $7 \cdot 8 = 56$, etc.) that are stored in the network have different degrees of associative strength that have a decisive influence on how long it takes to retrieve the information. If, for instance, the unit of knowledge $2 + 3 = 5$ has a stronger associative strength than $7 + 5 = 12$, it will take less time to retrieve the 5 in the first problem than the 12 in the second. The network structure denoting how the units of knowledge are organized in the network has a decisive influence on the solution of the problem, that is, on the strategy choice, the time it takes to solve the problems and so forth.

Ashcraft was motivated by his observation that the magnitude of the sum squared was a better predictor of adults' solution times than was the magnitude of the minimum number. To account for the predictive value of sum squared, Ashcraft hypothetized that adults represent addition facts in a form much like a standard addition table, with augends (first numbers) heading each column and addends (second numbers) heading each row. In this mental table, distances between columns and between rows would increase exponentially with increases in the absolute magnitude of the augend and addend. For example, the distance between the third and fourth rows would be greater than between the second and the third rows. Adults would locate the answer to each problem by traveling from the origin to the appropriate augend, traveling down to the appropriate addend, and then reading out the sum. Solution time would be directly proportional to distance traveled.

As indicated above, when Ashcraft analyzed the network structures according to his model, it was retrieval strategies and not backup strategies that were the focus of attention. Retrieval from the network was found to be the most common strategy in adults, and it showed greater and greater predominance over backup strategies across the elementary school ages.

Later the model was revised. The original suggestion about a table-based knowledge representation was abandoned, replaced by the proposal that "the strength with which nodes were stored and interconnected was a function of frequency of occurrence and practice" (Ashcraft, 1992, p. 86), so that, for example, the nature of instruction on arithmetic directly influenced the formation of the network structure itself, and the importance of education in the early childhood years became especially stressed (Ashcraft, 1990, 1992).

Siegler's distribution of associations model

Ashcraft's network model had two particular shortcomings. The first was that his relatively one-sided model focused on retrieval strategies, while backup strategies were given a disproportionate place. Secondly, the model did not go far enough in explaining the fact that many students quite often got incorrect answers even for simple problems: for example, for addition problems with one-digit numbers. Siegler developed his model with a view toward meeting these weaknesses (Siegler, 1987a, 1987b, 1988, 1989, 1990, 1991; Siegler & Campbell, 1989; Siegler & Shrager, 1984). What characterizes Siegler's model is that it accommodates retrieval strategies as well as backup strategies, and also that within it are clearly incorporated correct and incorrect answers. It is called *the distribution of association model* because within the model, errors, solution times, and strategy use are all a function of a single variable: the distribution of association between problems and potential answers.

Siegler's model includes *a representation* and *a process*. I first look at the representation and then show how Siegler came to explain the interaction between the process and the representation. Siegler and Shrager (1984) proposed that associations between problems and answers are formed each time a child encounters an arithmetic problem, regardless of the correctness or incorrectness of the answer.

If a child uses a backup strategy and arrives at 4 + 2 = 6, an association is produced between the problem and that correct answer. But the child can miscalculate so that he or she gets 7 as the answer for the problem, and an association will then be made between the problem and the incorrect answer. As indicated above, memory representation of arithmetic facts contains both correct and incorrect answers. Thus, problem-answer associations between problems and potential answers produce a dispersion that can be graphed as a curve. On this graph the potential answers to the problem are found on the x-axis and the associative strength on the y-axis (Siegler, 1987a, 1989, 1990, 1991; Siegler & Shrager, 1984). Problems like 2 + 2 = □ are rarely solved incorrectly. Thus, there are relatively many associations produced between problems and correct answers and relatively few associations between problems and incorrect answers. The associative strength is concentrated in a single answers. This will give "a distribution of associations" that takes a form of *a peaked distribution*. The associative strength between the problem and the correct answer is high, while it is relatively low between the problem and the other (incorrect) answer. Most associative strength is concentrated in the correct answer, and few, if any, interfering associations disrupt the retrieval process. Relatively more difficult problems, for example, the problem 5 + 8 = □, often have more incorrect answers. Siegler maintains that, in some instances, the associative strength between the problem and this correct answer is not essentially stronger than that between the problem and other incorrect answers. In this case the curve takes a form of *a flat distribution*. In the flat distribution, associative strength is dispersed among several answers, with none of them forming a strong peak. According to Siegler (1987), retrieval then is far more likely to access an incorrect answer.

