Analysing Instrumental and Pedagogic Situations in Preschools using the Didaktic Space

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Researchers rarely discuss methodological issues in regard to preschool mathematics education and if they do, they do not take their starting point from reconceptualisations of what mathematics might be for preschool children. This paper presents as an analytical tool the "didaktic space" that arose when responding to issues related to the analysis of data collected in a Swedish preschool. The issues that arose from categorising situations using Bishop's six activities required some reconsiderations of the methodology in relationship to the research questions. The paper discusses how methodological decisions can affect the analysis and the future possibilities that the didaktic space offers.

Methodological issues in understanding preschool mathematics

Clements and Sarama (2007) in reviewing literature on preschool mathematics education research identified the different theories (empiricism, neo-nativism and interactionism) that have been used to discuss how children learn and use mathematics–"mathematical ideas are represented intuitively, then with language, then metacognitively, with the last indicating that the child possesses an understanding of the topic and can access and operate on those understandings" (p. 464). However, most of the research that they reviewed assumed that preschool mathematics can only be understood in relationship to school mathematics. Such a starting point is problematic because of the many differences between school and preschool.

School students encounter mathematics in mathematics lessons and associated homework, that is, in situations clearly labelled and demarcated as mathematical. Therefore, it is possible to define school mathematics as a social practice as defined by Fairclough (2003):

> Social practices can be thought of as ways of controlling the selection of certain structural possibilities and the exclusion of others, and the retention of these selections over time, in particular areas of social life. (p. 23)

The same students might engage in mathematical activities in out-of-school situations; however, these are embedded in social practices very different to school mathematics (Lave, 1988), and are likely, in Bernstein's (1971) terms, to be more weakly classified and framed. The poor compatibility between the social practices of school mathematics and every-day life is one reason that every-day situations typically are not experienced as being connected to school mathematics.

With its long institutional and pedagogical tradition as an institution for the care and upbringing of young children (Roth, 2011), Swedish preschools also can be considered a social practice into which children are enculturated (Bishop, 1988) or become participants (Wenger, 1998). In learning terms, children learn to do preschool in the same sense that school students learn to do school. In a Swedish preschool setting with its strong tradition for perceiving children as learning through play, clearly demarcated situations include "fruit time", "circle time", "play" indoors or outdoors and others but not situations labelled "lesson" as is typical in a school setting, even though, in some preschools, situations labelled "mathematics" may occur.

Therefore, it is more difficult to attach the label of social practice to Swedish preschool mathematics. Whereas school mathematics is strongly classified and framed (Bernstein, 1971), thus making it easily recognisable as a social practice, this does not seem to be the case for mathematics in Swedish preschools as it is not delineated sufficiently to qualify as a social practice. Looking for mathematical activity characteristic of the social practice of school mathematics may give few results, and may not be appropriate, given the very different curricula (Skolverket, 2010; 2011). This raises the question of how to identify children – and preschool teachers – as being involved in mathematics, whatever that might be in preschools, even when they are not aware of it, as well of the meaningfulness of such identification. A research frame set by school mathematics could lead to the question of "where is the (school) mathematics" rather than, "in what ways are preschool children engaged in which mathematical activities". Therefore, the possibility for understanding the breadth of the mathematical activity in which children engage at preschool is reduced if we limit ourselves to only look for mathematical activity in situations labelled as such and in which all participants are aware of the label.

As a result of identifying the problem with viewing preschool mathematics only in relationship to school mathematics, we chose in our previous work (Johansson, Lange, Meaney, Riesbeck, & Wernberg, 2012; Helenius, Johansson, Lange, Meaney, Riesbeck, & Wernberg, 2014 forthcoming) to consider preschool mathematics as one version of Bishop's (1988) 6 mathematical activities – discussed in the next section. Here it suffices to say that this decision has required us to reflect more widely about issues that emanated from this

choice, such as who is doing the classification and for what purpose. In this paper, we discuss some of these issues in relationship to our research question "in what ways are preschool children engaged in which mathematical activities" produce other methodological issues that needed reflection.

