SUMMARY.

With a view to attempting to determine the proficiency in arithmetic in the elementary schools, in the spring of 1946 the author investigated 567 pairs of twins with a test battery comprizing 17 variables. The object of the investigation was to ascertain to what extent this proficiency was related to sex or was affected by heredity and environment, and what were the contributory and determining factors. The method of investigation employing twins as subjects was chosen, as it appeared to be the method best suited to elucidate, above all, the problem of heredity and environment.

The classification of the twins was carried out by means of a special questionnaire, which was answered by teachers partly with the aid of the parents. On the basis of these questionnaires and the teachers' reports the twins are classified as:

Identical twins, boys	86 pairs	
Identical twins, girls	79 »	165 pairs
Fraternal twins, boys	94 pairs	
Fraternal twins, girls	113 »	207 »
Unlike-sexed		195 »
	Total	567 pairs

According to Weinberg's formula, in the present case the number of identical twins should be: $567 - 2 \times 195 = 177$ as against the actual number 165, which represents a good agreement between the theoretical calculation and the number obtained empirically.

55 pairs out of the original 567 pairs of twins were excluded because the twins in these pairs had been taught in different grades. Another 132 pairs were eliminated, either because one of the twins failed to complete the test or because, although they belonged to the same grade, they were not in the same class division. This was

Factor IV. (p. 135), The factor appears to be identical with factor II for identical-twin boys. Variable 11 is missing but this is the only difference. The factor is interpreted as *school adaptation.* The author is unable to find any explanation for the presence of number analogies (8) in this connection.

Factor V. (p. 135). As in factor IV for identical-twin boys, paired associates (14) has here the highest loading. This is indeed the only variable that is significant, but variable 12, counting, tends towards significance: .295, and of the remaining variables it is only numerical memory (4) that has a fairly high value: .267. The factor seems to have a numerical memory character and is consequently classified as a *numerical memory factor.*

Factor VI. (p. 135). This factor has only one acceptable loading and no loading that approaches significance. The negative loadings of approximately — .250 for problem test (11), counting (12) and marks for arithmetic indicate that interpretation should be verbal rather than numerical, but in spite of this, the author prefers to leave the factor without attempting to interpret it.

Identical twins — Girls.

The rotated factor matrix is given in table 36, p. 136.

Factor I. (p. 138). The factor is of a purely numerical character. The presence of Simplex, while the C-test is missing is apparently due to the occurrence of a large number of purely numerical items in the first variable. Memory and report variables are on the border of zero loadings. The author will not interpret this factor, as he did in the case of factor V for identical-twin boys, as a numerical, perceptive factor, but rather as a *general numerical factor.*

Factor II. (p. 138). The author is fully aware that objections may be raised against the interpretation, but in view of the loadings of the two purely intelligence variables and the absence of perception and memory variables, he is prepared to venture on a hypothetical interpretation of the factor as a *general intelligence factor.*

Factor III. (p. 139). The dominance of the purely perceptive variables is evident here, and also the factor's relation to factor III for identical-twin boys and to factor II for fraternal-twin boys. That this factor has a larger scope than the two factors mentioned above, on account of the new variables that it contains, only appears to confirm its interpretation as a *general perceptive factor.*

Factor IV. (p. 139). In accordance with the reasons given for interpreting factors IV and V for identical-twin and fraternal-twin-boys respectively, this factor is interpreted as a *numerical memory factor*.

Factor V. (p. 140). The factor is of a purely numerical character. In view of the presence of number perception (6) and the relatively high loading in picture perception (5): .259, the factor is interpreted as a *numerical perception factor*.

Factor VI. (p. 140). The factor has a definitely numerical character but in view of, with the exception of paired associates (14), the low loadings of the component variables, the author prefers not to give any definite interpretation of the factor.

Fraternal twins — Girls.

The rotated factor matrix is given in table 39, p. 141.

