Number By Reasoning and Representations – The Design And Theory Of An Intervention Program For Preschool Class In Sweden

Görel Sterner & Ola Helenius
National Center for Mathematics Education, University of Gothenburg

We describe the design process for an intervention program in the domain of number in Swedish preschool class. A consequence of the design-feedback cycle was that the initial idea of combining a learning trajectory based approach with a socially driven teaching based on collective reasoning was revised. The resulting design keeps the emphasis on structured sequences of activities and children's and teacher's reasoning about representations, but moves the learning goals from individual sessions within the program to the level of the intervention as a whole.

Introduction

Preschool class has a unique position in the Swedish education system as the bridge between the informal learning that dominates in preschool, and the formal learning following in school. It is non obligatory but in practice almost all six year old children participate. This makes preschool class a potential arena for giving children opportunities to develop skills in mathematics to remedy mathematical difficulties and remove barriers for learning. In our discussions with preschool-class teachers, they often emphasize their need for support to develop mathematics instruction and to take advantage of findings from research. The purpose of the study, partially reported here, is to design and evaluate a mathematics intervention program in the Swedish preschool class built on structured instruction design, a concrete-representational-abstract learning pattern and children's collective reasoning. The overall effect of the intervention is measured on the level of children's learning by means of a cluster randomized control study reported in a forthcoming article (Sterner, Wolff & Helenius, manuscript). The present article deals with the design phase of this study, where the purpose is to fine tune the three guiding principles into a working practical realization. The research question is: Is it possible to combine such principles into a functional program, and if so, how can such a program be described? Hence this paper methodologically falls under the design research paradigm (Edelson, 2002). McKenney and Reeves (2012) argue that educational design research is based on five intertwined principles. They are:

> *Theoretically oriented.* Empirical testing is used to validate, refine, or refute hypotheses and conjectures that are embodied in the design.
>
> *Interventionist:* Educational design research strives to produce new theoretical understanding, to positively impact practice, bringing about transformation through the design and use of solutions to real problems.
>
> *Collaborative:* Educational design research is conducted in collaboration among a range of actors and educational contexts.
>
> *Responsively grounded:* The products of educational design research are shaped by participant expertise, literature, and especially field testing.
>
> *Iterative:* The insights and the interventions of educational design research evolve over time through multiple iterations of investigation, development testing, and refinement (pp 13-15)

The work reported here honor these five principles. It is *iterative* since the design and its implementation has been tested and developed over four feedback cycles. It is *collaborative* since researchers and practitioners with different background contributed both to design, evaluation, development and theorizing of the result. The obtained theoretical principles are implemented in a teacher's handbook, available for preschool teachers and this makes the work distinctively *interventionist*. What will be mainly emphasized in this article is the *theoretical orientation* namely, three initial design principles that built on different areas of research and theory, and was embodied in a specific teaching sequence presented in the teachers handbook. We will describe how both the embodiment – the actual teacher instructions – as well as the grounding principles changed as a result of how it was *responsively grounded,* through several cycles of field testing and additional consulting with the literature.