Mathematical activities

In his book, Bishop (1988)

> presented the case that six key 'universal' activities are the foundations for the development of mathematics in culture. … All cultures have necessarily developed their own symbolic technology of mathematics in response to the 'demands' of the environment as experienced through these activities. (p. 59)

The mathematical activities were Counting, Measuring, Locating, Designing, Playing and Explaining, which respectively, and in short, were answering questions involving quantification (how many? how much?); space and shape (where? what?); abstraction, hypothetical thinking and reasoning (how to? why?). According to Bishop, these activities are present in all cultures, albeit, in different forms depending on the particular social and environmental needs. He referred to the "internationalised discipline of mathematics" (p. 57) as Mathematics with a capital M and saw it as one "version" of the 6 activities. In the cases of Mathematics and school mathematics, the 6 mathematical activities are 'solidified' into distinct social practices (Fairclough, 2003). Seeing academic and school mathematics as social practices resonates with Bishop's conceptualisation of mathematics as a cultural activity. Both perspectives highlight mathematics as a human activity, which, rather than being one intellectual, non-material or even trans-human edifice, comprises a range of socially and culturally situated practices, each of which is characterised by a set of sayings, doings and relatings (Kemmis & Grootenboer, 2008) that affords and attributes purpose and meaningfulness to the activity.

As indicated earlier, we chose Bishop's 6 mathematical activities to be the "spectacles" with which to look for mathematics in preschools. We could identify all of Bishop' 6 mathematical activities in situations that were video-recorded in a Swedish preschool in 2011 (Johansson et al., 2012). One consequence of Bishop's conception of mathematical activities as embedded in cultural and, hence, social practices, is that the mathematical activity in a situation does not depend on being recognised by the participants. It is sufficient that the situation is recognised as involving a mathematical activity by the researchers.

Yet, the classification was not straight forward. Unlike MacMillan (1998) who also had used Bishop's 6 activities in preschool mathematics education research, in any one situation, we often could identify more than one

mathematical activity. Although a practical challenge, this did not require any rethinking about the classification.

In contrast, while doing the classification it became clear that we needed to consider the role the mathematical activity had in the situation. Sometimes it seemed to be the focus of the situation and at other times, it seemed to be an unrecognised tool for resolving a problem. An example of the first is where the teacher drew attention to the shape of leaves, collected by children because they liked collecting them. By highlighting the mathematics, the teachers turned the focus of the situation away from collecting leaves and on to the mathematical activity Designing. An example of the second type could be a situation where a child had filled a bucket of sand in order to make a sand castle. However, the bucket was too heavy to be turned over and so no castle could be produced. In this case, the child had to work out that to turn the bucket it had to be less heavy. This required the amount of sand in the bucket to be reduced, which was done by scooping some sand out. The mathematical activity Measuring was involved in solving the problem but was not the focus, or centre of awareness.

While the notion of mathematics as being comprised of 6 mathematical activities resolved one methodological problem, that of identifying the mathematics of preschool, it raised another issue. This required some rethinking because it seemed that the two different purposes did provide more details about how to answer our research question "in what ways are preschool children engaged in which mathematical activities".

Instrumental and pedagogic situations

We needed to find some way to discuss the different purposes and the affect that they had in responding to our research question. Subsequently, we chose to use Walkerdine's (1988) distinction between instrumental and pedagogic tasks.

> This classification used the designations *instrumental* and *pedagogic* to describe certain kinds of tasks at home and was a distinction originally devised in relation to practices involving *number* in the home. Instrumental referred to tasks in which the main focus and goal of the task was a practical accomplishment and in which numbers were an incidental feature of the task, for example in cake-making, in which the number *two* might feature in relation to the number of eggs needed and so on. In the pedagogic tasks numbers featured in a quite different way: that is, numbers were the explicit focus of the task. On such occasions the focus was predominantly the teaching and practice of counting. So, for example, a child might be asked to count her coat buttons for no other purpose than to practise the count. (Walkerdine, 1988, p. 81; italics in the original)

However, when Walkerdine tried to use the classification on parent–child interactions involving size relations, it was not so easy:

