

Voices on learning and instruction in mathematics

Editors

Jonas Emanuelsson

Laura Fainsilber

Johan Häggström

Angelika Kullberg

Berner Lindström

Madeleine Löwing

NATIONAL CENTER FOR MATHEMATICS EDUCATION



UNIVERSITY OF
GOTHENBURG

Contents

Introduction	1
Research at the University of Gothenburg	7
Research in mathematics education – an appendix of mathematics or of educational science? EVA JABLONKA	51
Research in the learning and teaching of mathematics – What can researchers in education and mathematics learn from each other? JEREMY KILPATRICK	63
Pre-school	
Mathematics in the thematic work in pre-school ANN AHLBERG	79
Learning from Alex's encounter with basic mathematical concepts ELISABET DOVERBORG AND INGRID PRAMLING SAMUELSSON	97
How toddlers meet mathematical structures in preschool MARIA REIS	113
Pre-school children's learning of number concepts in a game-based learning environment BERNER LINDSTRÖM, FERENCE MARTON, JONAS EMANUELSSON, MARITA LINDAHL AND MATTIAS PACKENDORFF	119
Primary school	
Communicating mathematics in primary school Ann Ahlberg	143
On curricula and the teaching process WIGGO KILBORN	159
On diagnostic test and arithmetic skills WIGGO KILBORN	175
National diagnostic test for grades 1–5 MADELEINE LÖWING, MARIE FREDRIKSSON AND SUSANNE FRISK	187
Bringing back mathematics into research on mathematics teaching JOHAN HÄGGSTRÖM	203

What does it take to learn negative numbers? ANGELIKA KULLBERG	215
Making sense of negative numbers CECILIA KILHAMN	227
The importance of instruction for second-language student's mathematical progress – a presentation of an ongoing dissertation project ÅSE HANSSON	235
Secondary school	
Mathematical creativity – is it assessed in teacher made tests? JESPER BOESEN	239
What justifies as a solution in a mathematics classroom? THOMAS LINGEJÄRD	257
Three different methods for solving quadratic equations WANG WEI SÖNNERHED	275
Head teachers' conceptions of high-ability students – methodological considerations LINDA MATTSSON	287
University and adult education	
Algorithmic contexts, intuitive ideas and formal reasoning – exploring students' understanding of concepts in calculus KERSTIN PETERSSON	301
Everyday calculus teaching TORBJÖRN LUNDH	315
Assessment in mathematics teacher education LISBETH LINDBERG AND ANN-MARIE PENDRILL	331
Prospective mathematics teachers' learning in geometry – the beginning of a longitudinal study on teacher education MIKAEL HOLMQUIST	351
Tacit rationality and mathematical models – two intertwining kinds of professional knowledge LARS GUSTAFSSON AND LARS MOUWITZ	365
Epilogue	381

Preface

The University of Gothenburg is prominent, both nationally and internationally, with respect to research in both the learning sciences and in mathematics as a scientific discipline. The University of Gothenburg hosts the National Center for Mathematics Education. We also host a nationally recognised research school for teachers where several doctoral students work with projects closely related to the teaching and learning of mathematics. Faculties, departments and these environments, taken together, give a solid foundation for the development of research in mathematics education. The work reported in this book reflects the co-ordination of different strengths at our university, strengths which will be developed in even more fruitful ways in further communication and collaboration.

We wish all contributors that have made this publication possible the best of luck in the continuation of their efforts to organise research in the learning and teaching of mathematics into a recognised field of scholarly inquiry. We also hope that their efforts, which this book illustrates, will prove useful in our educational programmes at different levels and in our communication with the practice of mathematics education. We also want to thank the principal of the university for supporting this work financially.

Bengt Johansson, National Center for Mathematics Education
Bo Johansson, Mathematical Sciences
Bengt Olsson, Education and Research Board for Teacher Training
Lars-Erik Olsson, Department of Education

Introduction

These articles are the product of a cooperative project among researchers from different faculties at the University of Gothenburg. The aim of this research anthology is to present a summary of current and previous research at the university and make it a platform for future research. The anthology gives a historical background for new research and new dissertations and is a manifestation of cross communication between groups and people at different faculties. The aim has been to strengthen the basis for continued cooperation and to create a solid environment for research on learning and teaching of mathematics.

An important part of the strategic work within each department at the university is to develop complete academic environments. Complete, in the sense that all three activities – research, education and collaboration with the surrounding society – should be fully developed and integrated. At the university an overall aim is to establish cooperation between research groups, departments and faculties and in the long term to carry out research that will be recognized internationally. This cooperation project therefore forms an essential part in supporting solid research environments and increased visibility of the researchers within the departments at the University of Gothenburg as well as on a national and international level.

Against the background of the results of the evaluations and scrutiny of teacher training in Sweden that has been presented during recent years, it is important that the University of Gothenburg conducts relevant high-level research in order to contribute to an increased basis of knowledge for the continued improvement of the contents and implementation of teacher training.