The representation also includes knowledge about strategies. Each time a strategy is used, the stimulation yields information about speed and accuracy. This information generates a strength for each strategy, both in general and on particular problems (Siegler & Shrager, 1984).

What is characteristic of the process as it operates on the representation as described in the previous paragraphs? The likelihood that the retrieval answer will be correct is, according to Siegler's model, dependent on whether the curve is peaked or flat. If the curve has a peaked form, few disturbing associations will break the retrieval process. Retrieval answers are quick and easy to spot. On the other hand, if the curve is flat, the retrieval process goes slowly. Attempts are made to find new retrieval answers but can be abandoned to the advantage of backup strategies. Flat distributions indicate a higher number of backup strategies and longer solution times. More precisely, the more peaked the distribution of associations, the more readily the answer can be retrieved from long-term memory, and therefore, the more likely retrieval processes will be used to solve the presented problem (Siegler, 1988).

The choice between retrieval and backup strategies is, in Siegler's model, a mechanistic choice. A backup strategy will be automatically realized if the pupil fails to produce an acceptable retrieval answer. To give up a retrieval search in order to replace it with a backup strategy is not the result of a deliberate metacognitive choice. Instead, the choice is based on two adjustable internal parameters: *(1) the search length time,* which indicates the maximum number of retrieval attempts a child will make before choosing an alternative strategy, and *(2) the confidence criterion.*

The confidence criterion is the name for a threshold for stating a retrieval answer. The value of this threshold indicates the likelihood of a retrieval answer. This likelihood is stated as the relationship between the associative strength between the problem and the answer held together by the associative strength between the problem and all the answers that are associated with the problem. More precisely, "the confidence criterion defines a value that must be exceeded by the associative strength of the retrieval answer before the child can state that answer" (Siegler & Shrager, 1984, p. 239). In other words, according to Siegler, the choice of strategy depends on whether the child has an inner assuredness that signals that the answer is correct.

The process that operates on the representation, as Siegler's original model described it, has three sequential phases: (1) retrieval, (2) subsequent elaboration of the representation, and (3) counting. First, a retrieval answer is attempted. If the child is sufficiently confident of it, he or she states it. Otherwise, he or she next generates a more elaborate representation of the problem. For example, in an overt elaboration the child puts up fingers to represent the problem's addends and tries again to retrieve an answer. As before, if the child is sufficiently confident of the answer, he or she states it. If not, the child finally switches to a new strategy - that is, to some algorithmic process such as counting fingers - to determine the answer (Siegler & Shrager, 1984).

Siegler later realized that the model was not flexible enough. This occurred especially with regard to the order in the process. In a later revision, he abandoned the idea that the child must, first and foremost, look for a retrieval answer. He also abandoned his previous thinking that backup strategies emerge only as result of a failed retrieval attempt (Siegler & Jenkins, 1989).

Strategy construction and development

One or many strategies

For a long time there was a widespread belief that children of a given age group used a single, age-typical strategy and that children in another age group used other strategies. For example in addition, a variety of findings, primarily based on chronometric data supported the view that young children consistently add in the same way. The most prominent of these models, which came out of Groen and Parkman's (1972) empirical work, has marked the debate concerning the development of task-specific strategies for more than a decade (Siegler & Jenkins, 1989).

Ashcraft (1982) demonstrated, as far as addition was concerned in first and second grade, that the best predictor for solution time was the size of the smallest addend. The best predictor for solution time for students from fifth grade to adulthood was the size of the sum squared. On this basis, Ashcraft concluded that first and second graders used the MIN strategy constantly, students from fourth grade and up used retrieval strategies, and third graders did something in between.

In the meantime, there were more and more research reports published that had as their point of departure verbal data in which the students themselves were given the opportunity to explain which strategies they had used. The research papers, which primarily were distributed through mathematics educators' reports of what children said they did when they solved arithmetic problems (Carpenter & Moser, 1982; Fuson, 1982), were critical of Groen and Parkman's hypothesis. Carpenter and Moser (1982) reported that first and second graders used the MIN strategy in fewer than half of the cases they had studied.

The results, published in research reports in the 1980s, documented that children often made use of a register of many different strategies - retrieval strategies and backup strategies which they varied during the problem-solving process (Siegler & Shrager, 1984). The results hardly came as a big surprise to mathematics teachers, who could observe the students directly in their work situation.

