Allgemeinbildung and digital technologies in mathematics education: a review

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The conception of Allgemeinbildung in Danish schools might be influenced by the inclusion of digital technologies in mathematics education. This paper presents the results of a literature review aimed at answering the potential and challenges posed by the inclusion of digital technologies in mathematics education in relation to students' Allgemeinbildung. The analysis is structured after Heinrich Winter's basic experiences regarding mathematical Allgemeinbildung. The review reveals both challenges and potentials. In particular, the handling of data, the role of outsourcing, and the lack of empirical research are discussed regarding the aim of Allgemeinbildung in the Danish school.

In Denmark, the school system is influenced by the *didaktik* tradition, in which aims are formulated with a focus on the philosophical perspectives of *Bildung* and the long-term human development in mind (Broström, 2022). The historical development of the Danish school is deeply influenced by the German Bildung tradition. Allgemeinbildung concerns growing up and managing oneself in a society under varying circumstances (Niss, 2021). The prefix *Allgemein* (common) states that the purpose of school is not (only) to prepare students for specific occupations but to prepare for life in a broader sense (e.g. Biehler, 2019). The double aim of both Allgemeinbildung and further education has been formulated since the first school law in 1814; however, the formulation has developed with society. Today, the formulation, besides its intention of further education, is formulated in terms of Allgemeinbildung and targets furnishing a democratic society with enlightened and empowered citizens (Danish Ministry of Education, 2019).

Mathematics teaching, as well as the other school subjects, must contribute to these aims (Danish Ministry of Education, 2019), and both Scandinavian and German literature discuss how mathematics

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education should contribute to this aim. In the Danish literature, modelling and the application of mathematics have been discussed as contributions to Allgemeinbildung (e.g. Blomhøj, 2001; Michelsen, 2016). This is also emphasised in critical mathematics education (e.g. Skovsmose & Nielsen, 1996). In the German literature, Allgemeinbildung has been discussed in detail since the 19th century, and the concept has developed over time, although it is often vaguely or ill-defined (Biehler, 2019; Horlacher, 2016).

In Danish schools, there has been a heavy introduction of digital technologies in teaching in general (Bundsgaard et al., 2019) and in mathematics teaching in particular, in both upper secondary school and primary school (e.g. Jankvist et al., 2019). In 2018–2021, a large-scale experiment was conducted on *Technology comprehension* as a school subject, both by itself or integrated into existing school subjects. However, how it will be implemented remains undecided. In terms of *digital emancipation*, this subject is also aimed at students' Bildung in the Danish context. One of the conclusions in the evaluation report is, nevertheless, the need to clarify the synergistic effects between existing subjects and Technology comprehension (Danish Ministry of Education, 2021).

Therefore, there is a need to explore the relationship between digital technologies in mathematics teaching and the aim of Allgemeinbildung. This paper aims to contribute to this exploration by presenting the results of a literature review of Allgemeinbildung concerning the inclusion of digital technologies in mathematics education. The goal is to answer the following research question: *According to the research literature, which potentials and challenges does the inclusion of digital technologies in mathematics education pose in relation to students' Allgemeinbildung?*

In the following sections, the term Allgemeinbildung will be characterised, and the connection to mathematics will be explained. Subsequently, the methods of the literature study will be elaborated. The analysis is structured into three parts based on Heinrich Winter's (1995) so-called basic experiences, which are elaborated below. Finally, a discussion and conclusion of the results will be presented.

Allgemeinbildung – the self, the world, and the transformation

According to the Norwegian educational researcher Lars Løvlie (2011), Bildung develops between two intersecting powers: between subject and object, between the self and the other, between the individual and society. Similarly, according to Danish educational researcher Alexander von Oettingen, Bildung is established on the grounds of experiences connecting the self and the world (von Oettingen, 2016). Both Løvlie and von Oettingen referred to Wilhelm von Humboldt and the liberal German

Bildung-culture. The development of competencies and practices regarding Bildung is a prerequisite; nevertheless, it is insufficient. The essence is the formative power of the subject matter, and this should be conveyed in such a way that the students can engage freely with the subject matter. The process concerns the formation of human beings in society and aims for subjects to see meaning in the way this formation takes place as a self-reflecting process. It connotes a "holistic self-enculturation" (Biehler, 2019; Neubrand, 2015; Vohns, 2018, 2021).