Negative trends are described, nationally as well as internationally, regarding the performance in mathematics of Swedish students. Consequently, an important task of the present government has been to reform teacher training with a vision that, a higher quality of teacher training will increase the status of the profession and attract better applicants.

Research in mathematics education

Research in mathematics education refers to a domain of knowledge related to the teaching and learning of mathematics at all levels and in different contexts. It also includes, for instance, the historical background,

the cultural development, and objectives and intensions underlying the organisation of mathematics education throughout the world (c.f. Sierpinska & Kilpatrick, 1998). The Swedish term *matematikdidaktik* is translated in a number of different ways internationally – *mathematical education* (UK), *mathematics education* (USA), *Didaktik der Mathematik* (Germany), and *didactique des mathématiques* (France). As noted by Kilpatrick (1995), there is a difference between, for instance, *Didaktik der Mathematik* and *mathematics education*. Where the German expression refers to the academic field, the Americans, in order to avoid the unpleasant word *didactic*, use *mathematics education* in reference to both the activity and the field of research.

The existing research is also very diverse. It stretches from pure basic research to creating specific material or methods and to the testing of didactical hypotheses for evaluation (see e.g. MathEduc Database, <http://www.zentralblatt-math.org/matheduc/classification>. Niss (2001), however, writes:

For a fair number of mathematical didactic researchers the focus is mainly on "pure" perspectives of basic research. We can however claim that the overall aim of the whole activity basically is to enhance and ameliorate the pupils' and students' mathematical learning and help them acquire mathematical competence.

(p. 25, authors translation)

Research in mathematics education is made up of the scientific activities of describing, analysing and better understanding the relation between human beings and mathematics. The processes studied are sometimes highly organised – for instance in compulsory schools or university departments of mathematics – and sometimes informal.

As in other countries, Swedish research in mathematics education has been developed within the university departments of either mathematics or pedagogy/education. Perhaps because of this dual identity, as well as the historical division of educational research and teacher education, research in mathematics education at the University of Gothenburg has several names. Labels such as *didactics of mathematics*, *pedagogy*, *mathematics oriented towards didactics*, etc. are used to describe the research. The work presented here has been framed as *research on learning and teaching of mathematics*. This title is not accidental – it is carefully chosen in order to avoid a discussion on the particular labels and hence to enhance a discussion on research issues and questions rather than the relative positioning of different research approaches. This approach has made it possible to jointly produce a publication where the authors see this field of research from different perspectives and bring different competencies and interests into the work.

Project background

At the end of 2005, Jonas Emanuelsson, Berner Lindström and Madeleine Löwing at the Department of Education took the initiative to organize a meeting between representatives for different milieus at the University of Gothenburg where research on learning and teaching on and about mathematics is carried out. At the meeting, a working group consisting of representatives from different faculties was appointed. This working group was given the task of investigating the possibility of creating a strategic plan for research on the learning and teaching of mathematics that included the various faculties. The group pointed out the need to make a survey of research activities, current and completed research projects, dissertations and a survey of existing staff, seminars and working groups within the university. The survey, which involved several faculties, constituted a basis for continued cooperation. Having created a sound platform for cooperation, the next step for the group was to develop a strategy for continued cooperation and to initiate a common series of seminars.

Another initiative by the working group was to organize a research symposium. As a part of this initiative an editing committee was asked to arrange the symposium with an objective of summarising current research activities and present them in an anthology. The purpose was that the creation of an anthology should strengthen the ties between different milieus active in mathematics education within the university. The anthology will also illustrate our collective knowledge within the field. The idea is that the articles will show different views on the process of learning, teaching and creation of knowledge, and on different research efforts and perspectives. In this way we can draw a map of the field and highlight the variety of what is being done and consequently uncover the areas that would benefit from firmer cooperation. We hope that this work can contribute to consolidating a solid academic environment across the different milieus, different research projects and approaches.

Research symposium

A research symposium on learning and teaching of mathematics was held 10–11 March 2008 at the Wallenberg conference centre at the University of Gothenburg. It was a great honour to us that Professor Jeremy Kilpatrick, University of Georgia and Honorary Doctor at the University of Gothenburg, and Professor Eva Jablonka, Luleå Technical University participated at the symposium.

At the symposium 25 different contributions were presented which all went through peer-review. All participants were assigned to read and make comments on other contributions and then led the discussions in different sessions. Beside the ambition to learn about each other's work this was a way to structure the efforts to raise the quality of the papers. The two days were filled with valuable discussions and interactions and the participants were pleased to get constructive responses on their own work and to learn about others'.