As indicated above, the conclusions derived from chronometric and verbal methods apparently conflicted. Siegler (1987a) was one of the first to analyze this conflict. He carried out a research project with children aged 5 to 7 years old. Chronometric data as well as data from verbal reports were collected for each child for each arithmetic problem. Use of chronometric research methods confirmed Groen and Parkman's hypothesis. But at the same time, the verbal reports documented that most of the children used several different strategies. These results, which showed that children used multiple strategies, were later confirmed by other research with preschool children (Geary & Burlingham-Dubree, 1989) and also with children with learning difficulties (Geary, 1990; Goldman, Pellegrino, & Mertz, 1988).

Aspects of strategy construction and selection

In his early publications, Siegler depicted the development of task-specific strategies as a sharp qualitative change from use of one strategy to the use of another (Siegler & Shrager, 1984). In his revised model, he has given up this standpoint. Now, the development is depicted as a gradual change so that the new strategies do not replace but function in harmony with the existing strategies. Thus, he claims that strategies can be formed on a basis of earlier existing strategies in such a way that new parts are imprinted into them. A critical point in this process is connected to the student's approaches of choosing the right segments of the existing strategies so that they can enter into the new strategies in an appropriate manner (Siegler, 1990; Siegler & Jenkins, 1989).

Siegler and Jenkins (1989) consider the process of strategy construction along a dimension of time and suggest a basic division of the process into two periods: *strategy discovery* and *strategy generalization*. "The strategy discovery period involves the time leading up to and including the first use of the new procedure. The strategy generalization periods involves the transition from having used the strategy once to using it in the full range of situations where it is the most effective approach" (p. 15). The first period can be characterized as an "aha period", where the strategy foregoes a relatively discontinuous change from an unknown strategy to a known one. Gradually, the child learns to use a strategy in several different relationships, for example, not only in connection with direct use of a specific concrete material but also in relationships where problem solving is based on use of earlier experiences without the concrete material physically present. A qualitative transformation takes place from that of a specific problem solution tool to that of a flexible resource that is useful inside a broader and broader area of function. The process in which this happens is designated as strategy generalization. Compared with strategy discovery, strategy generalization is, as a rule, more continuous and time consuming. But it is documented that strategy generalization can also happen relatively rapidly and discontinuously, as when a student for the first time understands how a strategy can be used to solve a mathematical problem (Siegler & Jenkins, 1989).

Strategy-selection mechanisms can be divided into two categories, associative and matacognitive. With *associative selection*, the child has a set of possible strategies, each of which is associated with a particular confidence level with which the task can be successfully performed (Siegler & Jenkins, 1989). There is no appeal to the child's understanding of, or declarative knowledge about, the problem. The selection is based on the child's stored knowledge about how good the answer provided by the strategy is. If the strategy, in the child's experience, can be relied on to provide a good answer, it is retained: otherwise it is replaced.

By contrast with associative models, *metacognitive selection* is based on an understanding of the problem. Several investigators have determined models based on understanding of the task, for example, VanLehn and Brown (1980) in the domain of subtraction, Gelman and Meck (1983) in the domain of counting, and Greeno and Johnson (1984) in the domain of arithmetic word problems. In these models children do not have strategies ready made for each situation. In contrast, they devise strategies based on their knowledge of relevant concepts and on the demand for the task. More precisely, declarative knowledge is used to construct strategies applicable to particular task contexts. VanLehn and Brown's model (1980) is what is called a *planning net model,* which is made up of directed graphs with nodes that include plans for the use of strategies. The links between the nodes include inferences concerning how well each strategy conforms to declarative knowledge. The two researchers have described and expounded their model from the point of departure of simple subtraction with the help of Dienes's logic blocks.

In essence, Greeno et al. (1984) based their model (also a planning net model) on the same philosophy as VanLehn and Brown, but the model is different in the manner in which it is to be implemented. Their point of departure was the observation that young children's efforts at counting, although prone to error, nevertheless reflect an implicit knowledge of the logic of counting. The researchers illustrate this phenomenon with the help of the following experiment. Young children counted a set of objects in a straight line beginning with the first one (from the left). Then, they were asked to count the objects making the second, the third, and so on, object the "one". Since these are unconventional counting procedures, it is unlikely that they would have been learned by rote skills. Despite the unconventional counting procedures and without any previously learned, skills the children succeeded better than chance. According to the researchers, this result indicated that the children understood something about the logic of counting and were in a position of thinking out a strategy that could be adapted to fit the constraints imposed.

