



Till läraren

Välkommen till Kängurutävlingen – Matematikens hopp 2023

Student

- Tävlingen genomförs under perioden 16 mars – 24 mars. *Uppgifterna får inte användas tidigare.*
- Sista dag för redovisning av antalet deltagare är den *31 mars*. Du får då tillgång till facit och ett kalkylblad där du matar in elevernas svar och sedan får du en sammanställning av klassens resultat.
- Redovisa resultatet senast *28 april*.
- *Tävlingen är individuell* och eleverna får arbeta i 60 minuter. De tre delarna ska genomföras vid *ett och samma tillfälle*.
- Eleverna behöver ha tillgång till papper för att kunna göra anteckningar och figurer. Linjal behövs inte.
- *Miniräknare eller sax får inte användas. Observera att telefoner, datorplattor och datorer inte heller får användas.*
- Läs igenom problemen själv i förväg så att eventuella oklarheter kan redas ut.
- Kontrollera att kopiorna blir tillräckligt tydliga så att nödvändiga detaljer syns.
- Besök *Kängurusidan* på ncm.gu.se/kanguru där vi publicerar eventuella rättelser och ytterligare information. Där finns också information om hur kalkylbladet fungerar.
- Samla in problemformulären efter tävlingen. Problemen får inte spridas utanför klassrummet förrän efter 28 april, men ni får gärna arbeta med problemen i klassen.

Mikael Passares stipendium

Mikael Passare (1959–2011) var professor i matematik vid Stockholms universitet. Han hade ett stort intresse för matematikundervisning på alla nivåer och var den som tog initiativ till Kängurutävlingen i Sverige. Mikael Passares minnesfond har instiftat ett stipendium för att uppmärksamma elevers goda matematikprestationer. Information om hur du nominerar elever kommer tillsammans med facit och kommentarer.

Lycka till med årets Känguru!

e-post: kanguru@ncm.gu.se

För administrativa frågor, vänd dig till Ann-Charlotte Forslund:

Ann-Charlotte.Forslund@ncm.gu.se

031–786 69 85

För innehållsfrågor, vänd dig till Ulrica Dahlberg eller Johan Häggström:

ulrica.dahlberg@ncm.gu.se

johan.haggstrom@ncm.gu.se



Svarsblankett

Markera ditt svar i rätt ruta

Uppgift	A	B	C	D	E	Poäng
1						
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SUMMA						

Namn:.....

Klass:.....

Kängurutävlingen – Matematikens hopp 2023

Student



Three points problem

1 What is the value of $\frac{7777^2}{5555 \cdot 2222}$?

- A 1 B $\frac{7}{10}$ C $\frac{49}{10}$ D $\frac{77}{110}$ E 49

[Greece]

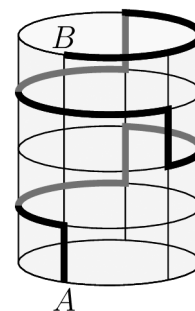
2 Julia rolls five dice. She rolls 19 points in total.

What is the maximum number of sixes she could have rolled?

- A 0 B 1 C 2 D 3 E 4

[Germany]

3 A cylindrical can has height 15 cm and the perimeter of its circular base is 30 cm. An ant walks from point A on the base to point B on the roof. Its path is either vertically upwards or horizontally along circular arcs around the can. Its path is shown with a thicker line (black for the path on the front of the can and grey at the back).



What is the length of the ant's path?

- A 45 cm B 55 cm C 60 cm D 65 cm E 75 cm

[Greece]

4 We call a positive integer n *two-prime*, if it has exactly three different divisors, namely 1, 2 and n itself.

How many different *two-prime* integers are there?

- A 0 B 1 C 2 D 3 E 4

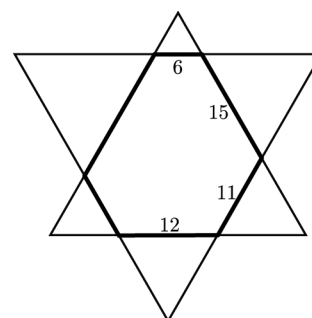
[Austria]

5 How many pairs of positive integers m and n satisfy the equation: $m + 2n = 2^{10}$?

- A $2^9 - 1$ B 2^9 C $2^9 + 1$ D $2^9 + 2$ E 0

[Montenegro]

6 Two equilateral triangles are put together to form a hexagon with their opposite sides parallel. We know the length of four sides of this hexagon, as shown in the diagram.



