Learning Subtraction Strategies From Principle-Based Teaching Activities

Håkan Sollervall
Linnaeus University, Växjö

Three teachers and a researcher have co-designed a teaching activity intended to support students' learning of two strategies for subtraction. The researcher focuses on the relation between theoretical principles, introduced to underpin the participating teachers' work, and the learning outcomes of their 33 students in grade 4. The principles are adapted by the researcher during three design cycles and negotiated with the teachers to meet emerging needs in the design process. The three teachers are fully responsible for planning, implementing, and evaluating an iterated teaching activity designed according to these principles. This study indicates positive effects of targeting low-achievers with teacher-led structured group activities, using guiding principles from self-regulation theory.

Introduction

Researching the teaching and learning of mathematics usually involves providing theoretically grounded descriptions of observed classroom activities or learning processes. This is often done without intervening in these activities and processes. In contrast, design research explicitly addresses the provision of opportunities for learning (Cobb, Confrey, diSessa, Lehrer, and Schauble, 2003). Design researchers engage in a cyclic process involving both instructional design and classroom-based research, encompassing all aspects of a teachers' work with planning, implementing, and evaluating teaching activities (Cobb, Stephan, McClain, and Gravemeijer, 2001). Such a comprehensive approach involves several different research tasks, implying high demands on the design researchers' cognitive, material, and social resources (Boote, 2010).

In the study reported in this paper, we have adopted a more modest and less demanding approach to design research. As researchers, we do not observe the classroom activities and we do not engage in qualitative analysis of the learning outcomes. Instead, we focus our attention on underpinning theoretical principles that may improve the learning outcomes. The researcher introduces and adapts theoretical principles to emerging needs in the design process, while the participating teachers are fully responsible for planning, implementing and evaluating an iterated teaching activity designed according to these principles.

Our study may be compared with a (preliminary) clinical trial, where specific treatments are introduced in an ecological context. Clinical trials are commonly used in medicine, for example to identify effects of various drug treatments. If a preliminary study indicates positive effects, repeated studies may be carried out for the purpose of confirming these effects. Although such positive effects may be confirmed, they are seldom explained. We follow a similar rationale in our research, that is, we attempt to identify positive effects (as improved test scores) of "theoretical treatments" adapted to meet emerging needs in the design process. Although the current study puts focus on identifying specific principles for treating a specific issue, it also investigates the potential value of using the methodology of principle-based clinical trials in mathematics education.

Research objectives

Our objective is to investigate possible connections between underpinning principles for teaching activities and students' test scores in relation to the specific learning object of these activities. As principles, we consider theories and theory-based methods that are introduced by the researcher and guide the teachers' planning and implementation of teaching activities. The principles are updated in a cyclic process, based on the students' intermediate test scores and the teachers' observations. Our research question follows.

- How does the flexible outcome-based adaptation of underpinning principles for the teaching activity affect the students' test scores?

The study involved three teachers and 33 students in grade 4 in Sweden. The learning object concerned contrasting, selecting, and applying two different strategies for subtraction, namely adding up (as in 304 – 298 = 2 + 4 = 6) and subtracting parts (as in 435 – 121 = 300 + 10 + 4 = 314).

Our conceptual framework – a bricolage of theories

Our study applies a *bricolage* of theories of different character and from different research traditions (Kincheloe, 2001). The bricolage approach, which fits within the Singerian inquiry tradition (Lester, 2005), has a long tradition in mathematics education research and challenges "the positivist epistemology of practice wherein practical reason is construed as the application of theory" (Cobb, 2007, p. 3). While a Lockean inquiry regards observations as evidence with respect to pre-defined theories, the Singerian inquiry "entails a constant questioning of the assumptions" (Lester, 2005, p. 463). Instead of generating research questions that fit a specific theoretical framework, our bricolage of theories is adapted to authentic questions and needs as expressed by the participating teachers. Our bricolage is also adapted to the specific learning object, which necessarily involves representing the two strategies for subtraction. In the next section, we briefly discuss theories about representation of mathematical objects. In the last section, we discuss meta-cognitive strategies for self-regulation. While theories of representation were included as principles from the beginning of the project, the theory of self-regulation became involved in the third cycle. In addition to describing these theories, we briefly account for how they were introduced – *but not how they were used* – as underpinning principles for the teaching activities.