The substance of Allgemeinbildung focuses on the fundamentals of understanding nature, culture, and society (Niss, 2019, p. 164 in Biehler, 2019). School subjects play an important role in this process. Each represents a specific approach to culture, knowledge, language, history, society, etc. (von Oettingen, 2016). Referring to Baumert (2002), school subjects can, according to Vollmer (2021), be seen as "historically grown organisational units" and serve as "[...] social and intellectual organisers of reality, of providing access to the world, of encountering and experiencing it" (p. 143). The question is what kind of approach mathematics as a school subject offers today.

Mathematics education and Allgemeinbildung

To investigate the kind of Allgemeinbildung offered through mathematics education, the Danish mathematics educational researcher Morten Blomhøj (2001) presented a profound work on this connection. He presented principles for mathematics education to contribute to students' Allgemeinbildung. Among these are specific competencies within the subjects of numbers, formalism, reasoning, problem-solving, and competence in creating and critically evaluating mathematical models. Mathematics teaching must create opportunities for students to connect their experiences to their potential application of mathematics. Fundamental and principle aspects of mathematics should be discussed. Based on their experiences, students should feel the need for the concepts and methods introduced to them, and the teaching should support the individual student's personal development (Blomhøj, 2001; Niss, 2021). These principles seem to address mathematics as a historically cultural world of its own, the application of mathematics in society, and the personal development of the individual student.

Another suggestion for an explication of the contribution of mathematics education to students' Allgemeinbildung is Heinrich Winter's basic experiences. Winter is a prominent voice in the German literature on mathematics and Allgemeinbildung (Biehler, 2019; Vohns, 2018). Winter (1995) specified that students in school should get access to create "[...] competencies and knowledge that are essential to every human being as an individual and as a member of society independent of his/her gender, religion, (future) profession, etc." (Winter, 1995, translated in Biehler, 2019, p. 153). He determined three basic experiences that mathe-matics teaching should allow students to create:

1 to perceive and understand the phenomena of the world around us in nature, society and culture in a specific way;

2 to get to know and to apprehend mathematical objects and facts represented using language, symbols, images or formulae as intellectual creations and as a deductively organised world of its own; and

3 to acquire by working on tasks capabilities of problem-solving which go beyond mathematics (heuristic competencies).

(Winter, 1995, p. 37, translated in Biehler, 2019, p. 153)

According to Neubrand (2015, p. 68), the first experience refers not to vocational education but to general (*allgemeine*) life. Neubrand quoted Winter (1995): "Applications of mathematics are not only interesting but also really indispensable for Allgemeinbildung when examples from real-life show how mathematical modelling works and what kind of enlightenment it can bring about" (own translation). Given Winter's work specifying the contributions from mathematics education to students' Allgemeinbildung, his basic experiences serve as an analytical lens in this paper.

The concern of Bildung is tightly connected to the German and Scandinavian didaktik tradition; however, the concepts of mathematical literacy from the English-speaking mathematics education community have some commonalities. It first occurred in the United States in the mid-1940s (Niss & Jablonka, 2020). In the latest *Programme for international student assessment* (PISA) framework, mathematical literacy is defined as an:

> individual's capacity to reason mathematically and to formulate, employ, and interpret mathematics to solve problems in a variety of real-world contexts. It includes concepts, procedures, facts, and tools to describe, explain, and predict phenomena. It helps individuals know the role that mathematics plays in the world and make the well-founded judgments and decisions needed by constructive, engaged and reflective 21st Century citizens. (OECD, 2019, p. 7)

According to Biehler (2019), mathematical literacy is a subset of Allgemeinbildung. Diverging from mathematical literacy, Allgemeinbildung addresses the development of individuals' personalities. A concern also embedded in the principles of Blomhøj (2001). Nevertheless, mathematical literacy and Allgemeinbildung converge on the theme of active citizenship and on preparing students as future members of society (Vohns, 2017).

