Galois Theory Through Textbooks

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The purpose of the article is to indicate how, in the historical perspective, Galois theory found its place in the textbooks. We follow a track from Galois' original writings, through the early presentations of his work, to today's rich choice of textbooks on this fascinating subject. We hope that this partly personal view may be useful for students and, in some degree, for instructors.

1 Galois

Évariste Galois published his first mathematical paper in 1828 when he was 18 years old. The paper was about continued fractions, but at the same time, he worked with the theory of algebraic equations. It was already known from the works of Paulo Ruffini (1799) and Nils-Henrik Abel (1824) that there exist algebraically unsolvable equations – equations whose solutions cannot be expressed by their coefficients using arithmetical operations (addition, subtraction, multiplication and division) together with extractions of roots. Such equations are called **not solvable by radicals**. It was known much earlier that for the equations of degrees less than 5, it is always possible to find formulae expressing the zeros in such a way. We say that these equations are **solvable by radicals**. For equations of degree 5 and higher, the existence of algebraic formulae for their zeros depends on the equation. Galois looked for a deeper understanding of differences between solvable and unsolvable cases. He wanted to find a simple method which would enable him to to decide whether a given equation is solvable. In order to achieve this, Galois looked at all those permutations of the solutions of a given polynomial equation, which do not change all polynomial relations between them (this assertion needs a closer explanation, which we omit). In this way, he defined what is called today, the Galois group of the equation. Galois realized that the properties of such permutation groups differ depending on the existence of algebraic formulae for the zeros of the polynomial. He succeeded to grasp these properties defining the notion of a solvable group. Then he proved that a polynomial equation is solvable if and only if its

(Galois) group is solvable. The results concerning solvability of algebraic equations were presented by Galois in two notes published before his death, in the famous letter to his friend August Chevalier, written during the night before the tragic duel, and in two papers, which appeared 14 years after his death (see [G]). In the first paper (*Mémoire sur les conditions de résolubilité des équations par radicaux*), he characterized all solvable groups which appear for polynomials whose degree is a prime number. This paper, as well as one of the notes mentioned above, contains an elegant result concerning such equations, which says that it is solvable by radicals if and only if every zero can be expressed rationally by any two fixed zeros of it (in modern terminology, the splitting field is generated by any two of the zeros). In the second (*Des équations primitives qui sont solubles par radicaux*), he works with the much more difficult case of polynomial equations whose degree is a nontrivial power of a prime number. Galois' purpose was a characterization of permutation groups corresponding to solvable equations.

Even if Galois' definition of the group was given in terms of permutations, it covers the general notion of finite groups. In several papers, Galois introduced many important notions related to groups, for example, normal subgroups, quotient groups and solvable groups. Galois studied also the notion of the field (without an abstract definition) and the last paper published during his lifetime, contains a construction of finite fields, which today are very often called Galois fields. Moreover, Galois was convinced that his work could be placed in a broader context of the future development of mathematics towards more conceptual and theoretical ideas beyond pure computations. He anticipated the importance of the algebraic structures (in his theory, groups and fields) and their role in the future developments in mathematics (see [K], p. 92):

"Go to the roots of these calculations! Group the operations. Classify them according to their complexities rather than their appearances! This, I believe, is the mission of future mathematicians. This is the road on which I am embarking in this work."

Galois' collected works fill about 60 pages, which are not related exclusively to the theory of polynomial equations. In spite of the modest volume, they affected the development of mathematics through many important ideas. The efforts of many mathematicians who followed Galois led to the modern version of his theory, which became a part of curricula in standard courses in algebra. But Galois' greatest contribution is probably his idea to use groups (and with that other similar algebraic structures) in order to solve very concrete, and otherwise difficult to handle, mathematical problems. His deepest results concerning polynomial equations, undoubtedly very interesting and important, are no longer in the center of mathematics. But the notion of group, which he articulated and used in an impressive way, evolved through the theory of permutation groups studied by many mathematicians to the general notion of group, which is one of the most powerful and important in the whole of mathematics. Galois saw many sides of this object and formulated several results about groups, which later inspired many mathematicians in their research.