The planning net models appear capable of explaining the development of counting knowledge. Futhermore, it raises important questions concerning how the strategy-acquisition processes relate to what kind of knowledge children have about number and counting. However, this approach is not valid when applied more widely, and in particular to acquisition of cognitive skills. Nevertheless, more recently, researchers have developed models based on domain-general processes or declarative knowledge (e.g., Anderson, 1983, 1989; Halford, 1993).

Anderson (1989) has developed a general cognitive explanation model of strategy selection. In his model it is the students' declarative knowledge that creates task-specific strategies. Furthermore, when strategies are first learned, they become assimilated into *production rules* that can operate automatically without activating the declarative areas of knowledge that trigger strategies. This reduces the demand on resources. The demand on resources can be further reduced by a process that Anderson calls *composition*. In this process a number of production rules are combined into a single rule. Once production rules are constructed, they are strengthened by associative learning mechanisms. Transfer of strategies into another domain depends on similarity. It is therefore a type of "identical elements" transfer process. If the transfer is to be made in new domains, it will be necessary to build up new production rules.

A model similar to Anderson's model was developed by Halford (1993). He pinpoints the ability to establish purposeful strategies as one of several central criteria indicating that understanding has been established. In Halford's model declarative knowledge functions as a guide with reference to construction of strategies and strategy development. This understanding functions not only with regard to developing new strategies but also regarding the placing of previously acquired strategies in new connections. Thus, his model emphasizes the guiding role of understanding in the selection of strategies in the problem-solving situation. Once strategies are developed, they can be applied to familiar situations without activating the understanding used in their development. More precisely, the model implies that strategies are part of what develops, but these strategies are built on domain-general and domain-specific knowledge acquired through everyday experience. According to Halford's model, the construction of new strategies can be based on associative processes, metacognitive processes, or both.

Assumptions about the role of domain-specific knowledge

Dependency upon context is central to issues in construction of children's strategies. Thus, the importance of domain-specific knowledge, that is, substantial knowledge of facts, has been given increased attention. As Chi (1978) demonstrated, a child with sufficient domain-specific knowledge can generate more advanced performance than an adult with lesser knowledge. More recently, Borkowski, Schneider, and Pressley (1987) presented a model called *The Good Strategy User Model.* A revised edition of this model came later, *The Good Information Processor Model* (Pressley, Borkowski, & Schneider, 1990). The researchers take as their point of departure two distinct forms of competence: *knowledge competence* and *strategic competence.* They also emphasized the importance of motivation in making possible the establishment of a functional interaction between the two forms of competence. Thus, in their model, effective strategy use is based on interaction of knowledge base, strategic factors and motivation factors.

According to Borkowski, Schneider, and Pressley (1990) domain-specific knowledge is an important component part in effective strategy use. They argue that "good strategy use" is not primarily dependent on strategic competence but rather on development and application of domain- specific knowledge. Their research on the interaction of domain-specific knowledge and task-specific strategies indicated that there are at least three ways that the knowledge base relate to strategy use: "Knowledge can either faciliate the use of particular strategies, generalize strategy use to related domains, or even diminish the need for strategy activation" (Schneider, 1993, p. 259). As indicated above, there is evidence in the literature that many instances of efficient learning occur without strategic assistance, and that rich domain-specific knowledge can even diminish the need for strategy activation (Bjorklund, Muir-Broaddus, & Schneider, 1990; Chi, 1978).

Undoubtedly, having detailed knowledge of a domain permits children to apply strategies more effectively. The quality of knowledge, however, does not uniquely comprise the critical factor for effective strategy use. Conclusions derived from a variety of findings emphasize the influence of the qualitative aspects of the knowledge base: how the knowledge is structured or represented, how the structure of knowledge representation changes with age, and how the structure affects processing performance (Ashcraft, 1992; Goldman, 1989; Ostad, 1992; Schneider, 1993; Tulving, 1983).

A sketch of the normal pattern of development

What characterizes strategy development as it manifests itself among mathematically normal children, that is, children without mathematics difficulties, as they move up through primary school? Developmental studies have revealed a general progression from immature to mature strategy use. This progression reflects a general learning mechanism that becomes increasingly effective with age and is probably mainly due to underlying changes in children's knowledge base, their processing efficiency, and their self-monitoring skills (Bjorklund & Harnishfeger, 1990; Burton, 1992).

According to the conventional view, the development of task-specific strategies takes place according to fixed patterns. For example, students' strategy use has a one-to-one relationship to the their age. The strategies that students employ up through the grades, therefore, are typical for their age (Siegler & Jenkins, 1989).