Background

Preschool children's mathematical knowledge when starting school is highly predictive of their later success in mathematics in compulsory school (Duncan et al., 2007). Children who start school with weak mathematical knowledge tend to experience further difficulties in a downward spiral (Morgan, Farkas & Wu, 2009; Geary, 2011). In recent years there has been a growing interest in early intervention in mathematics. A meta-analysis (Diamond, Justice, Siegler & Snyder, 2013) shows that interventions vary a lot regarding the mathematical content. Examples of targeted content include: *Relational arithmetic skills* e.g. seriation, classification and conservation of numbers (Malabonga et al., 1995), *counting and efficient counting strategies, addition and subtraction with objects/pictures, add one, subtract one, estimate numbers, read and write numbers* (Clark et al., 2011), and *number line estimation* (Ramani & Siegler, 2008). There are a few studies explicitly focusing on *number sense related to reasoning about numbers* (e.g. Nunes et al., 2007; Aunio, Hautamäki & Van Luit, 2005). Math-oriented early childhood curricula have been developed in collaboration between researchers and teachers, e.g. *Number Worlds* (Griffin, 2003; 2007) focusing on the central conceptual structure of whole numbers developed by Case and Okamoto (1996) and *Building Blocks* (Clements & Sarama, 2007; Clements et al, 2011). The program *Building Blocks* focuses both on numbers and geometry and a particular feature of this program is that each domain is structured along a research-based hypothesized hierarchical learning trajectory. The theory of hypothetical learning trajectories (HLTs) is usually connected to developmental and cognitive psychology and, more recently, developmental neuroscience (Consortium for Policy Research in Education, 2011; Simon, 1995). Typically, learning trajectories connects a theoretical idea about a particular learning process leading to some learning goal, as well as practical activities designed to take the learner through the process. One of the early proponents of learning trajectories define them as “made up of three components: the learning goal that defines the direction, the learning activities, and the hypothetical learning processes – a prediction of how the students’ thinking and understanding will evolve in the context of the learning activities” (Simon, 1995, p. 136). Instruction and instructional programs based on learning trajectories have often proved successful and in particular several intervention programs for preschool builds on learning trajectories (see Clements et al., 2011, for an overview).

Design process

The participants in the initial design process were researchers within the psychological and mathematical disciplines and other experts on mathematics education. Here we will describe what design principles the program built on and how these principles were realized in concrete instructions to the teachers. We then describe four testing cycles and how the feedback influenced design choices and the realization of them.

Initial design principles

The first design principle is that children should be provided with a structured sequence of activities. This has been shown to be particularly effective for children at risk for mathematical difficulties (Gersten et al, 2009). This way of choosing and sequencing activities is similar to Learning trajectory based designs (Clements et al., 2011). To help structuring the program for teachers, the activities are grouped in five themes designed to be carried out by teachers over ten weeks: *Sorting, classifying and patterns; Numbers, counting and patterns; Part-part-whole; Number line*, *Grouping and place value.* The ordering of the themes and how the content in the cross reference between themes is mainly based on Griffins research on the conceptual structure of whole number (Griffin, 2003, 2007). Due to space limitation we do not deal further with the details of the sequencing or present individual activities here.

The second principle concerned the Concrete – Representational – Abstract (CRA) model, a linear model where teacher and pupils start working with concrete objects and gradually advances to the use of visual representations and further on to abstract symbols (Witzel, Mercer & Miller, 2003). In terms of learning trajectory theory, each session contained elements designed to take children from a concrete manipulation stage through several phases of representations with for example dots, squares and other icons and towards some form of symbolic or abstract reasoning with symbols like written numerals. The effectiveness of teaching mathematics through a CRA sequence of instruction to students is well documented in the literature (e.g. Allsopp, 2007; Baroody, 1987; Clarke et al., 2011; Wintzell, 2003).

The third principle involved using children's reasoning about their work and about their documentation (drawings) of their work as the main vehicle for learning. In Vygotsky's theory (1978) the social interaction between children and adult is the main source for the development of advanced mental functions. All development in the child appears first at a social and then at an individual level. Language is viewed both as a cultural tool to develop and share knowledge within a social community, and as a psychological tool to structure the processes and content of one's own thinking. Examples of cultural tools are language, art, writing, numbering etc. (Vygotsky, 1978). Drawing on Vygotsky´s work Brooks (2005; 2009) argue that when drawing is used in a collaborative and communicative manner it exists at an interpersonal level. In our design, whole class collaboration and partner work function as activities on the social level while children's drawing also at one point function on an individual level. An underlying assumption here is that drawing facilitates children's reflection on the mathematical content they previously worked on in collaboration with teacher and peers, but from a different perspective, and that the interaction between the collective and the individual, contributes to the development of thinking (Vygotsky, 1978). Children's drawings are creative representations that connect back to the collective reality they were previously engaged in. In the follow-up activity their drawings once again turn into an activity on the social level. In the discussions about their drawings each child brings a personal dimension to the enterprise. No children are alike and even if the messages being transmitted can be considered the same, it will be perceived slightly different because the receivers are different (Bishop, 1991).