Factor I. (p. 143) This factor does not comprize memory tests, numerical identification and the marks for Swedish, a fact which, to a certain extent, restricts the interpretation. The component variables are of a predominantly numerical type. The Simplex test is, to a high degree, of a numerical character, and even the C-test contains items of a purely numerical nature. The low loadings bordering on zero for picture and number perception (5 and 6) exclude a perceptive interpretation. The number series (7) are mainly of an inductive character and items of this kind are well represented in the intelligence variables. The factor is interpreted as *a numerical factor with a tendency towards induction.*

Factor II. (p. 143). The two memory variables (14 and 4) have the highest loadings. The occurrence of the other two variables makes a definite interpretation more difficult, but it does not seem unreasonable to interpret the factor as a *memory factor of a numerical character.*

Factor III. (p. 143). The factor is definitively perceptive in character and is interpreted as a *general perceptive factor.*

Factor IV. (p. 144). The variables comprizing the factor presuppose recognition and memory. The factor is dominantly numerical in character and is interpreted as a *numerical recognition factor.*

Factor V. (p. 144). The factor contains all the variables that are found in factor II for the identical-twin girls, which the author, though with some hesitation, interpreted as a *g*-factor. In the case of the fraternal-twin girls, variable II also occurs, however, and its presence helps to confirm its interpretation as a *g*-factor. Consequently, as in the case of factor II for the identical-twin girls, the factor is interpreted as a *general intelligence factor*.

Factor VI. (p. 145). The structure of the factor with the report variables at the top, shows that it is related to factors II and IV for identical and fraternal-twin boys, consequently its interpretation as school adaptation appears justified. The above factor is, however, more comprehensive in its structure, and the additional variables are purely numerical. The Simplex test also contains many numerical items. School adaptation appears here to refer mainly to arithmetic, and its report variable (15) has also the highest loading. This subject appears to prove difficult for girls, but their ambition assists them in making efforts to master it. On account of the special efforts that are made by the girls, this factor is interpreted as *an ambition factor.*

Table 44 below shows the interpretations given the various factors for the different populations.

TABLE 44. *The factor interpretation for the different populations.*

Factors	Boys		Girls	
	Identical	Fraternal	Identical	Fraternal
General intelligence	I	I	II	V
School adaption	II	IV		
General perception	III	II	III	III
Numerical memory	IV	V	IV	II
Numerical perception	V		V	
General recognition	VI			
Numerical recognition			X	IV
General numerical factor			I	I
Ambition factor				VI
Uninterpreted		III, IV	VI	

It will be seen that three factors are common for the four groups investigated: a general intelligence factor, a general perception factor and a numerical memory factor. For the two latter factors there is no pronounced sex difference regarding the factorial structure, see table 45.

TABLE 45. *The structure of the factors interpreted,* (+) = *loadings bordering on significance.*

Test	B		G		B		G		B		G		B	G
	EZ	ZZ	EZ	ZZ	EZ	ZZ	EZ	ZZ	EZ	ZZ	EZ	ZZ	EZ	EZ
	I	I	II	V	III	II	III	III	IV	V	IV	II	V	V
1. Simplex	+	+	+	+			+							
2. C-test	+	+	+	+								+		
3. Form relation ...					+		+							
4. Numer. memory .									+		+	+		
5. Picture perception					+	+	+	+						
6. Number perception						+	+	+	+		+		+	+
7. Number series ...		+	+	+	+							+		
8. Number analogies	+	+		(+)				+					+	
9. Verbal analogies	+	+					+							
10. Numerical classification	+	+	+	+			+						+	
11. Problems	+	+		+									+	+
12. Counting........	+	+		+			+		+	(+)			+	+
13. Numerical identification													+	
14. Paired associates									+	+	+	+		
15. Marks for arithmetic			+	+										
16. Marks for reading			+	+										
17. Marks for writing			+	+										
	g-factor				General perception				Numerical memory				Num. perc.	