2 "French Period"

As we have already mentioned, Galois' collected works were published for the first time in 1846 by Joseph Liouville (see [G]). The first attempt to present Galois theory for broader mathematical community was a paper by Enrico Betti published in 1852 – 20 years after Galois' death [B]. The paper was written in Italian, which probably limited the number of readers. The first presentation of the Galois theory in a textbook form was the third edition of the book *Cours d'algèbre supérieure* written by Joseph Alfred Serret, which appeared in 1866. But the real turning point as regards the general knowledge about Galois theory and group theory, was the publication by Camille Jordan in 1870 of his book *Traité des substitutions et des équations algébrique* [J] on permutation groups and algebraic equations. This book evoked an interest in Galois theory and group theory in the whole mathematical world. In particular, many great German mathematicians were influenced by Jordan's work. Unfortunately, the text of Serret's book, which became standard at French universities and whose last edition appeared in 1929, was not changed after its third edition even when the understanding of Galois' ideas was growing all the time. Probably this together with the consequences of the Franco-Prussian War, which ended in 1871 and left a lot of tensions in relations between France and Germany, partly explains why during the second part of 19th and the beginning of the 20th century, the main developments related to Galois theory occurred in Germany.

3 "German Period"

The second half of 19th century was a time of very intensive developments in mathematics. Many algebraic structures, which in a natural way are related to Galois theory, like groups and fields, were studied in number theory, algebra and geometry. The notion of group crystallized to its modern form, thanks to developments in algebra, geometry, physics and chemistry. Algebraic number fields and their rings of integers were studied in connection with diophantine problems and Galois theory became a natural ingredient of these investigations. The general context of groups and fields, which is necessary for the modern formulations of Galois theory was developed by Richard Dedekind and Leopold Kronecker. Groups entered geometry in the famous Erlangen Program by Felix Klein in 1872. The first German presentation of Galois theory appeared in the article *Über Galois' Theorie der algebraischen Gleichungen* by P. Bachmann from 1881. But the most important contribution as regards Galois theory was the work of Heinrich Weber first in his article *Die allgemeinen Grundlagen der Galois'schen Gleichungstheorie* published in 1893, and then the chapters on Galois theory in his very influential textbook *Lehrbuch der Algebra* published in 1895. Weber presented Galois theory using the general language of groups and (arbitrary) fields. The theory of solvability of polynomial equations was of secondary importance and appeared as an application of the general theory. Weber's book was the main reference to Galois theory during the period of great activity in number theory, which at the beginning of 20th century led to successful descriptions of the arithmetical structure of abelian Galois extensions of number fields in the class field theory.

As a result of these developments, the theoretical edifice of Galois theory became more visible thanks to the work of those mathematicians who contributed to the great achievements of algebraic number theory. Several lecture series on algebra and number theory were given by Emil Artin and Emmy Noether at different universities. About that time, a new, more general (“abstract") context of different algebraic notions was created and this modern algebra increasingly entered the curriculum of university courses. Bartel Leendert van der Waerden followed several courses given by Artin and Noether, which resulted in his influential textbook *Modern Algebra* [W] published for the first time in 1930-1931 (from the fourth edition in 1955 as *Algebra*). A subtitle of the book contained the acknowledgement “Based in part on lectures by E. Artin and E. Noether". The book, whose contents were modified as the German editions as well as editions in many other languages followed, contains a chapter on Galois theory (chapter 8). Van der Waerden’s book played a very important role as practically the only textbook in modern algebra at a university level to approximately the middle of 20th century. It gave also a standard knowledge of Galois theory for those who did not follow a special course on this subject.

About the same time as van der Waerden’s book appeared, Nikolai Grigorievich Chebotarev[1] published in Russian his book *Basic Galois Theory* (1934). Two years later, he published a new book *Galois Theory* and in 1937 his classical book on Galois theory [T]. The edition of the German translation of Chebotarev’s book was planned for 1940 but the publication waited for another 10 years. The book is still very interesting as it contains material, which is not typically included in modern textbooks on Galois theory. Today, there is hardly a textbook for those who want to learn Galois theory for the first time, even if it were written with this intention. Chebotarev considered German textbooks (Weber, van der Waerden) in Galois theory as too abstract and wanted to present a text accessible for all students. Still the book is definitely interesting for those who want to be acquainted with the kind of problems and results of the theory during 19th and the beginning of 20th century.

