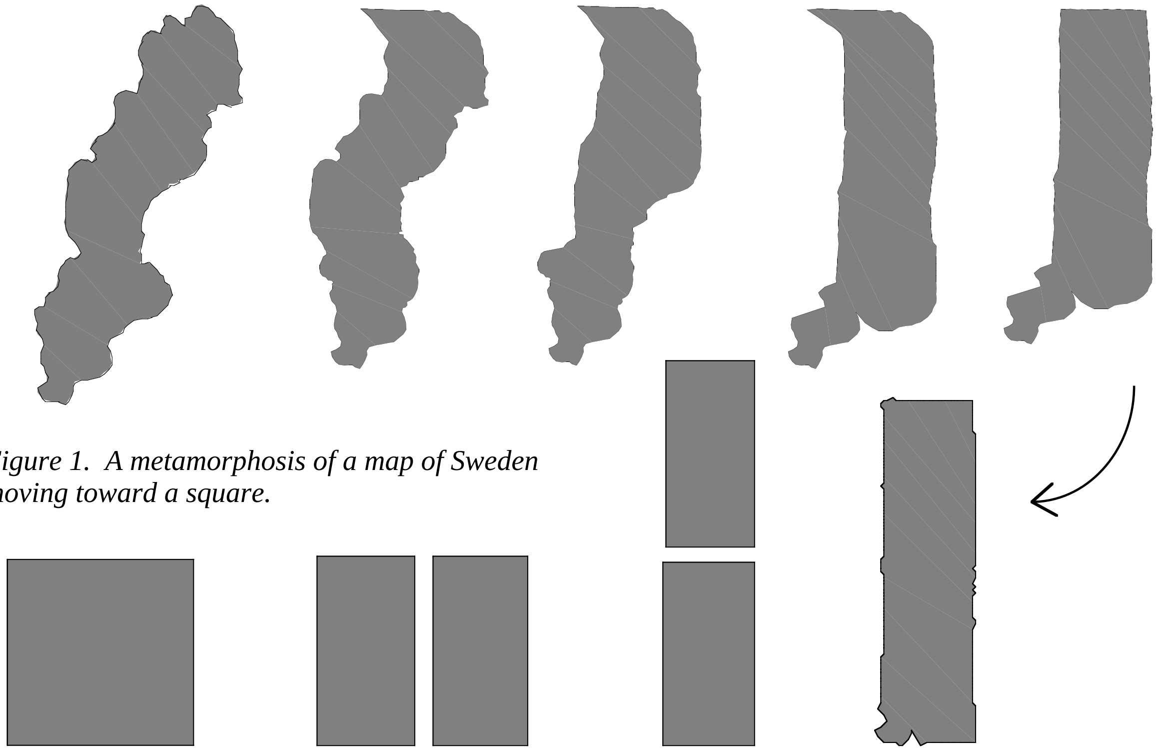

Figure 1. A metamorphosis of a map of Sweden moving toward a square.

Squaring Sweden

Robert Reys, Jennifer Bay, Ann Bledsole & Dee Cook

Här publicerar vi en artikel från matematikvänner i USA. Är det inte intressant att man i det stora landet i väster intresserar sig för skala och relationer mellan nordiska länders areal? Kanske kan texten användas för samverkan engelska – matematik?

Mathematical Explorations

Exploring the relative size of things in the real world helps develop number sense and encourages connections between visualizations and numerical benchmarks, which are particularly useful. For example, we often use our own height to estimate the height of another person, a ceiling or a tree. Likewise we may use an important date (birth of a child, wedding anniversary, special holiday or historical event) to recall on the time of another event. Such benchmarks develop from a variety of experiences and continue to grow and expand to incorporate additional personal experiences.

The size of one's country can be used as a benchmark. While every Swede knows that the country of Sweden is bigger than Denmark, the magnitude of the sizes may be less well known. A map of the Nordic countries can be helpful in providing gross visual comparisons, Figure 2.

However, maps can be deceptive and sometimes difficult to interpret, since the area of a country map represents an irregular shaped land mass. Let's explore some

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Figure 2. Map of the Nordic countries.

ways we might use the size of countries to promote the power of benchmarks, further develop number sense and establish some connections to geometry.