Mathematical representations

At the first project meeting in February 2012, subtraction was discussed from a structural perspective as a mathematical idea that needs to be mediated (or represented) by the use of artefacts (Ogden & Richards, 1923; Duval, 2006; Winsløw, 2003) such as tangibles, pictures, diagrams, symbols, and natural language. Representations can be transformed in two qualitatively different ways (Duval, 2006): as *treatment* within a specific representational system (e.g. the symbolic treatment 34 + 25 = 50 + 9) and *conversion* between different systems (e.g. converting three apples to the symbol 3). Ability to make conversions and coordinate different representations of the same object is needed for conceptual development (Winsløw, 2003) as well as problem solving (Janvier, 1987).

The participating teachers were well aware that the two targeted strategies for subtraction could be represented in a variety of ways. Examples were shared about possible ways to represent the two strategies, for example by making use of the number line or tangibles such as measuring tape or pearls on a string. In addition, the researcher introduced the so-called empty number line, commonly used completely without markers (Klein, Beishuizen & Treffers, 1998), thereby inviting the students to add markers and numbers (Fig. 1a). To further stimulate the students to discover the adding up strategy it was decided to make use of number lines with markers but without numbers (Fig. 1b).

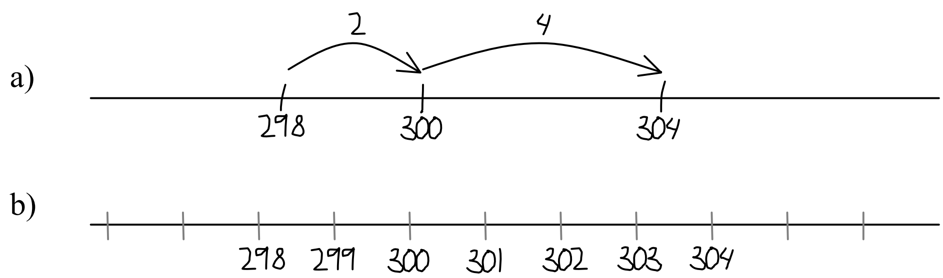

Figure 1. Two examples of empty number lines, completed by students.

Rather than asking the students to solve routine tasks by following instructions and making use of templates, it was agreed that the teachers should construct real-life problems inviting the students to work in small groups, exploring and modelling situations calling for them to compare or remove quantities by making use of provided artefacts. The teachers chose to avoid subtractions that do not fit well with respect to either strategy, for example 421 – 135. It was agreed to focus on the adding up strategy for terms that are close to each other (as in 304 – 298), and the by parts strategy when all the parts of the first term are larger than the corresponding parts of the second term (as in 435 – 121).

Self-regulation

At a team meeting during the third cycle of the design process, it was decided to draw on the theory of self-regulation. This decision was strongly influenced by the (rather disappointing) outcomes from the second cycle, where the poor test scores (for 13 out of 33 students) were interpreted as a consequence of the students not being able to distinguish between the two targeted strategies.

Mathematical problem solving or executing complex mathematical calculations often calls for ability to assess strategies and representations, select and implement a chosen strategy with a particular representation, monitor and control own performance of transformations, react on incorrect intermediate results, and reflect on the answer in relation to the original problem. Such meta-cognitive abilities are well aligned with the four phases of self-regulated learning: fore-thought, planning and activation; monitoring; control; reaction and reflection (Schunk, 2005). Each phase involves processes that can be related to cognition, motivation, behaviour, and context, or a combination thereof.

With respect to the two strategies for subtraction, the students are expected to assess that the subtraction 304 – 298 should be calculated by adding up, while 435 – 121 calls for subtraction by parts. Comparing strategies and thinking of different ways to represent these strategies (e.g. on a number line, or by splitting a number in its parts) is primarily a cognitive and contextual process in the phase forethought and planning. The students' mental representations may be externalized, but could just as well be managed internally (e.g. on a mental number line, or imagining a number being split in its parts). Still within this phase, the student has to select and activate a strategy with a chosen representation. This particular representation is often, but not always, externalized. The phases of monitoring and control require the student to engage in carrying out the strategy by transforming representations, either as treatments or conversions or combinations of both. Having achieved a preliminary answer, the student may reflect on its plausibility in relation to the original problem statement. For example, the student who transforms 304 – 298 to 194 could readily identify that the answer is incorrect by reflecting on the positions of 304 and 298 on a number line. Students who engage in systematic reflection during the transformation process may feel less need to engage in an overall reflection.

In our study, the teachers observed that not all students engaged in cognitive and contextual processes, particularly in the phase of forethought, planning, and activation. For this reason, the third design cycle specifically addressed such processes by targeting the low-achievers in structured teacher-led group activities.