> I found the exercise difficult. The usages did not always seem mutually exclusive and I was not convinced by my own categorisation. In addition, there appeared to be some exchanges that did not fit either of the classifications. In these exchanges the mother appeared to be *commenting* on an activity or on something which had been done or seen. In these cases the mother did not appear to be instrumental, in that the exchange was not actually part of a practical activity, but then neither was the purpose explicitly didactic. (Walkerdine, 1988, p. 86; italics in the original)

Hence, it could be that the designations instrumental and pedagogic was more suitable to classifying situations involving the mathematical activity Counting, maybe because features of the school mathematics version of Counting (counting, doing sums, practicing multiplication tables) figure so strongly in the public discourse about mathematics. In our video recordings, we succeeded in finding instrumental and pedagogic situations for each of the six mathematical activities (Johansson et al., 2012), thus suggesting that the first of the issues raised by Walkerdine was not relevant in relationship to our data set. We also did not have examples of the commenting that Walkerdine identified, perhaps because teachers in preschool settings are more likely to engage with children in a situation rather than just comment about what was going on.

However, another methodological issue did arise. This was one of perspective, that is, whose perspective of the situation was adopted in the analysis? Although our original assumption was that using Bishop's 6 activities would mean that the classification based on our researcher's gaze was appropriate, our reflections now made it clear that such an assumption was naive. Situations could be classified as either instrumental or pedagogical but would not necessarily appear the same to the participating children and teachers. In some cases, it seemed that a situation could be instrumental for the child but pedagogic for the teacher. In the how-to-turn-the-bucket-over situation, the child was engaged in the practical accomplishment of making a sand castle. Hence, the mathematical activity in the situation was instrumental for the child (i.e. IC). The teacher, watching the child, seemed to recognise the child's problem and supported the child working out the solution (taking out sand) by verbalising her interpretation of the child's tacit reasoning. Hence, it appeared to be a pedagogic situation for the teacher (PT), in which she supported the child's engagement with the mathematical activities of Explaining and Measuring. If she had just told the child to take out sand or done it herself, then we would have classified it as an instrumental situation for the teacher (IT).

From our reflections on the methodological issue of whose perspective, we decided to change our conceptions of situations being either pedagogical or instrumental to a classification that would allow for a more nuanced interpretation. Consequently, we decided to situate the classification of situations

in a two dimensional grid with the axes instrumental–pedagogic for the child(ren) respectively for the teacher (Figure 1). According to the analysis above, the how-to-turn-the-bucket-over situation would be located in quadrant ❹ (IC–PT). If the teacher had taken the sand out herself, it would be in quadrant ❶ (IC–IT). The grid spans a field of didaktic affordances and we hence label it "didaktic space".