What techniques can we use? One approach might be to transform the area of Sweden into a more commonly recognized geometric figure. Since this transformation must leave the area invarient, several different transformations are useful. The opening sequence of tesselations (Figure 1) illustrates how a map of Sweden might be transformed into a square with the same area. There are a number of resources detailing techniques of constructing different geometric figures which perserve the area and will tesselate (Bezuszka, Kenney & Silvey, 1977). A software package *Tessemania* (Kline, 1993) can be used to begin with a square and create different figures with the same area, each of which can tesselate. Figure 1 shows a sequence of figures which was actually generated by using *Tessemania*. We started with a square, which was then modified to create different figures with constant area. Figure 1 resulted from reversing the sequence of figures to show the evolutionary stages. Even though the final figure is not ”exactly” a map of Sweden, it provides a rough approximation. More importantly it reminds students that geometric figures may look different but have the same area.

Table 1
Land areas of Nordic countries

Denmark	43 069 km^2
Finland	337 032 km^2
Iceland	103 000 km^2
Norway	324 219 km^2
Sweden	449 964 km^2

Another approach would be to use the data of land masses as shown in Table 1. An examination of this table, confirms that Sweden has the largest land area. What are some other patterns/relationships that are suggested?

Here are several that come to mind:

Sweden is about ten times the size of Denmark. Finland and Norway are about the same size. Sweden is a bit larger than Norway and Iceland combined.

Encouraging students to create other comparisons is a natural way to extend and promote mathematical thinking.

Now let’s take the information from Table 1 and use it to construct a different visual representation of each Nordic country than is traditionally shown. Although we could generate a sequence of figures leading to a square for each of the countries (as was done for Sweden), let’s use a more direct method to construct a square to represent each country. This exercise provides several worthwhile mathematical explorations. First, it provides an authentic opportunity to examine what a square root means. Secondly, it provides some direct connections between arithmetic, geometry and number sense.

By definition, the square root of a number is a value that when multiplied times itself produces the number (Tate, 1994). The real world connection between a side of a square and its area is easier to grasp and ”squaring a country” provides this context. Suppose Sweden were in the shape of a square. What would be the length of one side of the country? Constructing a square for Sweden requires a thoughtful decision on what scale to use. The appropriate scale to use depends on how much space is available for drawing the square. For example, this journal page is small and would require a different scale than if the square was being made for a classroom bulletin board or to display on a floor or wall where more space is available.

Once the scale is selected, we are ready to investigate the dimensions of our square of Sweden and determine some approximate measurements. If the length of the side were 600 km, the square would be 360000 km^2 which is too small for the area reported in Table 1, whereas if the length is 700 km, the square would be 490000 km^2 and that would be a bit too much. Now either continued trial and error or the square root function on a calculator can be used to find that the square root is about 671 km. Using a scale of 1 mm to 10 km suggests that the length of the side of the square for Sweden should be nearly 7 cm, which fits nicely on this page. A square of Sweden is shown in Figure 3.

Now, suppose we want to show the land areas of the other Nordic countries as squares. Having Sweden as a benchmark, we can be confident that the length of the square for the other countries is smaller, and that the length of the side for the Danish square is the smallest. The construction of these squares could be facilitated by a variety of tools. For example, grid paper as well as a calculator and a ruler could be used. If computers are available, the *Geometer’s Sketchpad* could be used. Although this technology is powerful and exciting, it also requires careful use to insure that the desired products (in this case squares of appropriate sizes) result.

If you have the *Geometer’s Sketchpad* available, here are a few practical suggestions to get your students started. The construction of a square requires more than simply plotting four points and connecting the dots. Some necessary skills are knowing how to construct a

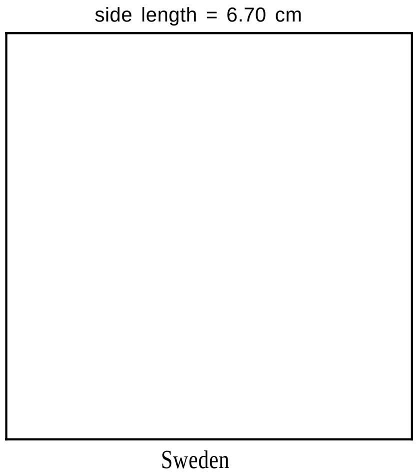

Figure 3. A square of Sweden

segment, perpendicular line and circles with a given center and radius. The arcs of the circles are used to construct equal length segments. Once a square has been constructed, measure the length of the side and then drag one of the vertices to create a square with the appropriate side length. Students can then measure the area of the figure and observe the relationship between the length of the side and the area of the corresponding square. As the vertex is dragged, the changes in the length of a side and corresponding area are displayed in dynamic fashion on the screen. As mentioned earlier, the scale of 1 mm to 10 km works well for the *Geometer’s Sketchpad*, (se även http://forum.swarthmore.edu/sketchpad/sketchpad.html).

Once the square roots have been found, then the squares for the Nordic countries can be constructed as shown in Figure 4. Another view results from superimposing them on each other as in Figure 5. Notice in either case, some differences are dramatically illustrated, whereas the visuals for Finland and Norway are very difficult to distinguish.

