The meaning of concept

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The overall purpose with my research project is to develop a theoretical framework for analyzing individuals´ conceptions of numbers. The first part of this project is a conceptual analysis of the notion of concept. The meaning of the word *concept* is not very precise and it is easy to become confused. It is not easy for teachers who do not really know the difference between *concepts* and *objects*, or *concepts* and *procedures*, to teach about concepts or to decide if students could use concepts to solve mathematical problems.

In research in mathematics education, the term *concept* is used in various ways. In some contexts, *concept* is imprecise and undefined, and in others the term is defined or at least explicated, but the explications are not coherent. For example; *concept* could be something subjective, each of us having our own concepts, or it could be inter-subjective, something that we have an agreement about. Some definitions even mix the subjective and inter-subjective view and there is a lack of an explicit or tacit common conception of what a *concept* is. Let me give two examples from texts often referred to:

According to Tall and Vinner (1981, p. 152), a concept has two associations. The first one is the *concept image* which is the total cognitive structure associated with the concept, built up by experience. The second association includes different types of definitions, both formal ones, which are accepted by the society of mathematicians, and individual ones, which different persons uses to describe their concept image. There is, however, an ambiguity in this view. Their use of *concept* seems to indicate that concepts are subjective but also, in formal definitions as well as in the concept image, inter-subjective.

Sfard (1991, p. 3) distinguishes between *concept*, which she uses when a mathematical idea is presented as a theoretical construct of the formal mathematics, and *conception*, which is the total cluster of internal representations and associations which are evoked by a concept. Another difference between Tall and Vinner (1981) and Sfard (1991) is that Sfard does not relate to the role of the definition in relation to the concept.

Piaget means that we create mental structures, both biological and conceptual, which can work as the meaning of a symbol (von Glasersfeld, 1995, pp. 82, 86, 109). Vygotskij (1999, p. 35), on the other hand, considers the meaning of a word to be the collective and social, inter-subjective, interpretation of a word. This meaning can be transformed to a general, abstract, idea in the form of a concept which is independent of individuals. Piaget and Vygotskij agree that a concept is a generalized, abstract meaning. The difference is about if the meaning and the concept is subjective (mental) or inter-subjective (communicative).

According to the Stanford Encyclopedia of Philosophy (Margolis & Laurence, 2012), there are three different, philosophical, views of what a concept is. The first one defines concepts_1 as mental representations. According to the second view, concepts_2 are abilities that are connected to cognitive agents. For example: the concept *number*, could be the ability to distinguish numbers from non-numbers and, from that, draw some conclusions about numbers. The third view says that concepts_3 are Fregean senses; concepts are identified with abstract objects, as opposed to mental objects and mental states.

A recent example of a philosopher who has written about mathematical concepts, is Jenkins (2008, pp. 120, 148), who thinks that concepts are sub-propositional mental representations, which are related to propositional mental representations in almost the same way as words are related to sentences. The structure of our concepts mirror the structure of the world and this mirroring occurs, according to Jenkins, because our concepts are grounded in experience of the world, they have an empirical ground.

The discussion at the conference will be about different views of concepts in mathematics education and in philosophy and I will raise the question how we could use the philosophical distinctions in mathematics education. Is there a need for a common view of concepts or is it feasible for us to have different views?

References

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