To investigate the potential and challenges of digital technologies in mathematics education for students' Allgemeinbildung, it seems appropriate to include both constructs in the study, as the two notions intersect (Biehler, 2019; Vohns, 2017). Both mathematical literacy and, in particular, Bildung have been on the agenda for decades (centuries for the latter). However, as the world and societies continually change, the discussion of what Bildung means must be revisited regularly. Concerning mathematical literacy, digital technologies are a subset of the tool concept (OECD, 2019). The role of tools was not a part of the definition from the beginning but was included in the 2012 round of the PISA assessment (Geiger et al., 2015), and is related to the substantial emphasis the PISA framework puts on mathematical modelling. Digital tools should be used to solve context problems and "describe", "explain," and "predict" phenomena (OECD, 2019, p. 75). In relation to Bildung, technology has, according to Løvlie (2003), transformed culture in ways that affect what Bildung is. Løvlie (2003) introduced the notion of *technocultural Bildung*. If Bildung concerns the self, the subject matter, and the transformation of the subject, technology changes how this transformation can be understood. The triad established by the classical ideal of Bildung – the self, the world, and the transformation – changes concerning the boundaries between humanity and technology (Løvlie, 2003). To revisit the concern of Allgemeinbildung in mathematics education in primary and secondary education in the new reality where digital tools play an increasing role, this paper presents a study of the literature.

Methodology

The review process

To address the research question, a hermeneutic literature review was conducted (Boell & Cecez-Kecmanovic, 2014). The review process is circular, consisting of searching, sorting, selecting, acquiring, reading, identifying, refining, and then back to searching. This process is appropriate, since the notion of Allgemeinbildung in mathematics education is broadly characterised and has varying definitions. Four databases relevant to mathematics education research were selected: Web of Science, Eric from Proquest, Scopus, and SpringerLink. The latter was also pertinent to securing German-language literature, which is relevant because of the tradition of Allgemeinbildung in German mathematics education research. Two different search strings were used: one concerning

Allgemeinbildung and one concerning mathematical literacy. Beneath is an overview of the number of hits in the different databases. Even though the number of hits in Web of Science and Scopus were both eight, the results were not identical.

Table 1. *Overview number of search results*

Search string	Web of science	Scopus	Eric PQ	Springer link	No. of results
digital technolog* AND allgemeinbildung AND mathematic*	0	0	0	15	15
digital technolog* AND "mathematical literacy"	8	8	3	79	98
Total					113

A total of 113 results were transferred to abstract screening. As criteria for inclusion, both digital technologies and notions of either Algemeinbildung or mathematical literacy must be mentioned in the abstracts. In the hermeneutic approach to reviewing the literature, the process of reading both abstracts and body text gave rise to revising the inclusion/exclusion criteria. In the reading of abstracts, the terms quantitative literacy and numeracy occurred. As these were used as synonyms or related terms to mathematical literacy, it was decided that they should be included because they could help answer the research question. In reading the main text, it was noted that sometimes mathematical literacy is just used as a synonym for mathematical knowledge (see also Jablonka, 2003). In these cases, the study would not help answer the research question, and were therefore excluded. If digital technologies were in the abstract together with *citizens* or *society*, a full-text screening was conducted. I determined whether the study addressed aspects of Allgemeinbildung or mathematical literacy without mentioning them in the abstract. They were included if mathematical literacy or Allgemeinbildung were explicitly addressed in the body text. Due to the low number of hits, at least in three of the databases, it was decided not to limit the timespan. These processes resulted in 12 studies. The references were used for snowballing, which resulted in four additional results. Overall, 16 papers were reviewed in this study.

Analytical approach – three basic experiences

To manage the broader research question of identifying challenges and potentials for students Allgemeinbildung when including digital technologies in mathematics education, Winter's (1995) specification of three kinds of basic experiences was used as an analytical frame. The research question is hence divided into three sub-questions asking for the potential and challenges of digital technologies to support the establishment of these basic experiences (BE):

BE 1: What potentials and challenges do digital technologies pose for students in *perceiving and understanding the phenomena of the world around us in nature, society and culture in a specific way*?