It has become increasingly clear that one important component of strategy development is the child's expanding knowledge base of task-specific strategies (Ostad, 1991). Thus, the main pattern for normal development is characterized, inter alia, by the fact that when the student gets older, new strategies are formed. Thus, the amount of knowledge of task-specific strategies increases, and the students gradually get a more varied collection of usable strategies. While a poverty of strategies characterizes the immature strategy user, a richness of strategies characterizes the mature strategy user. The challenges that children face when solving problems often vary from one situation to the next. Problems vary from one school grade to another, aids vary, the time the student has at his or her disposal varies, and so on. Furthermore, it is often the case that the cognitive conditions for solving problems also vary. Problems can originate in mathematics knowledge that the student understands to a greater or lesser degree, and the student can concentrate more or less in the work situation. In order to relate the use of strategies to various changing conditions, it is important for the students to have at their disposal a rich range of different strategies. This suggests that the functionality of their strategy use could be, in part, a function of the quantity of the student's strategy knowledge.

Another central feature in normal development is that the students' store of disposable strategies changes. Strategies that children used previously, for different reasons, become less relevant and are discarded in favour of new ones. In fact, the use of increasingly mature strategies cannot simply be characterized by substitution of one strategy, such as memory retrieval, for another less mature strategy, such as counting (Ashcraft, 1982). Rather, "development involves changes in the mix of existing strategies as well as construction of new ones and abandonment of old ones" (Siegler & Jenkins, 1989, p. 27). Empirical studies indicate that task-specific strategies undergo a variety of developmental stages during childhood. For instance, there is a development within the framework of backup strategies. Thus, in the operation of addition, the strategy use in the first grade is characterized by counting fingers (Gelman & Gallistel, 1978; Siegler & Jenkins, 1989; Siegler & Shrager, 1984). Later, and when counting is required, it is usually by means of verbal counting, rather than the counting-fingers strategies. Thereafter, the students gradually also become capable of finding the answer based on their knowledge of separate addition combinations, that is, they know the sum of the two addends without having to count (Ashcraft, 1992; Carpenter & Moser, 1982; Fuson, 1982). In general, empirical studies indicate that the frequency of backup strategies declines, while the frequency of retrieval strategies steadily increases (Ashcraft, 1982; Geary & Burlingham-Dubree, 1989; Siegler, 1978a; Siegler & Shrager, 1984).

In other words, the main pattern in normal development is characterized by diminishing use of counting and other backup strategies, while retrieval strategies gradually play a more central role. Accordingly, when backup strategies such as finger counting dominate the problem-solving process, it should be differently interpreted, depending on the age of the student. The strategies that are common in the first grade and that represent a natural link in natural development may be a symptom of defective development if the same strategies dominate problem solving, to the same degree, in fifth grade (Ostad, 1991).

In the course of development the cognitive mechanisms potentially contributing to the quality of the strategy-knowledge base change in the direction of more flexibility regarding the ability to adapt strategy knowledge to exterior and interior (cognitive) variations from one situation to the next (Ashcraft, 1982; Geary & Burlington-Dubree, 1989; Goldman, Pellegrino, & Mertz, 1988; Siegler, 1987; Siegler & Campbell, 1989; Siegler & Jenkins, 1989; Siegler & Robinson, 1982; Siegler & Shrager, 1984). When the student over a long period of time, for example, a two-year period, uniquely employs the same strategy without variation from one situation to the next, it may be due to a poverty of strategies. But it is also possible that strategy knowledge has not been appropriately stored. This phenomenon could be referred to as *strategy rigidity*.

A normal development pattern is also characterized by *more effectiveness* in the use of strategies. Older students have more correct retrieval answers. In addition, the processing of information seems to happen more quickly, so that the solution time gradually becomes shorter (Ashcraft, 1982; Geary & Burlington-Dubree, 1989; Siegler & Jenkins, 1989).

Most theoretical models that try to reveal the mechanism, that is, the single components that underlie effective use of strategies, have in large part been founded in general strategies and very little in task-specific strategies. The single components that comprise parts of these models are often thought of as being interactive even if the interaction between them is little known at the present time (Borkowski & Turner, 1990). It is a common suggestion that strategies are an important single component in effective use of strategies (Bråten, 1993); that strategic learning can improve effectiveness (Ashcraft, 1992; Siegler & Jenkins, 1989); that teaching of the use of alternative strategies can influence the choice of strategies (Pressley et al., 1987; Siegler & Shrager, 1984); and that specific knowledge of strategies, that is to say, knowledge of the effectiveness and areas of use, may contribute to more effective strategy use (Borkowski & Turner, 1990).