Hopefully this exploration helps further establish a personal benchmark, namely the size of Sweden, as well as connecting number sense, and an application of square root to geometry. Further opportunities for extending these ideas are limitless. How many Swedens are “equal” to Germany? China? Atlantic Ocean? the earth’s surface?

What if instead of representing the areas of the countries by squares, you chose to represent them by other figures, such as circles or equilateral triangles? Could they be represented by rectangles with a common length of one side? Once 2-dimensions seem too easy, how about ”Cubing the Baltic Sea.” (i.e., What would be the dimensions of a cube large enough to hold the water of the Baltic Sea?) Using dotty paper or cubes or both, students can explore the relationships of sides to volume (cube and cube root), and create other visual representations.

One of the powers of mathematics is that solving one problem stimulates generalizations and thinking that lead to more interesting and challenging activities. We hope that “Squaring Sweden” is a stimulus to let this power soar.

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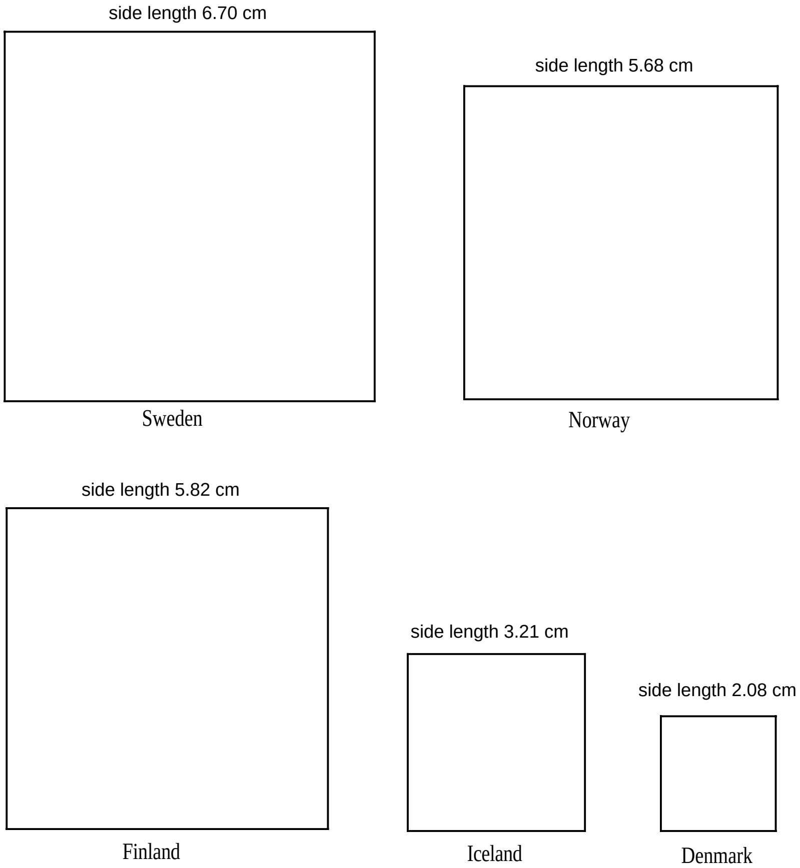

Figure 4. A view of the squares of Scandinavian countries.

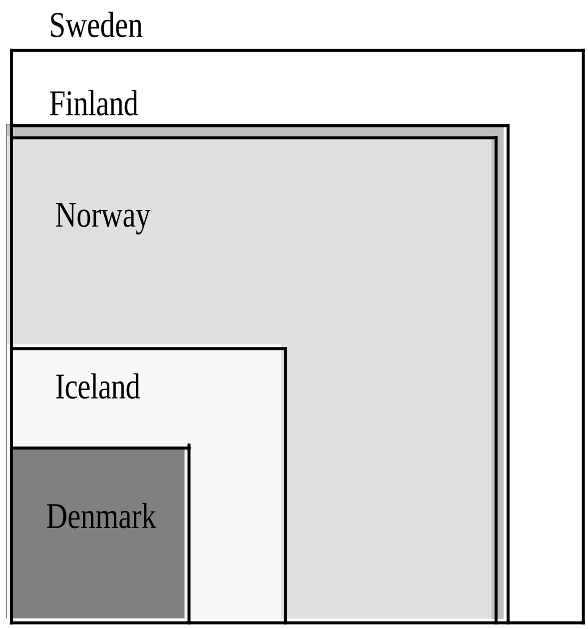

Figure 5. Another perspective of the squares of Scandinavian countries.