BE 2: What potentials and challenges do digital technologies pose for students *in getting to know and apprehend mathematical objects and facts represented using language, symbols, images, or formulae as intellectual creations and as a deductively organised world of its own*?

BE 3: What potential and challenges do digital technologies pose for students *in acquiring capabilities of problem-solving that go beyond mathematics*?

The potential and challenges of digital technologies

In table 2, the 12 studies are sorted based on each of the three basic experiences. Most of the studies contributed to answering more than one of the three sub-questions. The studies are also sorted according to whether they are theoretical, empirical, or review studies. This point is addressed in the discussion. Geiger, Goos and Forgasz' (2015) work is a literature

Table 2. *The studies sorted after Winter's (1995) three basic experiences*

	Theoretical	Empirical	Literature review
BE 1	Geiger et al. (2020) Gravemeijer et al. (2017) Keitel et al. (1993) Noss (1998) Stacey & Turner (2015) Steen (1999) Zevenbergen (2004)	Geiger, Goos & Dole (2015) Hoyles et al. (2010)	Geiger, Goos & Forgasz (2015)
BE 2	Geiger et al. (2020) Gravemeijer et al. (2017) Hischer (2018) Nocar et al. (2019) Stacey & Turner (2015) Steen (1999) Vohns (2021) Zevenbergen (2004)	Hoyles et al. (2010)	
BE 3	Geiger et al. (2020) Gravemeijer et al. (2017) Keitel et al. (1993) Peschek & Schneider (2002) Straehler-Pohl (2017) Zevenbergen (2004)	Hoyles et al. (2010)	

review although its focus is different from that of the present study; however, it has a section on numeracy and digital technologies that are useful for this study.

There is a twofold tension in the reviewed literature concerning the potential and challenges of digital technologies for mathematics education. First, technological development changes societies in ways that call for new demands for mathematics education. Second, technology offers tools for mathematics education, enhancing new opportunities to learn appropriate mathematical ways of thinking. In what follows, the three research questions will be explored under their respective headlines.

To perceive and understand the world around us

The analysis of the selected studies reveals a duality regarding the potential and challenges for students to perceive and understand the world around them. First, it is emphasised that digital technologies have changed society in a way that establishes new demands for mathematics teaching. Second, digital technologies offer new ways to comprehend phenomena by offering tools for teaching mathematics, which supports students in the exploration of this world.

In several of the studies, the challenge for mathematics teaching is focused on the increasing amount of data produced in society. Due to digitalisation, it is possible to create, store, and access large amounts of data. Learning to handle this has become an important challenge for mathematics teaching. Stacey and Turner (2015) state that the ability to handle these data becomes a part of mathematical literacy. Geiger, Goos and Forgasz (2015) stressed that mathematical teaching should address how statistical information can draw attention to important issues and how it can be used to influence public opinion. Steen (1999, p. 8) argued for the need to develop numeracy, as "data, graphs, and statistics both enrich and confuse our lives". In addition, Zevenbergen (2004) underlined that the technologising of society and workplaces influences how young people work mathematically. Geiger et al. (2020) and Gravemeijer et al. (2017) explicated these needs using the term *statistical literacy*, a topic-specific subconstruct to mathematical literacy. The notion has varying definitions; however, according to Geiger et al. (2020), it comprises fundamental statistical ideas along with representational skills, context knowledge, and disposition to adopt a critical stance.

Although these studies point to the fact that digitalisation creates new challenges for mathematics teaching, digitalisation also has the potential to handle and interpret the data in the world around us. Geiger, Goos and Forgasz (2015), found that digital technologies have the potential for students' development of numeracy capabilities, such as "collection, recording, and analysis of real-world data; comparing the features of relevant data sets; critiquing a situation or making judgements" (p. 538).