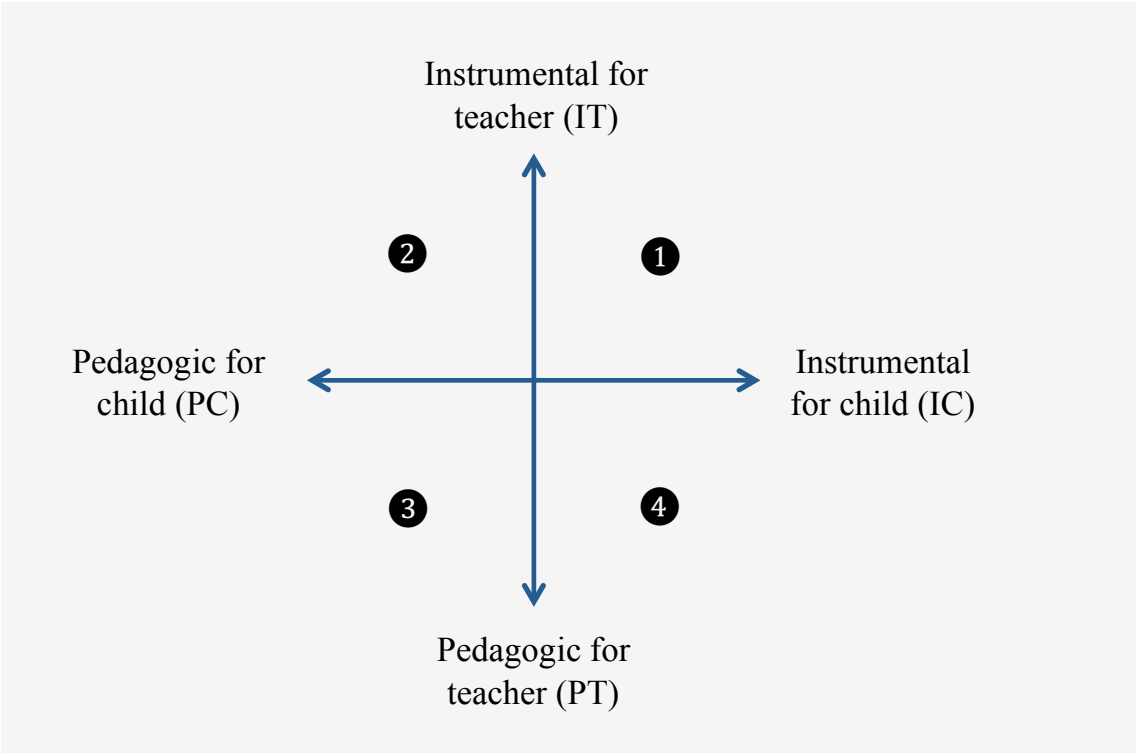

Figure 1. Didaktic space. The numbers refer to the quadrants.

The didaktic space

Each of the four quadrants in the didaktic space represents situations with a particular didaktic makeup. Situations would be located in quadrant one, when the teacher and the child(ren) were solving a problem, involving one or more mathematical activities. In these situations, none of the participants expected to teach or learn anything. Although one participant may be more knowledgeable about how to solve the issue the focus for all participants is on the resolution of the problem, not on the process of resolution, which opens up possibilities for teaching.

In quadrant two, the teacher may be focused on solving a problem whereas child(ren) would be focused on teaching the teacher or themselves about some aspect of a mathematical activity. Although there were few examples of this in the situations in our data set, in other data sets, it is possible to imagine a situation in which a teacher is focused on packing up materials, while the child is focused on learning about different attributes while doing it.

In the third quadrant, the focus for both the teacher and the child(ren) is on teaching/learning about a mathematical activity. Usually, the teacher is the one who has the role of teacher and the child(ren) the role of learner. However, there is a potential for the roles to be reversed. The teaching may just involve making children aware of a specific feature or more formally requiring a child to pay attention and to learn the material in a way that the teacher can recognise this. The PC–TP combination is characteristic of school mathematics. The gain from it is a strict focus on content, on "mathematics" but the loss may be the motivation and purpose of engaging in the mathematical activity.

In the final quadrant, the teacher's focus is on teaching the child(ren) about the mathematical activities. However, the child's focus is on resolving a problem. In our data set, we had many situations that we could classify as belonging to this quadrant.

Apart from providing characterisation of situations with distinct didaktic make-up, another advantage of the didaktic space was that it allowed us to track changes in the focus of the teacher and/or the child(ren) within a specific situation. The dynamic nature of the interaction could then be described. In the future, this may allow us to determine whether the appearance of a specific mathematical activity or combination of activities might be related to the instrumental or pedagogical foci of the teacher and/or the children. Thus the didaktic space provides us with a way of conceptualising the "field of choices".

In the following sections, we re-analyse situations from our earlier work (Johansson et al., 2012) using the didaktic space model as an analytical tool.

Counting leaves

In an outdoor activity, the teacher had the children be pretend magpies and collect five leaves to place in hoops, which represented their 'pantries". This example was chosen for reanalysis because it showed a common situation in which child's focus appeared to be different from that of the teacher.