Undoubtedly, the most influential textbook on Galois theory was Emil Artin’s book whose first version, in the form of lecture notes from different universities in Germany and the United States, was published in 1938 as *Foundations of Galois Theory*. Later, the book was published in 1942 as *Galois Theory* and is still in use as a textbook reprinted several times on the basis of the second corrected edition from 1944. Artin, sometimes with Emmy Noether, delivered a number of lectures on Galois theory in Germany before he emigrated to the United States in 1937 (Emmy Noether emigrated in 1933). Artin’s book presents the theory in a very elegant way, which all modern textbooks follow today. In Artin’s presentations, the central feature is a connection between two kinds of field extensions: splitting fields of polynomials and fixed fields for (finite) automorphism groups of fields. Investigations of relations between splitting fields and automorphism groups of fields (Galois groups) became the main topic in Galois theory. The correspondence between field

[1] There are different spellings: Chebotaryov, Chebotarov or Tschebotaröw

extensions and groups (often called Galois correspondence) is in the center of the theory. Applications to the theory of equations are only an application of the theory even if this aspect was treated similarly much earlier, for example, in Weber's book. Artin's book gives a very concise and lucid presentation of the subject and is still attractive as a textbook for beginners. Unfortunately, it has some drawbacks on the practical side. After reading Artin's book, one could have an impression that Galois theory is a very theoretical subject, missing non-trivial examples and interesting problems. In fact, there are only very few concrete examples in the book and when the choice of the textbooks was limited, a teaching assistant working with the students, was forced to work a lot, in order to find good examples and interesting, not unrealistic problems. Moreover, the book was in use a long time before computers became available and as well as before many algorithms useful in Galois theory were implemented in different powerful computer packages like Maple, Pari or Sage (and many many more, which already are forgotten and replaced by more powerful ones). Now it is possible both to learn and teach the subject combining the elegance of the theory with unlimited possibilities to exemplify both the theoretical and practical aspects of it, using computers. Unfortunately, there are very few textbooks which seriously combine these two aspects, but there are many textbooks in which good examples and exercises can be found.

4 1950 – 1970

During a period of about 30 years, Artin's book was very often used as a textbook when courses in Galois theory were given at universities. This, in many ways, seminal text, can still be recommended as a first contact with the subject (after an introductory course in abstract algebra). Moreover, a few years after the end of WWII, a period of greater stability and the need for new contents of traditional courses in algebra became evident. Several new textbooks in algebra were published. Some of them as general textbooks in algebra, which contain at least one chapter on Galois theory and some as textbooks dedicated to this specific subject. For several reasons, the first category dominated as there was a need for forming a framework for modern curricula on the subject at leading universities. Galois theory became an obvious part of standard textbooks in algebra. Later more ambitious courses in algebra had to be cut down (after 1968 – 1970), which created a market for simplified texts on abstract algebra and more specialized textbooks in Galois theory in which pedagogical aspects became more important. First, we discuss some of the more influential general textbooks in algebra published after 1950. Then we will talk about the textbooks of the "new generation" after the turbulent years around 1968.

In 1953 the textbook *A Survey of Modern Algebra* [BM] by Garrett Birkhoff and Saunders Mac Lane was published. The last chapter of this book contains a very concise presentation of Galois theory. Similar presentations, more or less complete, could be found in a few other books in algebra published during the same decade, for example, in Nathan Jacobson's book *Lectures in Abstract Algebra* (3 volumes, 1951-1964). In later editions, the book changed its title to *Basic Algebra* [Ja] published 1974-1977. It contains a rather comprehensive, reliable and easy to read presentation of Galois theory. About the same time, Paul Moritz Cohn published his textbook *Algebra* [Co] with a similar profile and equally good presentation of Galois theory in the second volume. Probably the most influential textbook in algebra of all times is "Lang's algebra" – Serge Lang's *Algebra* [L] – first published in 1965 with several later editions (in particular the third edition from 1993). It contains a very clear and comprehensive presentation of Galois theory with a lot of nontrivial exercises in chapter VI. In this category of general textbooks in algebra, it is impossible not to mention Israel Nathan Hersteins' book *Topics in Algebra* ([H], 1964), which was very popular as a textbook before the era of Lang's Algebra and at many universities remained for many years to come.

