Till läraren

Välkommen till Kängurutävlingen – Matematikens hopp 2024
Student

- Tävlingen genomförs under perioden 21 mars – 5 april. *Uppgifterna får inte användas tidigare.*
- Sista dag för redovisning av antalet deltagare är den *12 april*. Du får då tillgång till facit och ett kalkylblad där du matar in elevernas svar och sedan får du en sammanställning av klassens resultat.
- Redovisa resultatet senast *30 april*.
- *Tävlingen är individuell* och eleverna får arbeta i 60 minuter. De tre delarna ska genomföras vid *ett och samma tillfälle.*
- Eleverna behöver ha tillgång till papper för att kunna göra anteckningar och figurer. Linjal behövs inte.
- *Miniräknare eller sax får inte användas. Observera att telefoner, datorplattor och datorer inte heller får användas.*
- Läs igenom problemen själv i förväg så att eventuella oklarheter kan redas ut.
- Kontrollera att kopiorna blir tillräckligt tydliga så att nödvändiga detaljer syns.
- Besök *Kängurusidan* på ncm.gu.se/kanguru där vi publicerar eventuella rättelser och ytterligare information. Där finns också information om hur kalkylbladet fungerar.
- Samla in problemformulären efter tävlingen. Problemen får inte spridas utanför klassrummet förrän efter 30 april, men ni får gärna arbeta med problemen i klassen.

Mikael Passares stipendium
Mikael Passare (1959–2011) var professor i matematik vid Stockholms universitet. Han hade ett stort intresse för matematikundervisning på alla nivåer och var den som tog initiativ till Kängurutävlingen i Sverige. Mikael Passares minnesfond har instiftat ett stipendium för att uppmärksamma elevers goda matematikprestationer. Information om hur du nominerar elever kommer tillsammans med facit och kommentarer.

Lycka till med årets Känguru!

e-post: kanguru@ncm.gu.se

För administrativa frågor, vänd dig till Ann-Charlotte Forslund:
ann-charlotte.forslund@ncm.gu.se
031–786 69 85

För innehållsfrågor, vänd dig till Ulrica Dahlberg eller Johan Häggström:
ulrica.dahlberg@ncm.gu.se
johan.haggstrom@ncm.gu.se

Svarsblankett

Markera ditt svar i rätt ruta

Uppgift	A	B	C	D	E	Poäng
1						
2						
3						
4						
5						
6						
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8						
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					SUMMA	

Namn:..

Klass:..

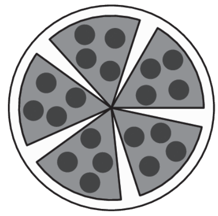
Three points problem

1 Which of these integers is two less than a multiple of ten, two more than a square, and two times a prime?

A 78 B 58 C 38 D 18 E 6

[*United Kingdom*]

2 A young kangaroo cut a pizza into six equal slices. After eating one slice, he arranged the remaining slices with equal gaps between slices.

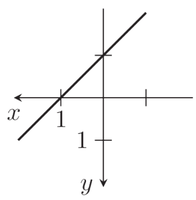


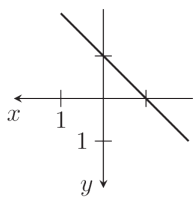
What size is the angle of each gap?

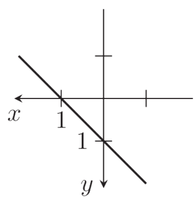
A 5° B 8° C 9° D 10° E 12°

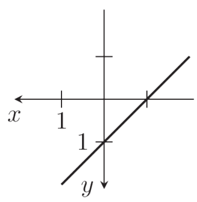
[*Germany*]

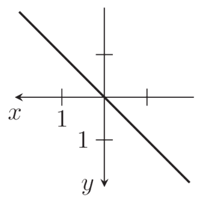
3 Juuso has an unusual habit of drawing the xy-plane with the positive coordinate axes pointing left and down. What would the graph of the equation $y = x + 1$ look like in a coordinate system drawn by Juuso?

A


B


C


D


E


[*Finland*]

4 Kaito has manipulated a die. The probabilities of rolling a 2, 3, 4 or 5 are still 1/6 each, but the probability of rolling a 6 is twice the probability of rolling a 1.
What is the probability of rolling a 6?