Methodological considerations

The design process was documented by the researcher and one teacher, separately taking notes about progress and decisions. These notes were primarily used to keep the development project on track, but proved sufficient for supporting recall of associated events of relevance for the research study. The test scores were documented by the teachers during the project in tabular form.

The development of the students' test scores have been illustrated in line diagrams from where three different groups of students have been identified and characterized (Fig. 2). This simple approach has been manageable for our small sample of 33 students.

Regarding methods for organizing the design process, we have partly been committed to the current study being carried out within a development project arranged as collegial interaction with external expertise, as recommended by Timperley (2008). Three mathematics teachers have collaborated with one researcher in mathematics education. While Timperley (2008) addresses professional development of teachers, similar co-design approaches are well established in the research domain (Penuel, Roschelle & Shechtman, 2007). These approaches may be compared with theory-oriented design research as pursued by Cobb et al. (2001), with limited involvement of teachers, and the practice-oriented learning study approach (Marton and Pang, 2006) where teachers may collaborate without any guidance of external expertise.

In our principle-based approach, the teachers have been responsible for planning, implementing, and evaluating their own practices (Stigler & Hiebert, 1999; Clarke & Hollingsworth, 2002). The researcher has not engaged in qualitative analyses of implemented teaching activities and learning outcomes. The learning outcomes have been quantitatively measured on a traditional test prepared by the teachers, according to their own standards and not influenced by the principles. The researcher has only been responsible for introducing the theoretical principles and engaging in collaborative discussions with the teachers, with focus directed at motivating and exemplifying the principles, evaluating teaching outcomes and negotiating further actions based on these outcomes. The researcher participated in two preparatory meetings in spring 2012 and three additional meetings during autumn 2012, when the teaching activities were implemented with 33 students in grade 4. All of these meetings took place at the school in question. Between these meetings the teachers worked on their own to plan, implement and evaluate the teaching activities.

Results

The study was carried out in autumn 2012 with two classes in grade 4, each with 17 students. One student did not participate in any part of the study, and so the study consisted of 33 students. In the first and second cycles, the teaching activity had similar pedagogical arrangements. The two classes were taught separately, one hour per session, with most of the time spent on the students solving subtraction problems that called for making conversions between representations. The problem solving sessions were arranged with 3-4 students working together and the teacher walking between the groups to answer questions.

Intermediate outcomes influencing the design process

While evaluating the second implementation of the activity, the teachers discussed how to interpret the test scores. They agreed that 12 correctly answered problems – out of the total 17 problems – was a satisfactory result, but also noted that 13 of the 33 students had not achieved at least 12 points on either post-test 1 or post-test 2. On post-test 2, two students were close (11 points) but the remaining 11 scores were in the range from 2 to 8 points. These unsatisfactory results were discussed with the researcher. It was argued that the low achieving students may not have been involved in all aspects in the group work, possibly adopting passive roles and letting the other students dominate in the group work. The researcher and the teachers agreed to specifically target the 13 students and stimulate (force) all of them to become involved in the problem solving processes. It was agreed that the teacher should meet the (new) groups one at a time and spend 2x30 with each group, leading the work by asking questions to make sure all students become involved in all aspects of the problem solving process. An additional student attended although she had previously achieved a satisfactory result. For this reason, her test score on post-test 3 is not included in the diagrams. The 14 students were divided into four groups of 3, 3, 4, and 4 students, respectively. The 2x30 minutes per group were divided into 30 minutes each, on two consecutive days.

Development of the test scores

The test scores from post-test 3 show a substantial improvement for 9 of the 11 students, whose test scores increased from 2-8 points to 12-17 points (Fig. 2).