Geiger, Goos and Dole (2015) provided evidence of the positive influence of digital tools on students' development of skills, mathematical knowledge, dispositions, and orientation towards using mathematics critically. Geiger, Goos and Dole (2015) empirically investigated how digital tools can support numeracy teaching and learning and presented a model that integrates the context with mathematical knowledge, dispositions, and tools. These relations are embedded in a critical orientation "[...] as the fundamental purpose of numeracy in practice is that it empowers individuals with the capacities to evaluate and to make judgements and decisions about their options and opportunities in the lived in world" (Geiger, Goos & Dole, 2015, p. 1123). In their design-based research study, students investigated the overall question of what level of physical activity is required to maintain good health. They used different digital tools to track, gather, and represent data relevant to their question. In addition, the digital tools gave rise to reflections on what measures were necessary and a critical examination of their situation.

Although the selected studies reveal the potential for mathematics teaching to include digital technologies to reach a goal of mathematical (or statistical) literacy, some studies also introduce a paradox about mathematics teaching. School mathematics is not preparing students for the challenges of their future work and everyday lives. According to Gravemeijer et al. (2017) and Keitel et al. (1993), manual calculations are the main focus of school mathematics; however, they are calculations carried out by computers in real life. Similarly, Geiger, Goos and Forgasz (2015) found a large amount of literature problematising the bridge between school mathematics and the mathematics used in workplaces. Mathematics is context-bound and deeply interwoven in modern workplace practice, and the mathematics taught in schools is poorly transferable to these situations. They also found that there is limited research on students' development of numeracy capabilities concerning digital tools in school (Geiger, Goos & Forgasz, 2015). Noss' (1998) warning about reducing mathematics to its utility is noteworthy.

For mathematics teaching to support students in perceiving and understanding the world around us, the selected studies suggest an emphasis on the development of statistical literacy, where digital tools can support the investigation of data. However, the studies also point to a lack of coherence between mathematics teaching and what is needed in real life. In addition, they point to a lack of research in this regard.

To know and to comprehend mathematical objects and facts

The findings in the literature under the second basic experience also comprise a duality. On the one hand, the studies reveal that the digitalisation of society and mathematics education has changed the mathematical objects and facts that are needed to know and comprehend. Indeed, this establishes a challenge for mathematics teaching in adapting to these changes. On the other hand, digital technologies provide new possibilities for operating on mathematical objects. On a general level, Stacey and Turner (2015) explained how technology supports a change in how objects are represented and offers new possibilities for students to operate on them. More specific suggestions regarding which mathematical objects become increasingly important to address in the mathematics classroom are in line with the strong emphasis on data and statistics, as explained above.

Gravemeijer et al. (2017) pointed to two subjects gaining importance. One is space geometry due to the possibility of 3D printing. The other one relates to statistics, and they emphasise ”[...] big ideas such as variability, sampling, error, prediction, and the distinction between signal and noise. [...] Related aspects are data collection and data displays (graphs, frequency tables, and pie charts)” (Gravemeijer et al., 2017, p. 117). Geiger et al. (2020) found that variation and expectation, distribution, informal inference, and sampling constitute essential mathematical and statistical knowledge. Similarly, Zevenbergen (2004) emphasised the competence to make sense of an expanding amount of information represented in graphs, tables, and statistics. Hoyles et al. (2010, p. 171) also highlighted concepts related to understanding the mathematical models underlying different financial products, such as ”understanding percentage interest rates and their effects over time”, ”understanding growth [...] and the present value of money ”, and ”interpreting graphs and charts; making estimates and predictions of costs of loans”.

Gravemeijer et al. (2017) interpreted working with computers in itself as a translation from a phenomenon in reality to numeric values. Therefore, mathematics teaching should focus on the mathematical concepts that enable students to understand these processes and capture which information is lost in translation. This requires an understanding of measures and measurement, including uncertainty, repeated measurement, mean, measurement error, data creation, and sampling, as well as understanding variables, co-variation, and functions. Besides their connection to handling data, these concepts also enhance quantity and number, dimension, and shape, and pattern functions and relationships.

