Design research on local instruction theories in mathematics education

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Over the last decades, the view, that teachers have to transmit knowledge, has been replaced with the view that students have to construct knowledge while being supported by teachers and textbooks. It is, however, not immediately clear how to guide and support students in such processes in the case of mathematics education. In response to this problem, design research emerged as a method for developing theories that can function as frameworks of reference for teachers.

Mark that the notion that people construct their own knowledge does not offer a pedagogy. For it implies that students will construct their own knowledge whatever form instruction takes. It does, however, point to the question of *what* it is the students construct. Or, what we want them to construct. This brings us to the question: What do we want mathematics to be for our students? Following Freudenthal (1971) we argue that students should experience mathematics "as a human activity", as the activity of doing mathematics. According to Freudenthal students should be supported in reinventing mathematics, which fits nicely with the constructivist mantra of students constructing their own knowledge. But how to help students invent or construct what you want them to invent/construct?

In answer to this problem, Simon (1995) coined the term, "hypothetical learning trajectory" (HLT), which refers to choosing tasks with an eye on what they might bring about, envision the mental activities of the students, and anticipate how their thinking might help them to develop the mathematical insights you are aiming for. Being hypothetical, the learning trajectory of course has to be put to the test. When the HLT is enacted, one has to observe students, analyze and reflect upon their thinking, and adjust the HLT. Following this line of thought, we have to support teachers by helping them to design HLT's, not by offering them scripted textbooks. For, if we want students to reinvent mathematics by doing mathematics, teachers have to adapt to how their students reason and help them build on their own thinking. To do so they need a framework of reference to base their HLT's on. We may offer them such frameworks in the form of "local instruction theories"—and corresponding resources. A local instruction theory consists of theories about both the process of learning a specific topic and the means to support that learning. The goal of the kind of design research I am discussing here is to develop local instruction theories.

Design research typically encompasses of the following three phases.

1. *Preparing for the teaching experiment*; in this phase, the researchers clarify the theoretical intent, the background theories, the starting points of the students, and the instructional goals; and design a conjectured local instruction theory. Here I want to stress the importance of a sound instructional design theory, as the quality of the research highly depends on the design. The theory of realistic mathematics education that grew out of Freudenthal's adagio of mathematics as a human activity qualifies as such a theory.
2. *Conducting the teaching experiment;* during the teaching experiment the researchers design and adjust instructional activities on the basis of the evolving local instruction theory. In relation to this we speak of *micro design cycles*, which are very similar to Simon's (1995) HLT: (1) anticipate in advance what the mental activities of the students will be when they will participate in some envisioned instructional activities, (2) try to find out to what extend the actual thinking processes of the students correspond with the hypothesized ones (3) reconsider potential or revised follow-up activities. During the teaching experiment the researchers have to assemble data that allow for the systematic analysis of the learning processes of the students and the means by which that learning was generated and supported.
3. *Retrospective analysis;* since the instructional sequence and the local instruction theory are revised and adapted during the process, a reconstruction of both the instructional sequence and the local instruction theory that are the product of the teaching experiment is needed. Further the teaching experiment may be framed as a paradigm case of more encompassing phenomena, such as: the proactive role of the teacher, the classroom culture, the role of symbols & tools. Here we may use Glaser and Strauss's (1967) the method of constant comparison. By first establishing what happened in a three step procedure; identifying patterns emerging from the data, describing them as conjectures, and looking for confirmations and refutations—in whole dataset. Secondly, by establishing, why this happened; in a similar procedure aiming at finding explanations/causal mechanisms. By first describing them as conjectures, then looking for confirmations and refutations.

Mark that the data analysis needs an interpretative framework to translate observed phenomena in empirical data. In relation to his we may refer to Yackel & Cobb's (1996) emergent perspective. From a methodological perspective, we may further point to the methodological norm of *trackability*, which we take from ethnography: Outsiders should be able to retrace the learning process of the researcher(s). Here we follow Smaling (1992) who points out that the classical methodological norm of *reliability* actually refers to replicability—which in qualitative research translates into virtual replicability. This fits with the goal of offering teacher an empirically grounded theory, which they may adapt it to their own situation by designing HLT's is tailored to their students, and their goals.

References

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