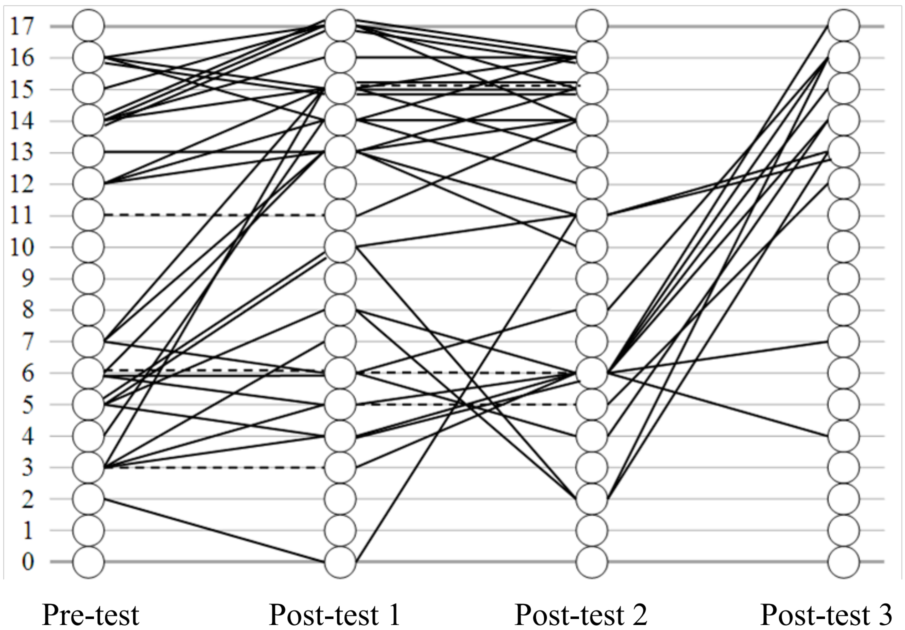

Figure 2. All test scores of the 33 students.

In addition, the two students who had 11 points on post-test 2 both achieved 13 points, thus also satisfactory. We would like to emphasize that the third cycle implementation was not "teaching to the test", but focused on the same conversion problems as in the two previous implementations. The results of all tests are shown in Figure 2, where the lowest level corresponds to 0 points, the next level 1 point, and so on, up to the maximum level 17 points. A dashed line indicates that the student did not take one of the two tests.

It should be noted that the results improve only slightly from the pre-test to post-test 1. The median for difference in test scores is 2 points. Between post-test 1 and post-test 2, the median is 0 points. The major improvement comes, as already mentioned, for the 13 students between post-test 2 and post-test 3 where the median is 8 points.

Based on the criterion that 12 points in any of the first two post-tests is considered satisfactory, we can distinguish three groups of students: the 14 students who had satisfactory results already on the pre-test, an additional 6 students who achieved satisfactory results on either post-test 1 or post-test 2, and the remaining 13 students who did not reach the 12 points on either test. (Due to the limited effect of the second implementation, we do not distinguish between students who achieved satisfactorily on the first and second post-test.) We report the test scores for these three groups separately, in line diagrams (Fig. 3).

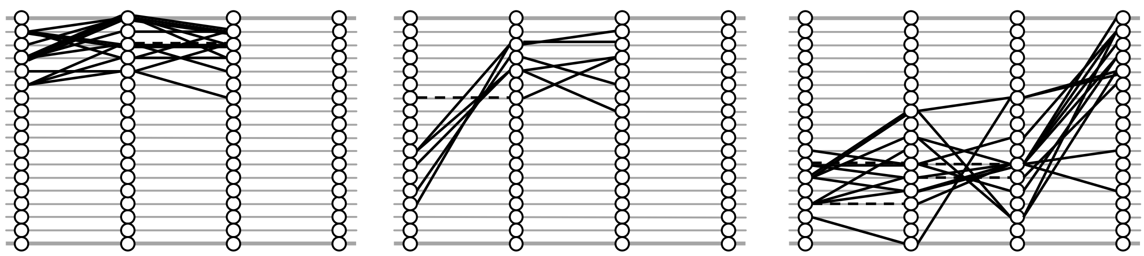

Figure 3. Test results for the three subgroups of students.

We can readily see (Fig. 3, left pane) that the 14 students who scored well already on the pre-test improved slightly on the first post-test, but their scores dropped slightly on the second post-test. Similarly, the 6 students who achieved 12 points or more on the first post-test (Fig. 3, middle pane) show a similar lack of improvement on post-test 2. The remaining 13 students (Fig. 3, right pane) show slightly improved scores on the first and second post-tests, but the substantial improvement came on the third post-test.

In retrospect, it seems as if the second teaching session did not add much for any group, while the first session resulted in a substantial improvement for 5 students (the sixth student was absent the first session but achieved 14 points on post-test 2). The third session contributed to making 11 out of 13 students achieve satisfactorily, with a median improvement of 8 points. Overall, 31 out of

33 students achieved 12 points or more on at least one of the post-tests. The major improvements occurred on the first and third post-tests (Fig. 4).