For the general intelligence factor, however, such a difference does exist, and the distinctly different structure for boys and girls is seen particularly in the high loadings for the report variables for the girls, while the loadings for these variables for the boys are approximately zero. Consequently, there appears to exist a closer connection between intelligence and ambition, as the latter finds expression in the reports, in the case of the girls as compared with the boys. On examining table 44, it will be seen that the factor which is interpreted as *school adaptation* and which is characterized by high factor loadings in the mark variables is only found in the two boys' groups. From this fact it is possible to draw the conclusion that the school adaptation, which in this investigation is an independent factor in the case of the boys, is on the other hand so integrated with intelligence in the case of the girls, that it does not find any independent expression but becomes apparent in the structure of the general intelligence factor.

Another difference in the structure of the intelligence factor for the two sex groups is that form relation (3) is contained with a significant loading in this factor only in the boys' groups. This fact is in good accord with the result previously obtained by the author: that there is a significant difference between boys and girls as regards this very variable.

For both the groups of identical twins, factor V has been interpreted as a numerical perception factor, and this factor is not found in any of the groups of fraternal twins. This probably indicates the existence of a certain hereditary relation; it is the only occasion in the factor analysis where it has manifested itself. Table 45 shows the significant factor loadings, and as is apparent, the factorial structure for the boys is so much more extensive than that for the girls, that some degree of caution must be observed when interpreting the factors as identical. In both cases, however, their purely numerical character is fully evident. The intelligence, as well as the mark variables have low loadings — and this also applies to the purely memory variables. Verbal analogies (9) approach a zero loading in both cases. Form relation (3) is partly of a perceptive character and its zero loading for both groups is consequently remarkable. On the other hand, the loadings for picture perception (5) are not significant, but lie considerably higher: .252 and .259 respectively, a fact which should confirm its perceptive interpretation. The manifest differences in the factor loadings for number analogies (8), numerical classification (10), and numerical identification (13), may be assumed to be due to a sex difference, and hence factor V in the two identical-twin groups may be considered to be affected by heredity, while at the same time exhibiting a certain amount of sex variation.

done in order to obviate such evident differences in environment as being taught by different teachers must signify. Moreover the pairs of twins in the two highest grades, 7 and 8, were excluded so as to prevent the deviation in the age class being too great. Thus the number of the remaining pairs, whose achievements form the basis for the present investigation, were:

Identical twins, boys	66 pairs	
Identical twins, girls	62 »	128 pairs
Fraternal twins, boys	66 »	
Fraternal twins, girls	75 »	141 »
Unlike-sexed		111 »
	Total	380 pairs

The tests used in the investigation were: (see also p. 76 ff.).

1—2. The Simplex and C-test; two general intelligence tests.
3. Form relation; a Swedish version of NIIP's Form Relation test.
4. Numerical memory
5. Picture perception
6. Number perception
7. Number series
8. Number analogies
9. Verbal analogies
10. Numerical classification
11. Problems
12. Counting
13. Numerical identification
14. Paired associates
15. School marks for arithmetic
16. School marks for speech and reading
17. School marks for writing and grammar

Heredity-Environment variance.

The elaboration of the test results was carried out according to the following different methods: the calculation of

1. The percental deviations of marks in the pairs.
2. The mean absolute deviation of marks.

3. The variance quotient according to Dahlberg's formula, which is based on the degree of deviation. If the variability exhibited by the fraternal twins, and which is the result of heredity and environment, is denoted by σ_{am}; and the variability among identical twins, which is due to environmental factors, by σ_m, and finally if the variability due to heredity is denoted by σ_a, then the latter may be obtained by means of the equation $\sigma^2_{am} = \sigma^2_a + \sigma^2_m$.

   The relation between heredity and environment is then

$$\frac{a}{m} = \frac{\sigma_a}{\sigma_m} = \sqrt{\frac{\sigma^2_{am}}{\sigma^2_m} - 1};$$

4. Intra-class correlations. If both variables are denoted by X' and X'' and the mean for all the pairs of twins in a group by $\bar{X}$, then $\bar{X} = \frac{1}{2N}\sum_1^N (X' + X'')$

   The degree of deviation is then obtained by the formula $\sigma^2 = \frac{1}{2N}\left[\sum_1^N (X' - \bar{X})^2 + \sum_1^N (X'' - \bar{X})^2\right]$ and subsequently intra-class correlations are calculated from $r_i = 1 - \frac{\sigma_d{}^2}{2\sigma^2}$; where σ^2_d is the variance of the paired differences, and σ^2 is the variance for all the individuals.