Other network activities

Apart from the symposium and the work with this anthology, several things have occurred the last couple of years. The work of the group has resulted in the initiation of a scientific discussion by representatives of different specialized research groups with focus on the learning and teaching of mathematics. Another example is in the area of the PhD-education. PhD students have taken the initiative to create a network called FLUM (*Forskning om Lärande och Undervisning i Matematik – Research on Learning and Teaching of Mathematics*). This is a working seminar involving students from different faculties. The aim is that these seminars shall support PhD students with research interest in mathematics education but who pursue their work with different methodological and theoretical perspectives, with different points of departure and within different academic traditions. Representatives of the different research subject concerned have expressed a will to give support to the seminar, which however, is independently owned and run by the PhD students.

Pointing forward

The research symposium in 2008 was the first in a series of seminars involving different faculties within the University. The working group will continue to initiate seminars common to the university which will be a meeting place for discussions about research within the field, to which local as well as national and international researchers will be invited. These seminars are thought to form an arena for on-going cross-scientific discussions about the learning and teaching of mathematics. The ambition is to create sufficient prerequisites to establish, in the long term, a solid research environment involving different faculties at the University of Gothenburg. It probably will take some time before such an environment can be considered a "centre of excellence". Much work and other resources are needed, but we see this anthology as a first step.

Short presentations of the articles

In the next section we give an overview of research carried out at the University of Gothenburg and then follows the papers from the research symposium.

Our invited guests, Eva Jablonka and Jeremy Kilpatrick, are the authors of the first two papers. From different perspectives they try to describe the relations between mathematics as an academic field and mathematics education, giving an important background to the rest of the articles.

The rest of the articles are grouped based on the ages of the intended learners. The first of these sections contains four articles about pre-school mathematics. The first article, written by Ann Ahlberg, deals with how children can meet mathematics in preschool and how to stimulate children's curiosity, desire and skills in solving problems of a mathematical nature. In the next article, Elisabeth Doverborg and Ingrid Pramling Samuelsson describe different pre-schools as environments for young children's learning. It contains a description of children's initial understanding of different mathematical concepts. The third article in this section is written by Maria Reis. She describes how pre-school children develop early mathematical understanding in social integration with peers and adults. The fourth article, by Lindström et al. describes a quasi-experimental study of the effects of a computer game on pre-school children's learning of basic number concepts.

The second section contains eight articles related to mathematics in primary school. Here, Ann Ahlberg describes and analyses how primary school children in a school context understand and handle arithmetic word problems. The following three articles are all about assessment. The first one, written by Wiggo Kilborn, deals with the construction, interpretation and use of diagnostic tests. In the next article, Kilborn describes classroom studies as a tool to follow up the teaching process from the curriculum to the outcome of teaching. The third article on assessment is written by Madeleine Löwing, Marie Fredriksson and Susanne Frisk and deals with their experiences on constructing and trying out a new national diagnostic test.

The following articles on primary school mathematics are more focused on specific mathematical contents. The article written by Johan Häggström treats algebra and describes what is possible for students to learn about simultaneous linear equations in relation to the handling of the mathematical content in the classrooms. The two following articles deal with negative numbers. Angelika Kullberg describes how four teachers teach about addition and subtraction of negative numbers and Cecilia Kilhamn has interviewed twelve students in grades 6–9 about their

reasoning when treating negative numbers. Finally, Åse Hansson presents a study on factors that influence second language student's mathematics achievement.

The third section contains four articles about secondary school mathematics. The first article, written by Jesper Boesen, deals with the fact that pupils often meet just a familiar type of tasks, and for that reason miss the opportunities to develop their problem solving skills. In the next article, Thomas Lingefjård offers analyses of social interaction patterns that have proved to be successful when characterizing secondary school classrooms in teaching and learning of mathematical modeling. Next, Wang Wei Sönnerhed presents and analyses three different methods to solve quadratic equations. Finally, Linda Mattsson presents an account of the methodology and method for exploring "head teachers" conceptions of high-ability students in the Swedish upper secondary school.

The fourth and last section deals with university and adult education. The section begins with an article by Kerstin Pettersson about student's understanding of concepts in calculus, including the concepts function and derivative, requiring proof by induction. In the second article, Torbjörn Lundh describes his experiences of teaching mathematics at an introductory university level. Lisbeth Lindberg and Ann-Marie Pendrill then, show how authentic tasks can promote higher-order learning in connection to teacher students' visit to an amusement park. In the fourth article, Mikael Holmquist shows that the way teachers understand geometrical concepts influence their teaching and consequently what their pupils might learn. Finally Lars Gustafsson and Lars Mouwitz discuss the nature of mathematics learned in specialized work environments and the validation of such knowledge.

References

- Niss, M. (2001). Den matematikdidaktiska forskningen karaktär och status. In B. Grevholm (Ed.), *Matematikdidaktik: ett nordiskt perspektiv* (pp. 21–47). Lund: Studentlitteratur.
- Sierpinska, A. & Kilpatrick, J. (Eds.) (1998). *Mathematics education as a research domain: a search for identity* (an ICMI study). Dordrecht: Kluwer Academic Publishers.