Björn:	Jag kan ränka, en, två, tre, fyra, fem	Björn:	I can count, one, two, three, four, five.
Lärare:	Fem, bra! Nu har ni fem stora löv i ert skafferi	Teacher:	Five, great! Now you have five large leaves in your pantry,

Originally, we classified this as an instrumental, Counting situation because the child initiated the counting, possibly to check if he had accomplished the task. We still classify it as instrumental for the child (IC). The teacher, however, had planned the situation so that the children would participate in Counting and thus learn something about the number 5. Thus, for the teacher it was a pedagogic situation (PT) even if she could not predict that the child would initiate the counting. In the didaktic space, it would be situated in quadrant ❹.

Walking along the bench

This second example illustrates how foci can change as a situation develops. Whilst playing outside, a toddler climbed on a bench and walked back and forth along it. The second picture in Figure 2 shows the child requesting assistance to get down, by raising her arms to the teacher. When the teacher did not pick the child up immediately, the child clambered down, after first gauging how far down she had to go.

Exploration of space is a feature of Locating. In this situation, the child seemed to have initiated her own learning about the spatial relations of being *up on* the bench *above* the ground, walking *along* the bench, *back* and *forth*, looking *down* to the ground. Hence, although there was no teacher actively involved, we originally considered that the focus had a pedagogic purpose.

In the re-analysis, we pay more attention to the sequence of events. At first, the child did seem engaged in a pedagogic Locating situation (PC). As the teacher watched the child engage in Locating, the situation also seemed to be pedagogic for the teacher (PT). This part of the situation is located in quadrant ❸ (PC-PT).

Then the situation turned into an instrumental situation of Measuring for the child (IC) because she wanted to get down and now had a problem to solve. The child estimated the distance to the ground and compared it with her sense of her own size and climbing capability. First, she asked the teacher for assistance by stretching out her arms. The teacher declined the child's request to be lifted down, probably because she wanted the child to engage in the problem of getting down by – physically and intellectually – combining her understandings of

Figure 2. Walking along the bench

Locating and Measuring. Thus, it was a pedagogic situation for the teacher (PT). The situation now is located in quadrant ❹ (IC–PT).

The child then bent down and the teacher offered her assistance, perhaps because she decided the challenge of getting down was too much for the child. We interpret this as a change from pedagogic to instrumental for the teacher (IT). The child, however, declined the teacher's offer and climbed down without assistance so the situation can be said to be in quadrant ❶ (IC–IT).

Thus, during this situation, we see a move – in terms of the model – from ❸ (PC–PT) to ❹ (IC–PT) to ❶ (IC–IT).

Conclusion

This paper has explored the issue of how to respond appropriately to the research question "in what ways are preschool children engaged in which mathematical activities". As discussed earlier, other preschool mathematics education research which takes its understanding of mathematics from a school mathematics perspective can be considered problematic because of differences at the level of social practices. Our initial analysis of situations in a Swedish preschool (Johansson et al., 2012), using Bishop's (1988) 6 activities seemed to provide a more appropriate way to discuss the mathematics that children were participating in. However, it then became obvious that categorising situations from the researcher's perspective did not provide us with a sufficient detailed understanding of what was occurring in the video data. Although Walkerdine's (1988) distinction between pedagogic and instrumental purposes for situations seemed helpful in raising this issue, it then raised the issue of whether it was the children or teacher's focus in the situations that should be the basis for our analysis. The development of the didaktic space as an analytical tool has provided us with a more nuanced response to our initial research question. Nevertheless, these questions continued to remind us that our analytical choices influence what we can discuss when describing the mathematics of Swedish preschools.

It also seems likely that the didaktic space may solve some other methodology issues when researching the mathematical activities in which children engage. In Swedish preschools the curriculum is quite clear that children are not expected to reach any pre-set agenda of mathematical objectives. Instead, the objectives are about what the preschool should make available to children (Skolverket, 2010). As well, play has a central role in conceptions of how learning should occur. A research methodology such as the didaktic space provides a way to interpret dynamic situations in Swedish preschools that can be useful in identifying the impact of the professional development initiatives now being provided to preschool teachers. This is because it provides a way of analysing data on what occurs in preschools both before and after an intervention

of this kind, without relying on formal assessment of young children's mathematical knowledge.

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