Textbooks on Galois theory were not too common as the qualities of Artin's book were indisputable. In 1960, a book *Foundations of Galois Theory* by M.M. Postnikov was published in Russian (later in Russian under the title *Galois Theory*). The English edition was published in 1962 (see [P]). The spirit of the book reminds a little of the book by Chebotarew. It also contains several interesting topics, which are no longer considered as standard in introductory texts. Unfortunately, the book does not contain exercises. A less known textbook by the great mathematician Richard Brauer *Galois Theory* appeared in 1958. This valuable text contains several less usual topics like a discussion of Galois theory related to modular equations.

5 After 1970

Broadened university recruitment and, as a consequence, changes in curriculum as well as the way in which more advanced courses in mathematics were formed, created a need for simpler, more "student-friendly" texts with many explicit examples and exercises of different complexity. This new trend resulted in many textbooks with somewhat diversified profiles. It is not realistic to present all of them without an evident risk of omitting some valuable positions. We will focus on this extremely rich choice of textbooks in Galois theory published after 1970 guided rather by personal taste and some degree of coincidence than a desire of completeness.

One of the oldest, in this new category of textbooks, is *Classical Galois Theory* by L. Gaal [Ga], which appeared in 1971 and was popular at American universities. Gaal's book is intended as a kind of "workbook" for self-study with many examples. Probably some students will find the book very useful. A very popular textbook by I.N. Stewart [S] was published for the first time in 1972 and is still in use at many universities. Giving courses in Galois theory, I used this book several times aware of some of its drawbacks, but nevertheless after trying to work with other texts, I returned to it anyway mostly because of its favorable reception. On a long list of books, which can be used as textbooks, let us mention in chronological order: C. R. Hadlock ([Ha], 1978) H.M. Edwards ([E],1984), J.R. Bastida ([Ba], 1984), D.J.H. Garling ([Gar], 1986), J. Rotman ([R], 1991), P. Morandi ([M], 1996), J.-P. Escofier ([E], 1997), D. Cox ([C], 2004), J. Swallow ([S], 2004), J.M. Howie ([H], 2005), J. Bewersdorff ([Be], 2006), S.H. Weintraub ([We], 2006), S.C. Newman ([N], 2012).

Galois theory has remarkable features as a subject of a course in mathematics. There is an interesting problem, which is easy to formulate and comprehend even by an inexperienced student (how to solve polynomial equations?). It is possible to demonstrate easy, but non-trivial partial solutions (quadratic, cubic, quartic equations). There is a very interesting history behind it. The solution of the problem (quintic and higher degree equations) requires a suitable notional edifice, which in a natural way motivates an abstract mathematical glossary (from equations to groups, rings and fields). The degree of abstraction can be adapted depending on the previous knowledge and character of the class (e.g. future teachers, PhD students, applied mathematicians etc). Even if some classical applications, like solvability of polynomial equations by radicals, are no longer at the center of the whole discipline, the general theory has still its own challenging unsolved problems and is very important in many other areas of mathematics and its applications. Of course, there is also something as unusual in mathematical subjects as the well-known personality related to it and the history surrounding his mathematical achievements. All this makes the subject fascinating for an informed student and attractive for an ambitious teacher.

One of the first decisions is, however, a choice of auxiliary material and a textbook that may be part of it. In many cases, the teacher decides to use his or her own notes. This is an usual phenomenon and the variety of different texts on the web is impressive. We will return to this point later. Probably, in most of the cases, the teacher wants to choose a suitable textbook. This choice is governed by many factors – mathematical quality of the text, conformity of the scope of the book to the local plan, availability, price, tradition, accessibility with regard to the previous knowledge, good exercises and so on. But there are also psychological aspects: Is the textbook “reader-friendly"? Is it too concise or maybe a dud? Could I, as a teacher, do things my own way even if in principle I follow the book? Can students easily maneuver between using the textbook and their own notes from the lectures? It is not easy to answer all these questions but sometimes the best book is such that creates a degree of freedom both for students and their teacher. This freedom means also that a student has a possibility to go beyond the limits of the compulsory scope of the course in a natural way – just opening a new page in the book.

Taking into account all these aspects, it is very difficult to give specific recommendations. It is clear that different textbooks satisfy these requirements differently. Let me, however, come with my preference, even if I never had an opportunity to use it in practice as a teacher, since the book appeared a very short time before I had my last class in Galois theory. This is the textbook *Galois Theory* by David A. Cox, which was published for the first time in 2004.