Realization of design principles

To make the design principles into a teachable program, we developed a "teacher's guide" (Sterner, Wallby & Helenius, 2014). The first principle was realized by means of organizing the guide in the themes and for each theme give concrete and explicit instruction of activities to carry out. Each theme involves around ten sets of activities (sessions). The mathematics sessions were organized in a structure with six phases:

- *Counting rhymes:* A lesson starts with children and teacher gathering in a circle on the floor, counting in chorus up and down on the counting string. When a child, standing in the middle of the circle, pointing rhythmically at each child while all count together, the circle that children and teacher form is the very representation of the counting (Freudenthal, 1991).
- *Initial activity:* The teacher introduces the current task and the work is done collectively in class by using concrete objects like blocks, sticks, buttons, dices,
- *Partner work:* Children then work with partners or in small groups on similar and extended activities as they did earlier in class, using different objects or other representations.
- *Whole-class discussion:* Children and teachers come together to a joint monitoring and discussion of pair work.
- *Children's documentation:* Children create drawings as documentations of what they have done so far. The drawings are new representations that form the basis for future collective activities and discussions with teachers and peers in the next phase.
- *Follow-up activity:* Children's drawings are the starting point for further reasoning about the concepts they have worked on and connections, differences and similarities among the representations of those concepts.

Through these phases the CRA principle is realized by means of the initial work with concrete objects followed by subsequent representations of those objects when the children make their documentations. In the discussion phase, even if a child does not have an abstract idea about some concept targeted in the session, the teacher can use other children's reasoning and representations to shift the discussion towards the abstract. This means that a drawing that, from the child's point of view, started out as a representation of concrete objects and relations, may be discussed by others as a representation of abstract structures or concepts helping all children to extend their understanding towards the conceptual. In this sense, the way that the CRA model is realized in the six phases is intended to interplay with the third principle concerning the role of reasoning and social interaction.

First testing cycle.

In the first phase of the iterative stage of the design process, sessions and themes were tested by six preschool class teachers and children in their classes. One thing we learned from the collaboration with the teachers in the first cycle was to carefully choose the concrete materials to be used in the activities. In one activity the children are expected to investigate and reason about how to move soft toys between delimited quantities in order to make those quantities equivalent. The teacher experienced that the activity did not work at all since children's attention was drawn to the soft toys – everyone wanted as many as possible and they forgot all about solving the number problem. We later found this phenomenon described in the research literature (DeLoache, 2000). The more children are attracted to the physical attributes of the representation the harder it seems to be to see the symbolic information and to stick with that. In terms of our principles, this relates to the realization of CRA-principle in relation to principle of explicit structured activities. For the "C-phase" in CRA to increase the possibility of discernment of the abstract structures that are built into the particular activity, the objects should not have attractive physical or emotional attributes.

Second and third testing cycle

In the second, and later also in the third, cycle six new teachers were recruited to the team. In both cycles, a researcher (the first author of this paper) and the teachers met at seven seminars where the mathematical content and the teaching strategies were discussed. In the time between those seminars the teacher tried out the activities in their classes and documented their experiences. Teacher's documentation then became the basis for in-depth discussions at the following seminar.

A problem that emerged during the second cycle was difficulties to make all children to participate in the discussions, to express their views and suggest solutions. The teachers felt uncertain on how to pose open questions that would take the discussions and children's thinking further. We decided to complement the material with examples of questions such as: How do we know that...? What is similar and what is different in these solutions? How do we know that we have found all solutions? What will happen if we change...? How do you think Thomas thought when he made this pattern? More importantly, we also introduced a puppet into the pedagogy that sometimes came and asked questions and contributed to the reasoning in the group. The puppet has at least three equal important functions:

1. Children's ability to imagine the puppet as a "real" person help to bring out the playfulness in mathematics and "trick" them to teach the puppet and express their own views.
2. The puppet asks questions and makes statements that triggers the children's desire to reason about concepts and relationships between concepts, come up with hypothesis, provide explanations and propose solutions.
3. Using the puppet's questions and statements, the teacher can help children turn their attention to certain mathematical aspects and phenomena.