A few studies are related to digital literacy’s potential challenges for mathematics education. Digital literacy (or digital Bildung) as an objective in mathematics education is discussed. This discussion relates to recent curricular reforms and the public debate about implementing digital literacy. Nocar et al. (2019) recognised the great potential for mathematical and digital literacy to complement each other. Their findings are in the context of curricular reforms in the Czech Republic. In the German debate, Hischer (2018) and Vohns (2021) criticise the terminology of *Digitale Bildung* (equivalent to the notion of digital literacy). Their criticism concerns the meaning of Bildung, which is tied to the subject and human consciousness (Hischer, 2018). Bildung is more than learning and requires reflective processes and an awareness of self-development (Vohns, 2021). Hischer (2018) established a model that combines methodology, proficiency, and reflection related to implementing media in mathematics education. Vohns (2021) pointed to the constraints around digital Bildung through seven theses, some of which relate to diverging understandings in policy, mathematics education, and mathematics communities. Social implications and the development of "critical digital literacy" are, according to Vohns (2021), underexposed.

To acquire capabilities of problem-solving

The third basic experience is slightly different from the first two, as it addresses a more heuristic competence of problem-solving. Technologies change how problem-solving can be applied in mathematics teaching. It frees mental space, as tidy calculations can be carried out by the computer, so students can instead focus on more heuristic competencies. The literature offers some suggestions for re-theorising mathematics teaching to embrace these potentials. However, there is also disagreement as to whether the outsourcing of mathematics to a digital tool is mostly an advantage or a disadvantage.

For re-theorising, Zevenbergen (2004) suggested that mathematics education should promote more global problem-solving aspects instead of calculations and accuracy. Gravemeijer et al. (2017) suggested three overall mathematical competencies for mathematics education to focus on: 1) applying and modelling, such as recognising where mathematics is applicable and translating practical problems into mathematics; 2) understanding, as conceptual mathematical understanding is needed to understand the underlying and hidden procedures of digital technologies; and 3) checking, which is necessary to evaluate the computer-based results, not by repeating the calculations by hand but by checking if the results seem plausible. Hoyles et al. (2010) focused on the usefulness of focusing on the interrelatedness of mathematical and technological skills. They derive the term "techno-mathematical literacy" from mathematical literacy to cope with the needs of modern working life and to address the specific needs for mathematics in the contexts in which mathematics is expressed through an artefact (Hoyles et al., 2010). Again, statistics and finance are enhanced, and the problem-solving aspects are outlined here. Hoyles et al. (2010) identified the core statistical ideas for the data-driven workplace, such as "interpret data, to transform into a trend or a problem-solving analysis [...] to use the information in a well-constructed argument". Moreover, regarding financial problem-solving, the need for a conceptual mathematical understanding of underlying models, the ability and confidence to make financial decisions, and developing a critical stance are stressed (Geiger et al., 2020; Hoyles et al., 2010). According to Geiger et al. (2020), digital technologies help facilitate students' exploration, development, design, interpretation, manipulation, and critique of graphical representations.

In the discussion of outsourcing, there is a kind of disagreement regarding whether outsourcing mathematical procedures to digital technology is an advantage or a disadvantage for mathematics teaching. Some scholars hold a positive view of outsourcing mathematics to digital technologies in the contribution to students' Allgemeinbildung (Peschek & Schneider, 2002). Others warn about outsourcing and hiding mathematical processes (Keitel et al., 1993; Noss, 1998). From the perspective of Peschek and Schneider (2002), Allgemeinbildung is focused on the layperson's capability to communicate with experts. It is not the primary task or possibility of the school to educate all people to be experts in a given subject; instead, it is essential to become capable of communicating with experts. They interpret the outsourcing of mathematical knowledge to the computer as equivalent to outsourcing to experts. It is not a demand for laypersons to know and comprehend the underlying mathematics, regardless of whether this knowledge is embedded in the computer or an expert. Instead, basic mathematical skills and reflective knowledge become essential for students' Allgemeinbildung.