A 1/4 B 1/6 C 7/36 D 2/9 E 5/18

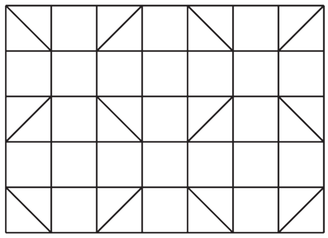
[*Germany*]

5 Which of the expressions below has the same value as $16^{15} + 16^{15} + 16^{15} + 16^{15}$?

A 16^{19} B 4^{31} C 4^{60} D 16^{60} E 4^{122}

6 Beaver wishes to color the squares and triangles of the following figure so that no two neighbouring figures, even those sharing a single vertex, are the same color.

What is the least number of colors needed?



A 3 B 4 C 5 D 6 E 7

[*Finland*]

7 There are 6 glasses on a table with their open ends up. In any one move, we turn over exactly 4 of them.

What is the least number of moves required to have all glasses upside down?

A 2 B 3 C 4 D 5 E 6

[*China*]

8 Hanna started with the number 1 and multiplied it by either 6 or 10. She then multiplied the result by either 6 or 10, and continued this procedure many times.

Which of the following cannot be one of the numbers she obtained?

A $2^{100} \cdot 3^{20} \cdot 5^{80}$ B $2^{90} \cdot 3^{20} \cdot 5^{80}$ C $2^{90} \cdot 3^{20} \cdot 5^{70}$

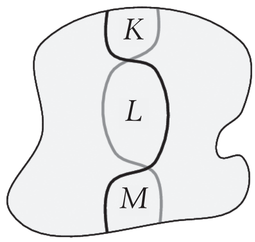
D $2^{110} \cdot 3^{80} \cdot 5^{30}$ E $2^{50} \cdot 5^{50}$

[*Greece*]

Four points problem

9 A black trail and a grey trail cross a park, as shown. Each trail divides the park into two regions of equal area.

Which of the following must be true about the areas K, L and M?



A $K = M$ B $L = K + M$ C $L = \frac{1}{2}(K + M)$ D $L = \frac{2}{3}(K + M)$ E $L = \frac{3}{5}(K + M)$

[*Greece*]

10 Exactly one of these statements about a certain positive integer n is true. Which statement is true?

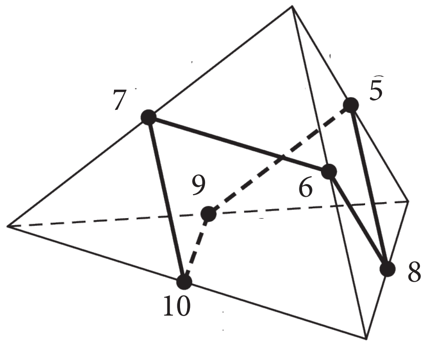
A n is divisible by 3 B n is divisible by 6 C n is odd

D $n = 2$ E n is prime

[*Greece*]

11 In a tetrahedron, the midpoints of the edges have been marked and connected with black lines to form a closed 3D polygon (a hexagon).
The numbers at the midpoints indicate the length of the respective edge, e.g. point 7 is the midpoint of an edge with length 7.

What is the perimeter of the closed hexagon?

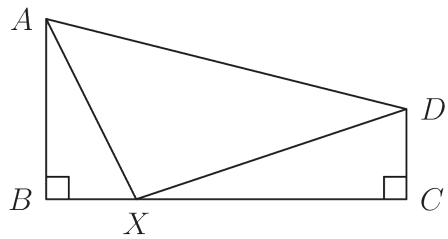


A 19 B 20 C 21 D 22 E 23

[*Greece*]

12 A quadrilateral *ABCD* has two right angles at *B* and C, where $AB = 4$, $BC = 8$ and $CD = 2$. Point *X* lies on *BC*.

What is the minimum value of $AX + DX$?