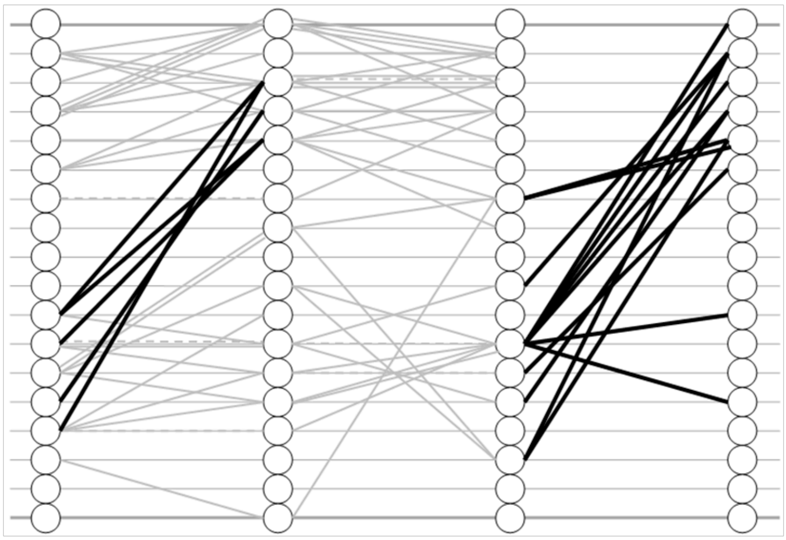

Figure 4. Highlighting improved results for two groups of students.

Discussion

Our principle-based approach, particularly the flexible adaptation of theoretical principles to emerging needs in the design process, has contributed to providing new and unexpected insights into a well-known problem, namely how we can teach students to select and use efficient strategies for subtraction. If we instead would have committed to work with a pre-defined theoretical framework, we would probably not have been able to obtain the reported findings. We have illustrated that involving principles of self-regulation, implemented through teacher-led structured activities in small groups, lead to substantial improvements of test scores for low achieving students. The introduction of this particular theory depended both on the particular context, suggestions from the teachers, and the researcher's "improvised" judgment of an appropriate treatment for the low achievers. The theory and its possible implementation were negotiated with the teachers, before they engaged in the detailed planning process. Although theories of self-regulated learning have previously been recognized as being relevant for mathematical problem solving, we could not foresee the substantial positive effects of a treatment based on scaffolding self-regulation.

However, we readily acknowledge that involving self-regulation principles was only one part of the treatment in the third cycle. In order to address the low-achieving students' cognitive and contextual processes, particularly during the phase fore-thought, planning and activation, we decided to change the pedagogical arrangements and implement teacher-led structured activities in small groups. Although it may be argued that any teacher can do a better job under such favourable conditions, the teacher still has to arrange "good" activities for the students. In the current study, our strategy has been to characterize such a good activity, for a particular group of students and their teachers, in terms of principles that guide the teachers' planning and implementation of the activity.

While classroom-based design research is often interpreted as describing recommended practices, our principle-based approach avoids this replica trap by completely avoiding descriptions of the classroom activities. Instead, we invite teachers to plan and implement teaching activities based on confirmed theoretical principles. This may be a fundamental issue for research dissemination in the learning sciences. Encouraging teachers to identify and carbon-copy so called "best practices" draws focus away from designing even better practices and may impede further improvement. Furthermore, copying practices without being informed about underlying principles may cause instability and possibly complete loss of focus on part of the teacher if the implemented activities do not proceed as intended. Rather than attempting to encapsulate current teaching practices as static recommendations for the future, we suggest a dynamic process of professional improvement based on flexible adaptation of confirmed theoretical principles. The limited involvement of researchers in a principle-based design process allows schools to involve researchers at a reasonable cost and could also stimulate a substantial number of similar studies. With maturation, such an approach could result in the encapsulation not of best practices but of best principles, not as a general set of principles for all learning objects (c.f. Kirschner, Sweller, and Clark, 2006) but a few core principles for each (type of) learning object (Dede, 2006).

Identifying theoretical principles that meet the demands in a complex design process is not an easy task. The selection of theoretical principles necessarily depends on the researcher's theoretical preferences, understanding of relevant principles, and available resources such as literature and colleagues. Despite the inherent subjectivity in the principle-based approach, we believe it is important that researchers sometimes go beyond neatly organized research programmes and engage in rather unstructured exploration of the authentic problems that teachers face in their classrooms. In our case, we are satisfied in having designed a treatment for learning subtraction strategies that proved to be successful for 33 students and their three teachers at a school in Sweden.

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