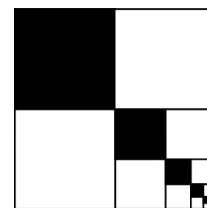
What is the perimeter of the hexagon?

- A 64 B 66 C 68 D 70 E 72

[Mexico]



- 7 A square with area 84 is divided into four squares. The upper left square is coloured black. The lower right square is again divided into four squares, and so on. The process is repeated an infinite number of times. What is the total area that is coloured black?



A 24 B 28 C 31 D 35 E 42

[Netherlands]

- 8 Each of the integers from 1 to 9 is to be placed in one of the 9 boxes in the picture so that any three numbers in consecutive boxes add to a multiple of 3. The numbers 7 and 9 have already been placed

In how many different ways can the remaining boxes be filled?



A 9 B 12 C 15 D 18 E 24

[Mexico]

Four points problem

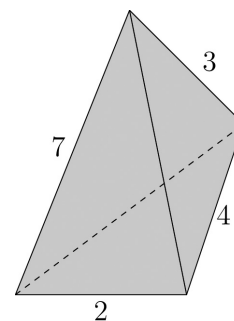
- 9 What is the units digit of the product $(5^5 + 1) \cdot (5^{10} + 1) \cdot (5^{15} + 1)$?

A 1 B 2 C 4 D 5 E 6

[United Kingdom]

- 10 A triangular pyramid has edges of integer length. Four of these lengths are as shown in the diagram.

What is the sum of the lengths of the other two edges?



A 9 B 10 C 11 D 12 E 13

[Greece]

- 11 What is the sum of the digits of N if $N! = 6! \cdot 7!$?

A 1 B 2 C 4 D 8 E 9

[Türkiye]

- 12 The graphs of the functions $y = x^3 + 3x^2 + ax + 2a + 4$ all pass through the same point, no matter what value of a is chosen.

What is the sum of the coordinates $(x + y)$ of that point?

A 2 B 4 C 7 D 8 E none of the previous

[Greece]



- 13 We are given five numbers a_1, a_2, a_3, a_4, a_5 whose sum $a_1 + a_2 + a_3 + a_4 + a_5 = S$.
For each $k, 1 \leq k \leq 5$, we know that $a_k = k + S$.

What is the value of S ?

- A $\frac{15}{4}$ B $-\frac{15}{4}$ C -15 D 15 E none of the previous

[Greece]

- 14 How many pairs of integers m and n satisfy the inequality $|2m - 2023| + |2n - m| \leq 1$?

- A 0 B 1 C 2 D 3 E 4

[Greece]

- 15 There are 23 chairs in a row along one wall in the ballroom. One dancer sits on each chair, either a boy or a girl. Every dancer has at least one girl next to him/her.

How many boys can there be at most among the dancers?

- A 7 B 8 C 10 D 11 E 12

[Hungary]

- 16 For some integer n we have $n^n = 5^{(5^6)}$.

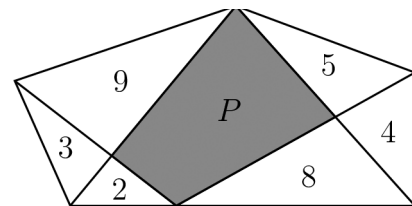
What is the value of n ?

- A 5^{30} B 5^6 C 5^5 D 30 E 11

[Greece]

Five points problem

- 17 A pentagon is dissected into smaller parts, as shown.
The numbers inside the triangles indicate their areas.
What is the area P of the shaded quadrilateral?