Using a puppet in the pedagogy in this way has previously been described in research (Freeman, Antonuccia and Lewis, 2000). This change related to how our third principle about children's reasoning was realized in the teacher instructions in the handbook. We concluded that for the reasoning sessions to be productive, the handbook did not only need to contain explicit activities, but also explicit tools and routines that could support teachers to carry out productive reasoning sessions.

Stage 4 analysis

In the fourth cycle eight teachers participated. Seminars were conducted in a similar manner as in stage 2 and 3. It was not until now it became apparent to us that teachers felt frustrated and uncertain of how to proceed with a subsequent session when all children did not reach what the teachers perceived as the learning goals of the present session. For example, when children documented their experiences from the work on part-part-whole relations of number seven, some children visualized the combinations by making drawings of concrete objects in two colors in different combinations. Other children drew the combinations by using dot number patterns and still others used mathematical symbols to represent different combinations like “7 0” and “6 1” with an empty space between the numerals for each combination.

On the one hand the problem seemed to be that some children when expected to use e.g. dots, circles in the representational phase, they preferred to use abstract symbols like numerals that belonged to the abstract phase. On the other hand the teacher had an idea of the group moving through the representations all together in an attempt to make sure that each child in the end reached the abstract phase and abstract understanding of every concept they had worked on. The difficulties that the teachers experienced was: 1. Children did not reach the abstract level at the same time or some children kept on using iconic representations for a long time. 2. Some children spontaneously used abstract mathematical symbols during the representational phase and the teachers meant it simply wasn’t tenable to tell the children that they had to wait to the abstract phase before they could use mathematical symbols and to share their ideas with peers.

In our discussion with the teachers we decided that instead of making sure that all the children reached a particular goal at the end of a session, it was emphasized that the primary role of the teachers was to make sure each child got opportunity to present their own representations of the activity, and have it and it’s relation to other children’s representations reasoned about in the group. In this way children's differing views and ways of expressing themselves about the activity and the concepts that were in focus in a particular session became an asset in the discussion. It was also emphasized that the relations between the mathematical themes, meant that the concepts children met were reinvented several times in different mathematical contexts.

This adjustment effectively ties all our three principles together. In essence, we place the principle that the children should be given opportunity to reason about their representations of the activity or the concept above the principle that each session should take children through the CRA stages. But the two other principles will in fact mean we can recover also the CRA-principle. Due to the emphasis on collective reasoning, even children that did not themselves reach the abstract stage in a particular session, will be part of a discussion where abstract ideas are represented. Moreover, the sequencing of the session means that the same concept is handled many times, so children will get further possibilities to reach the abstract level through the program.

Discussion

This study used both literature and experience to investigate how three design principles could be combined to support teaching mathematics in preschool class in Sweden. Findings from the field testing confirmed that these principles offered relevant support but also revealed some challenges. The first testing cycle made us make the description of activities more detailed with respect to exactly what objects to use to increase the possibilities for children to attend to the underlying abstract structures of the activity. The second/third testing cycle made us complete the teacher material with more detailed instructions, tools and routines for how to make the reasoning sessions more productive. Both these changes concerned the embodiment of the CRA principle and the principle of reasoning about representations respectively.

The discovery in the fourth testing cycle however, was of a different nature. As pointed out, our program has many similarities with a learning trajectory design. Even though it is not required theoretically, in such designs individual activities often come with learning goals. In addition to the structured design, our program build on sessions involving collective reasoning about children's individual representations of collectively experienced activities. In our testing, we found that the idea of sequenced learning goals tied to such sessions created a conflict with the idea of collectiveness. When rethinking our design, we concluded that each session was better seen as an instance to get a particular type of experience.

This design is the result of work both from researchers as well as from teachers and their children. The effectiveness in terms of overall student outcomes is currently analysed. It would be an interesting exercise for future design work and research to examine in what sense these design principles are transferable to other contexts, like other areas of mathematics or other ages of students.

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