5. The relation between the mean and deviation for the intra-paired differences for all the individuals within the respective populations of twins.

   Husén (1948, 1949) has determined the tendency of co-variance, by computing for each of the two groups of twins the mean, M_d, for the intra-paired differences. The differences between these means may be looked upon as due to genetic influences acting on the test results. However, since the absolute magnitude of these means is also dependent on errors of measurement, they must be related to the standard deviation, σ, for the whole group of twins, and consequently $M_{d/\sigma}$ is the comparable figure that may be employed. The lower this number is, the greater will be the tendency of co-variance within the group of twins.

1. The calculation of the *percental occurrence of deviations in the marks* shows that both as regards boys and girls, fraternal twins

are more frequently found to have different reports than identical twins.

2. Also as regards *the mean absolute deviation of marks,* fraternal twins exhibit greater deviations than identical twins. No particular difference was observed between the boys' groups and the girls' groups.

3. Even Dahlberg's method shows that there is a clear dependence on heredity as far as the three report variables are concerned. The quotient (Table 10. p. 90) is somewhat larger for boys than for girls for arithmetic (15), speech and reading (16), but the conditions are reversed for writing and grammar (17), a result which is in accord with the conception that girls have a greater innate ability for speech and boys a greater innate ability for arithmetic as far as the school marks may be considered as significant expressions of these special types of ability. The higher quotient obtained by the boys for speech and reading is probably due to the fact that girls are more easily influenced by tuition in these subjects, inter alia, on account of their greater interest for the purely dramatic, which is by no means an unimportant factor in reading well. The boys evince less interest for this, and their usual reluctance to »react» causes them in this subject to be less affected by environment.

As regards the other variables we find (Table 10. p. 90) that the greatest dependence on heredity is shown by the intelligence variables (1 and 2), number series (7) and problems (11). The variance quotient for both the intelligence tests is higher for girls (1.43 and 1.80) than for boys (.99 and 1.18) which signifies that the girls' intelligence is less affected by environment than that of the boys. On the other hand the quotient for number series and problems is higher for the boys than for the girls; this appears natural in view of the special »numerical» character of both these tests.

The variance quotient for the other variables is less affected by heredity. The quotients vary between .48 and .83, the latter figure refers to counting (12). This lower value for counting as compared with the value for solving arithmetical problems, bears out the well-known fact based on experience, that there is a greater possibility of teaching the pupils to become proficient in counting than in applied arithmetic.

4—5. The material has also been treated with regard to the tendency of co-variance, which is represented on the one hand by the intra-class correlations, r_i, and on the other hand by the relation between the mean and deviation of the intra-paired differences for all the individuals in the respective populations of twins, $M_{d/\sigma}$. The results are shown in tables 13, 15 and 16 (p. 93, 96, 97).

The intra-class correlations for the whole material, table 13 p. 93, are — with one exception: paired associates (14) — higher for identical than for fraternal twins. The degree of significance, C. R. = Critical Ratio, for the differences between the two correlations, which is calculated according to Fisher's method, is shown in table 11. p. 91. The differences between the intra-class correlations and hence also the stronger genetic influence on the test results, are significant, (the Critical Ratio is > 1.96), as is apparent in eight of the subtests.

The mean for the intra-paired differences is, table 13. p. 93, throughout lower for the identical pairs, and the same applies, with the exception of test 14 — to the quotients $M_{d/\sigma}$, which, similarly to the high intra-class correlations in this group of twins, indicate a greater genetic influence on the test achievements of this group.

By dividing the material into a boys' group and a girls' group we find, table 15 (boys), p. 96 and table 16 (girls), p. 97. that significant differences, table 14, p. 95. between the intra-class correlations exist, for both the sex groups, as regards the intelligence variables (1 and 2) and for both marks in Swedish (16 and 17). The girls' group does not show any other significant differences, while for the boys' group these differences are observed in connection with four other variables: form relation (3), number perception (6), number series (7), and problem solving (11), while numerical classification (10) has a C. R. value which borders on significance. The above facts agree well with the accepted view that boys have a greater innate aptitude for arithmetic and for the factors connected with it.