Keitel et al. (1993) called for the enhanced use of digital technologies in mathematics teaching; however, they are somewhat critical of outsourcing. Computers as black boxes serve as abstractions in society. These abstractions must be questioned as to why both mathematical and technological knowledge, as well as reflective knowledge, are important, and both the effects and side effects of computer use must be considered. In line with Keitel et al. (1993), Straehler-Pohl (2017) stressed the need for the recovery of critical distance concerning demathematisation of the school agenda. For this purpose, he expanded the need to let students reflect on the opportunity to reject the use of mathematics. Further, the critique of capitalism should be brought into the agenda of mathematics classrooms (Straehler-Pohl, 2017). Keitel et al. (1993)

proactively suggested the following roles of the tool: it can serve as a means to check the use of algorithms and evaluate their appropriateness concerning the context, to play with different types of modelling and data, and to play with different types of modelling in various contexts. Students must also reflect on the reliability and acceptance of solutions. Finally, students must critically reflect upon the use of computers, both in the classroom and in society.

Noss (1998) also warned about outsourcing the role of the thinker to the computer. Technology augments the pitfall of rendering the underlying model invisible. However, technology also has the potential to enhance a new learning culture that supports the development of relevant mathematical knowledge. He suggests two elements: "[...] tools which bring the model to life (e.g. graphs, variables, and parameters) and the means to express its structure (e.g. numerical, algebraic, or geometrical tools)" (Noss, 1998, p. 5).

Discussion

The review identifies various novel challenges and opportunities in relation to the development of students' Allgemeinbildung. These include the ability of students to engage with and comprehend the world they around them, their comprehension of mathematical concepts and facts, as well as changes in the demands and opportunities for acquiring problem-solving skills. Notably, a pervasive theme throughout the review is the impact of an increasingly data-rich world. As a result, statistical literacy has become increasingly necessary for coping with the numerical information that is generated by digital technologies. Digital technologies, besides creating the demand for statistical literacy, also establish an opportunity to promote students' exploration of data (Geiger et al., 2020). This idea of focusing on fundamental statistical ideas relates to the warning of trivialising the application of mathematics. Focusing on how the applications of mathematics really work when using them to understand and perceive the world around us seems vital to contributing to Allgemeinbildung. This is in line with the modelling focus in the Danish literature on mathematics and Allgemeinbildung. Returning to the implementation of *Technological comprehension* in the Danish educational context and the potential synergy between mathematics education and digital empowerment, the development of statistical literacy and fundamental statistical ideas could be seen as an anchor point. Although the aim of digital empowerment is made explicit in the syllabus of Technology Comprehension, Vohns' (2021) warning that Bildung is more than learning should be carefully taken into consideration in the implementation of the new school subject.

A major hurdle in informing the implementation of Allgmeinbildung in mathematics teaching with digital technologies is the lack of empirical studies. Only one empirical study showed the potential for the inclusion of digital technologies in creating and handling data (Geiger, Goos & Dole, 2015). Several studies offer concrete suggestions on how the digitalisation of working life has changed and how it is reshaping what mathematics is needed (Gravemeijer et al., 2017; Hoyles et al., 2010; Noss, 1998; Zevenbergen, 2004). However, the need to understand how the goal is implemented in concrete teaching practice remains unfulfilled.

Returning to Løvlie's technocultural Bildung, the desire to let tools solve problems for us is a natural part of being human. In the results from the review, it is interesting that outsourcing mathematical procedures to digital tools simultaneously is problematic and has potential. Arithmetic skills become superfluous, as the tools can easily outmatch them. However, conceptual understanding, problem-solving, and decision making have become increasingly important (Geiger, Goos & Forgasz, 2015; Gravemeijer et al., 2017; Noss, 1998). Several studies have addressed the issue of mathematics becoming both invisible and pervasive because of digitalisation (Gravemeijer et al., 2017; Hoyles et al., 2010; Keitel et al., 1993; Noss, 1998; Straehler-Pohl, 2017; Zevenbergen, 2004). The stance of both Noss (1998) and Gravemeijer et al. (2017) points towards the importance of letting digital technologies play a role in mathematics education, which allows highlighting the understanding of the underlying procedures. The role of digital technology in mathematics education should be one in which it is used to check the use of algorithms and to investigate different types of modelling in various contexts.