- A 15 B $\frac{31}{2}$ C 16 D 17 E 18

[Greece]

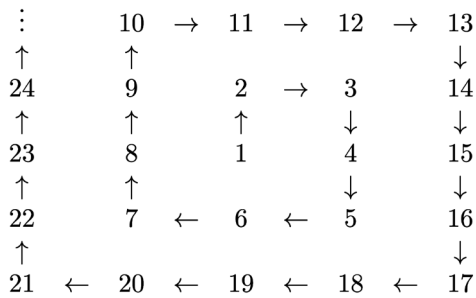
- 18 How many positive integers are factors of $2^{20} \cdot 3^{23}$ but are not factors of $2^{10} \cdot 3^{20}$?

- A 13 B 30 C 273 D 460 E none of the previous

[Greece]



19 A spiral of consecutive numbers is created, as shown, starting with 1.



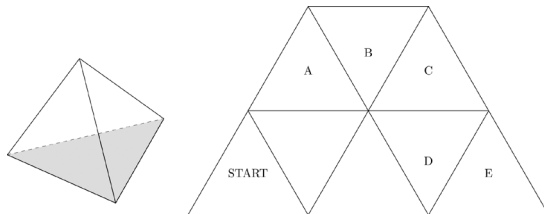
When the pattern of the spiral is continued, in which arrangement will the numbers 625, 626 and 627 appear?

A $\begin{matrix} 627 \\ \uparrow \\ 626 \\ \uparrow \\ 625 \end{matrix}$ B $\begin{matrix} 626 \rightarrow 627 \\ \uparrow \\ 625 \end{matrix}$ C $625 \rightarrow 626 \rightarrow 627$

D $\begin{matrix} 625 \rightarrow 626 \\ \downarrow \\ 627 \end{matrix}$ E $\begin{matrix} 625 \\ \downarrow \\ 626 \\ \downarrow \\ 627 \end{matrix}$

[Serbia]

20 A block in the shape of a regular tetrahedron has one face shaded.



The shaded face of the block is placed on the board on the triangle labelled START. The block is then rolled from one triangle to the next by rotating it about one edge.

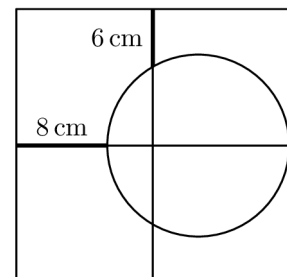
On which triangle will the block stand for the first time again on its shaded face?

- A A B B C C D D E E

[Poland]

21 The large square in the diagram is dissected into four smaller squares. The circle touches the right hand side of the square at its midpoint.

What is the side-length of the large square?



- A 18 cm B 20 cm C 24 cm D 28 cm E 30 cm

[Germany]



22 Two functions f and g on \mathbb{R} satisfy the system of equations:

$$f(x) + 2g(1-x) = x^2 \text{ and}$$

$$f(1-x) - g(x) = x^2.$$

What function is f ?

A $x^2 - \frac{4}{3}x + \frac{2}{3}$

B $x^2 + \frac{4}{3}x + \frac{2}{3}$

C $-x^2 - \frac{4}{3}x + \frac{2}{3}$

D $x^2 - 4x + 5$

E there are no such functions

[Greece]

23 What is the greatest common divisor of all numbers of the form

$$n^3(n+1)^3(n+2)^3(n+3)^3(n+4)^3$$

where n is a non-zero natural number?

A 2^93^3

B $2^33^35^3$

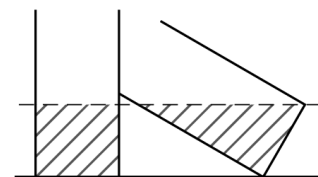
C $2^63^35^3$

D $2^83^25^3$

E $2^93^35^3$

[Greece]

24 Two identical cylindrical water tanks contain the same amount of water. One water tank is standing upright, and the other is leaning against it, and the water level in each of them is the same as in the picture. The bottom of each of the cylinders is a circle with area $3\pi \text{ m}^2$.



How much water does each tank contain??

A $3\sqrt{3} \pi \text{ m}^3$

B $6\pi \text{ m}^3$

C $9\pi \text{ m}^3$

D $\frac{3\pi}{4} \text{ m}^3$

E it's impossible to determine from the information given