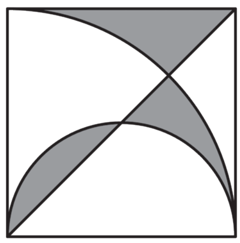


A $9\sqrt{2}$ B 12 C 13 D 10 E None of the previous

[*China*]

13 A diagonal, a semicircle and a quadrant are drawn in a square of side 6 cm.

What is the area, in cm^2, of the shaded part?



A 9 B 3π C $6\pi - 9$ D $10\pi/3$ E 12

14 $0 < p < q$. Which of these expressions is the largest?

A $\frac{p+3q}{4}$ B $\frac{p+2q}{3}$ C $\frac{p+q}{2}$ D $\frac{2p+q}{3}$ E $\frac{3p+q}{4}$

[*United Kingdom*]

15 How many three-digit numbers are there that contain at least one of the digits 1, 2 or 3?

A 27 B 147 C 441 D 557 E 606

16 With digits *p, q, r* och *s* we make a positive decimal number $\overline{pq,rs}$, which is the average of the two-digit numbers *pq* och *rs*. What is the sum of $p + q + r + s$?

A 14 B 18 C 21 D 25 E 27

[*Australia*]

Five points problem

17 Andre has six cards with one number written on each side of each card. The pairs of numbers on the cards are (5, 12), (3, 11), (0, 16), (7, 8), (4, 14) and (9, 10).
The cards can be placed in any order in the blank spaces of the figure.

$\square + \square + \square - \square - \square - \square = ?$

What is the smallest result he can get?

A -23 B -24 C -25 D -26 E -27

[*Czech Republic*]

18 Kangaroo solves the equation $ax^2 + bx + c = 0$, and Beaver solves the equation $bx^2 + ax + c = 0$, where *a, b och c* are pairwise distinct non-zero integers. It turns out that the equations share a solution. Which of the following must be true?

A The common solution must be 0

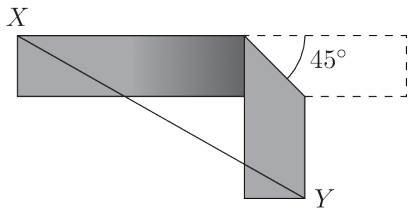
B The quadratic equation $ax^2 + bx + c = 0$ has exactly one real solution

C $a > 0$ D $b < 0$ E $a + b + c = 0$

[*Australia*]

19 I have a strip of paper that is 12 cm long and 2 cm wide. I make a crease across it at 45° and then fold it, so that the two parts of the strip are aligned in a right angle, as shown.

What is the smallest possible length, in cm, of XY?

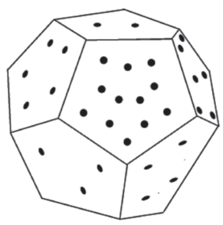


A $6\sqrt{2}$ B $7\sqrt{2}$ C 10 D 8 E $6 + \sqrt{2}$

[*Australia*]

20 Rasika has several unbiased 12-sided dice, each with faces labelled 1 to 12. When rolling all the dice at once, the probability of rolling a 12 exactly once is equal to the probability of rolling no 12s.

How many dice does Rasika have?



A 8 B 9 C 10 D 11 E 12

[*Australia*]

21 A polynomial $p(x)$ satisfies the relation $p(x+1) = x^2 - x + 2p(6)$ for every real x.

What is the sum of the coefficients of p?

A -40 B -6 C 12 D 40 E None of the previous

[*Greece*]

22 $2^x = 3$, $2^y = 7$ and $6^z = 7$.

Which of the following gives the relationship between x, y and z?

A $z = \frac{y}{1+x}$ B $z = \frac{x}{y} + 1$ C $z = \frac{y}{x} - 1$

D $z = \frac{x}{y-1}$ E $z = y - \frac{1}{x}$

[*Australia*]

23 A function $f: \mathcal{R} \rightarrow \mathcal{R}$ satisfies $f(20-x) = f(22+x)$ for all real x.
It is known that f has exactly two roots.

What is the sum of these two roots?

A -1 B 20 C 21 D 22 E 42

[*Greece*]

24 A special four-digit number $\overline{abcd}$ satisfies the equation $\overline{abcd} = a^a + b^b + c^c + d^d$.

What is the value of a?

A 2 B 3 C 4 D 5 E 6

[*Switzerland*]