On the basis of the results obtained, it is difficult to draw any definitive and absolute conclusions with regard to the heredity-environment variance. It is quite clear, however, that it is the hereditary aptitude that forms the actual basis and affords certain possibilities, which may be utilized and developed to an extent that varies very considerably, but which also sets certain limits that may not be exceeded even under the most favourable external conditions. There are pupils in our school classes, who, in spite of the best tuition and of great diligence on their part, either make very little or no progress at all, for example, in arithmetic. They never acquire a proper conception of what it is all about; while other pupils find, from the very outset, that the line of thought in this subject is easy and self-evident. Although the teachers devote much effort, and may possibly even give their best work, to the former category of pupils, they consider that the latter have no chance of becoming as proficient in this subject as their more gifted comrades. If a pupil has little aptitude, there is a very restricted possibility, even with the aid of the most appropriate pedagogical methods, of intensifying the tuition.

The difference between boys and girls.

The author has investigated 181 pairs of unlike-sexed twins; of these, 120 pairs were taught together, and 61 pairs were taught separately. The mean of the difference between boy and girl is shown in table 19. p. 103. What is most evident here, is the great superiority of the boys in form relation (3), a result which entirely agrees with those of other investigators, namely, that the ability *to conceive and judge spatial relations and relations of magnitude* are much more developed in boys than in girls.

In the group that was taught together, the girls have higher means for paired associates (14) than the boys, and also in the marks for speech and reading (16); the latter result is in complete accordance with experience obtained regarding the greater facility, and interest that girls manifest for speech and reading where they can find scope for their more pronounced dramatic and emotional dispositions as compared with those of boys.

If instead of taking the mean of the differences between boy and girl in the unlike-sexed fraternal pairs of twins as our point of departure, we compare the means for the achievements and calculate the values for σ^2 and C. R. then to a great extent the same results will be obtained as in the former case. Even on this basis of calculation there exist, table 20. p. 106, significant differences (C. R. = 4.20) in the boys' favour for form relation (3), and in the girls' favour (C. R. = 2.04) for the marks for speech and reading (16). On this basis of calculation the girls also have the best achievements for paired associates (14), but the difference here is not significant. The almost general tendency — although this is only significant in a single test — for the girls to have higher means for their achievements than the boys in the unlike-sexed pairs of twins, is found to be still more pronounced if we study the group of fraternal twins of the same sex. According to table 21, p. 106, it will be seen that in the latter case there exist significant differences in the girls' favour for five of the variables: picture perception (5), number perception (6), counting (12) and the two marks for Swedish (16 and 17).

Finally, if we make a comparison between sex differences for the groups of identical twins, table 22, p. 107, we find that in not less than nine variables there is a significant difference in the girls' favour: the two intelligence variables (1 and 2), number perception (6), verbal analogies (9), counting (12), paired associates (14), and the three mark variables (15, 16 and 17). From the results obtained, it seems reasonable to draw certain conclusions with regard to the hereditary factors connected with sex, but this, however, is not warranted, because, in accordance with the statistical method employed here — in contrast with what was done in a preceding chapter — no attention has been paid to differences *within* or *between the pairs* but only to the purely individual achievements.

The fact that the girls' mean achievements are throughout better than those of the boys, may be assumed to be due, on the one hand, to the more pronounced ambition of the former, and on the other, to their general lead in mental development, in comparison with boys, for the period of development that is covered by the investigation. That the better achievements of the girls are not similarly accentuated within the different groups of twins may be presumed to be due to, inter alia, the difference in environment, which, even from an educational point of view, undoubtedly exists for individuals comprizing the various groups of twins. The more dissimilar environment, which no doubt exists for pairs of twins of the same sex, in comparison with that of identical twins, and which is still more the case for unlike-sexed, fraternal twins, when compared with the remaining groups of twins, appears, in accordance with the results obtained here, to exert an equalizing effect on this sex difference.