Different meanings of critique are addressed in the literature. Geiger et al. (2020) indicated that contexts, mathematical tools, and dispositions are activated through critical orientation. Critical orientation is described as using mathematical information to make decisions and judgements, add support to arguments, and challenge others' arguments or positions. Gravemeijer et al. (2017) emphasised critical thinking and communication as essential to making sense of mathematics that is black-boxed by digital technologies. They also emphasised self-reliance and self-confidence when dealing with mathematics in everyday life and citizenship. Keitel et al. (1993) and Straehler-Pohl (2017) offered a more extensive conception of critique. The capacity to judge and understand what they call "frozen" mathematics in the "black boxes" created by digitalisation is crucial.

Volmer's (2021) view of school subjects as historically grown organisational units that serve as worlds of their own gives rise to considerations about a balance between working within mathematics itself and its application. Noss (1998) warned about focusing entirely on the utility of mathematics. In the same line, Winter's (1995, p. 37) second basic experience states that students should experience mathematics "as intellectual creations and as a deductively organised world of its own". Some research shows how this can connect to the idea of Bildung (e.g. Neubrand, 2000); however, the potential of including digital technologies in this process should be explored further.

Both mathematical literacy and Allgemeinbildung contain many facets that might be addressed in studies that are not included in this review. For instance, could reflectiveness or critique be on the agenda but not mathematical literacy or Allgemeinbildung explicitly. There might also be studies handling mathematical literacy or Allgemeinbildung in which digital technologies only have an implicit role. The choices regarding the review method were made to establish a balance between a limited amount of literature and the opportunity to identify patterns and trends in the results.

Most studies in this review are related to mathematical literacy, numeracy, or quantitative literacy. Only a few studies were explicitly related to Allgemeinbildung in mathematics education. The definition of mathematical literacy in the PISA framework aims at engaged and reflective citizenship. However, the components of Allgemeinbildung include the whole formation of human beings in a society, and the reflective process that makes subjects see the meaning in this formation is embraced in the notion of Allgemeinbildung. Recalling Blomhøj's (2001) principles of mathematics teaching, which are based on students' experiences and care for their personal development, seems vital in that respect. These aspects of Allgemeinbildung might be subject to further development when including digital technologies in mathematics education.

Because it was decided not to narrow down the time span in the inclusion criteria, some of the studies in the review were old, for instance Noss (1998) and Keitel et al. (1993). However, these studies contributed to some more philosophical aspects of digitalisation that were valuable to reflect on the research questions together with the more updated studies.

Conclusion

This paper aims to discuss the potential and challenges of including digital technologies in mathematics education concerning students' Allgemeinbildung. The review results clearly show that digital technologies pose several challenges and potentials for mathematics education. Digitalisation changes the world around us in a way that establishes new demands for mathematics education to contribute to Allgemeinbildung.

As digitalisation creates a large amount of data, learning to handle this data becomes a demand. However, digital technologies can serve as tools to explore these phenomena. The development of statistical literacy might serve as an anchor point in the synergy between mathematics education and the development of digital empowerment.

Digitalisation affects which mathematical objects and facts students must comprehend. Tidy calculations become redundant. In addition to concepts relating to data handling, variables, co-variation and functions, quantity and number, dimension and shape, and patterns, functions, and relationships are essential. Mathematics teaching should balance the aim of handling the application of mathematics in a digitalised world. However, it is also vital, concerning Allgemeinbildung, to acknowledge mathematics as a deductively organised world of its own. The potential of digital technologies in this regard should be the subject of further research.

The outsourcing of mathematics to digital technologies poses both a challenge and a potential for mathematics education. It frees students to explore general aspects of modelling and problem-solving. However, it increases the demand for the development of a critical stance. Further research is needed regarding Bildung as a process in which the individual sees meaning as a reflective process and the role of digital technologies in mathematics education.

Finally, there is a crucial need for more empirical research exploring the synergy and intersection between mathematics education, digital technologies, and Allgemeinbildung.

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