Cox’ book satisfies many of the requirements and wishes formulated above. First of all, it starts with a chapter on solving of the polynomial equations with rich historical background. At the same time, already at the beginning, the Author tries to explain how mathematical ideas change depending on the character of those problems which have to be solved. Considerations of this type are formulated in *Mathematical notes*, which appear in the whole text and give valuable insights concerning theoretical concepts. The book is meant for undergraduate students and in principle independent of the previous knowledge on groups, rings and fields. Those are either supplied in the text or in an appendix starting on the level of complex numbers. The contents covers the majority of standard topics discussed in the Galois theory courses. Personally, I miss a little the normal bases, Kummer theory and some facts related to Hilbert's Theorem 90, but these topics are usually not discussed in undergraduate texts. The Author has other preferences as regards material which goes beyond standard courses. Two distinctive features of the book are a modern presentation of Galois' main contributions and a discussion of numerical methods in connection with computations of Galois groups. As regards the first point, there are proofs of the results on irreducible solvable equations of prime and prime-squared degrees as presented by Galois in his two main papers published after his death. There are not many places in the literature, where these results are discussed. Moreover, it is done in a highly readable way. The second point is the presentation of the computational methods, which essentially were started already by Lagrange and continued by Galois in the form of the theory of resolvents. This was a somewhat obscure very old subject, which dominated the old presentations of Galois theory. Emil Artin succeeded in an elegant presentation of Galois theory without resolvents. But in the age of more and more powerful computers, it appeared that the resolvents give not unrealistic possibilities to compute Galois groups. The book contains a chapter on computing of Galois groups and examples related to practical use of the symbolic packages Maple and Mathematica. There are examples of usage of these packages not only in connection with computing the Galois groups, but also in connection with other computational tasks related to the theory. The book contains some other exciting and more or less unusual topics in the standard texts, which are presented in a lucid way like Abel's results about geometric constructions using the lemniscate, Galois theory applied to origami and a more classical, but often forgotten "Casus Irreducibilis" (impossibility to express real roots of irreducible cubic by real radicals). This choice of topics in some way provides this modern textbook with a spirit of Galois theory before Artin's book appeared. Moreover, there is a wealth of good exercises with hints to some of them. The book is well organized and contains suggestions concerning a choice of material and possible student projects.

A very nice text on Galois theory is J.-P. Tignol's book *Galois' Theory of Algebraic Equations* [T]. In this book, a presentation of Galois theory is rather a pretext than the purpose. The Author wants to show how mathematics is created in order to solve concrete problems – in this case how mathematical theories evolve in order to deal with solving algebraic equations. This aspect of Galois theory confirms its great value as a very useful ingredient of general education (including mathematics). The book is a source of many interesting mathematical and historical facts related to Galois theory even if it is hardly a textbook for beginners where a broader knowledge of Galois theory is the purpose. But the idea of this book is really very pleasing and a course based on it could be appealing to many categories of students.

Usually, courses in Galois theory follow some introductory course in what is called abstract algebra discussing fundamental algebraic structures and often some applications of them. Many textbooks in "Abstract Algebra" contain some elements of Galois theory. Such concise and simplified presentations are very useful as a general orientation on the subject, for example, for those who want to study Galois theory on their own. Sometimes, they may be considered as fully satisfactory sources of information depending on the purpose of studies. The chapters on Galois theory in the classical textbooks in algebra, which we discussed earlier are more like concise textbooks on Galois theory of today. Among the textbooks published after 1970, let me mention M. Artin's inspiring textbook *Algebra* [AM] from 1991. Another textbook is Lang's *Undergraduate Algebra* [Lu] from 1984, which presents on about 20 pages a concise but perfectly readable presentation of Galois theory.

Finally, let us mention an amazing source of knowledge on Galois theory which can be reached through the internet. An abundance of good lecture notes, historical texts and articles on more specialized topics related to Galois theory seems to be endless. In this category, let me mention lecture notes by J.S. Milne [M], which some of my graduate students used with great satisfaction, as well as different notes related to algebra and number theory including Galois theory by K. Conrad [Con]. As regards historical facts, which were also useful in this article see F. Brunk [Br] and J.J. O'Connor and E.F. Robertson [OR].

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