For the age, and type of schooling with which the author's investigation is concerned, it appears, if only the results obtained for unlike-sexed, fraternal twins are taken into account, that to a great extent, no differences exist — with the exception of form relation — between the achievements of boys and girls in the abilities that the tests employed are intended to measure. If, however, the results for the unlike-sexed groups of fraternal and identical twins are taken into account, then the means for the individual achievements will practically throughout show a better result for the girls than for the boys. This seems to contradict the usual idea that boys are better at arithmetic. Other factors besides purely ability-factors may, however, play a part here. The greater diligence and ambition of the girls, their greater interest in learning, and their higher development curve at this age, may be assumed to equalize the effects of differences in ability. The better mean achievements of the girls appear to be related to their environment in such a way that their superiority increases when the environment is similar.

Factor analysis.

For the factor analysis, in the preceding statistical treatment of the material, the subjects have been divided into four groups: identical twins and fraternal twins, boys and girls. This was done in order to try to ascertain whether hereditary factors and sex differences could be shown to exist in the factor pattern. The analysis, which was carried out by Thurstone's successive approximation method, was based on the correlation tables on pages 124—125.

Identical twins — Boys.

The rotated factor matrix is shown in table 30, p. 126.

Factor I (see also p. 128) is interpreted as *the general intelligence factor*. The highest loadings are here found to be the intelligence tests. The other variables represented in the factor are built up of the type of items, which, as a rule, form a part of group intelligence tests. It is worthy of note that number series, which is generally assumed to be intelligence-saturated, only had a loading of .262. The author is unable to account for this relatively low loading. Memory, in this case rote memory, is not included in the factor, nor are the marks for arithmetic and Swedish, which thus do not appear to be *g*-loaded here.

Factor II (p. 128). The factor is interpreted as a *school adaptation factor with reference to arithmetic and Swedish.*

Factor III. p. 130) The factor appears to be of a perceptive character, but in view of the dominance of variables 5 and 3 and since it only contains one numerical test, the author wishes to interpret it as *a perceptive factor of a more general character.*

Factor IV. (P. 130). The highest loadings are here in the two memory tests. Number perception (6), is certainly not a memory test, but its treatment presupposes nevertheless that the subject remembers the group of number in the first column when comparing with those in the second column. Furthermore it is incontestable that proficiency in counting requires a considerable degree of numerical memory. Consequently, the factor is interpreted as a *numerical memory factor*. This interpretation is also confirmed by the low loadings in the other variables, e. g. the problem test (11), in which numerical memory does not play an appreciable part. The loadings of the intelligence variables are here also approximately zero, a fact that also stresses the rote character of the memory in question.

Factor V. (p. 131). The purely numerical character of the factor is evident. Intelligence and report variables are missing here. Memory variables, verbal analogies, form relation or picture perception are also not present. Consequently, only tests of a purely numerical character remain. It is true that the loading in number series (7) is not significant, but it is nevertheless .288. Besides the numerical aspect, the perceptive aspect also appears to be involved and the factor is interpreted as a *numerical, perceptive factor, a factor for numbers and numerical relations.*

Factor VI. (p. 131). For this factor it appears that an identification mechanism is mainly required. This applies to variables 5 and 13. Form relation (3), which also requires a certain ability of recognition, that is to say, the ability to identify among a number of different parts, the one that fits into the principal figure, tends towards significance: .294. The occurrence of numerical memory (4) in this connection, cannot be explained by the author. In spite of the fact that the occurrence of this variable makes interpretation more difficult, the author nevertheless considers that the factor can be interpreted as a *recognition factor*, though not of a numerical but, a *more general character.*

Fraternal twins — Boys.

The rotated factor matrix is given in table 33, p, 132.

Factor I. (p. 134). The factor has variables all of which are common with those for factor I for identical-twins boys. In addition to this, there is number series (7). The factor is interpreted as a *general intelligence factor.*

Factor II. (p. 134). The perceptive nature of the factor is evident and it is interpreted as a *general perceptive factor.*

Factor III. (p. 134) The factor is very heterogeneous in character, and the author does not attempt to interpret it.