Summary and conclusion

As shown by the varied contents of this article, the term strategy has changed substantially, from its original connotation, a rehearsal device, to its present connotation, loosely a non-obligatory procedure that serves a goal-related purpose. Mental models developed by Groen and Parkman, Ashcraft, and Siegler in the area of arithmetic have participated fully in this change of connotation.

A substantial body of empirical work has been devoted to examining the acqusition and development of strategies by mathematically normal children. These include a focus on strategies as a function of subject characteristics. A variety of findings, primarily based on chronometric data, supported the suggestion that strategies of individuals vary with age and ability, but also that a single individual - by definition having a particular age and level of ability - will often use different strategies on different occasions.

Futhermore, a useful way to conceptualize the strategic differences among individuals is in terms of *strategic flexibility,* which reflects the quality of the strategy knowledge. In a specific content domain, for example, in the area of arithmetic, the first grade child as compared with the child in the seventh grade may represent, process, and access strategic information inflexibly (and thus inefficiently). That is, "subroutines of information may be less readily available both for the combination of encoded stimuli into meaningful form and for the comparison of old problems and solutions with new ones" (Kolligian & Sternberg, 1987, p. 11).

However, several investigations have determined that the strategy-use differences among individuals cannot be explained simply in terms of the construct of strategic flexibility. Recent research has shown that domain-specific knowledege, that is, substantial factual knowledge, is an important component in effective use of strategies. Accordingly, I suggest the existence of important individual differences in the richness of domain-specific strategy knowledge. More precisely, I argue that the amount of factual knowledge the child possesses about the various strategies, and how and where to apply them, might be reflected in problem solving through the range of variation in the strategies used.

As indicated above, a central theoretical viewpoint in this article includes aspects of strategy variability as a fundamental characteristic of mathematical cognition. Researchers have attributed the ability to acquire and apply mathematical knowledge to both memory retrieval and procedural skills. In summary, a normal course of development of task-specific strategies has shown an obvious progression over time from immature, inefficient counting strategies, through verbal counting, and finally to arithmetic fact retrieval as children move through primary school (Ashcraft, 1992; Carpenter & Moser, 1982; Siegler & Jenkins, 1989).

A growing body of research has provided useful information regarding the strategy use of mathematically disabled children. As compared with that of their mathematically normal peers, these children are characterised by the frequent use of inefficient problem-solving strategies, rather long solution time, and frequent computational and memory-related errors (Geary & Burlingham-Dubree, 1989; Geary, Widaman, Little, & Cormier, 1987; Goldman et al., 1988).

The above suggestions have to a large degree been restricted to chronometric models including the study of simple addition within the framework of relatively small groups of children, and the samples of children have been studied almost exclusively through results achieved on a single mathematical test. Not enough consideration seems to have been given to the fact that, for the youngest age groups, the difficulties encountered during the test may have been of a relatively short duration. Thus, it is possible that the researchers may have operated with heterogeneous samples, composed partly of children with temporary difficulties and partly of children with difficulties of a more permanent nature. Futhermore, it could be argued that most of the studies so far have focused more or less exclusively on single age-groups and on the youngest age-groups in particular, that is, children up to the ages of 6-8 years. Left unanswered, therefore, was whether the differences between mathematically normal and mathematically disabled children could be seen throughout the elementary school years. To address these issues, I designed a study with a longitudinal perspective in 1989 comparing children with and without mathematics difficulties as they moved up through primary school. The first report of the study has been published (Ostad, 1997). Subsequent reports will help elaborate our understanding of the development of strategic competence.

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Strategisk kompetanse:
Diskusjon av oppgavespesifikke strategier i aritmetikk

Artikkelen fokuserer på termen strategi og på strategibruk i elementær aritmetikk. Den gir et overblikk over klassiske strategiteorier og over empirisk forskning knyttet til såvel tilegnelse som anvendelse av oppgavespesifikke strategier. Det blir særlig lagt vekt på å synliggjøre karakteristiske trekk i det utviklingsmønsteret som nedtegner seg blant de elevene som har en normal utvikling. På tvers av alle de nevnte emneområdene er hovedhensikten med artikkelen å tilveiebringe en teoretisk referanseramme for operasjonalisering av forskjeller, hvis de eksisterer, mellom elever med og uten matematikkvansker med henblikk på det utviklingsmønsteret som nedtegner seg i de to gruppene opp gjennom grunnskolealderen.

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