Till läraren

Välkommen till Kängurutävlingen – Matematikens hopp 2021
Student, för elever i gy kurs 4 och 5

– Tävlingen genomförs under perioden 18 mars – 15 maj. *Uppgifterna får inte användas tidigare.*

– När du redovisar antalet deltagare får du tillgång till facit och ett kalkylblad där du matar in elevernas svar. Du får då en sammanställning av klassens resultat. Sista dag för redovisning av antalet deltagare är den *15 maj.*

– Redovisa resultatet senast *20 maj.*

– *Tävlingen är individuell* och eleverna får arbeta i 60 minuter. De tre delarna ska genomföras vid *ett och samma tillfälle.*

– Eleverna behöver ha tillgång till papper för att kunna göra anteckningar och figurer. Linjal behövs inte.

– *Miniräknare eller sax får inte användas. Observera att telefoner, datorplattor och datorer inte heller får användas.*

– Läs igenom problemen själv i förväg så att eventuella oklarheter kan redas ut.

– Kontrollera att kopiorna blir tillräckligt tydliga så att nödvändiga detaljer syns.

– Besök *Kängurusidan* på ncm.gu.se/kanguru där vi publicerar eventuella rättelser och ytterligare information. Där finns också information om hur kalkylbladet fungerar.

– Samla in problemformulären efter tävlingen. Problemen får inte spridas utanför klassrummet förrän efter 20 maj, men ni får gärna arbeta med problemen i klassen.

Mikael Passares stipendium
Mikael Passare (1959–2011) var professor i matematik vid Stockholms universitet. Han hade ett stort intresse för matematikundervisning på alla nivåer och var den som tog initiativ till Kängurutävlingen i Sverige. Mikael Passares minnesfond har instiftat ett stipendium för att uppmärksamma elevers goda matematikprestationer. Information om hur du nominerar elever kommer tillsammans med facit och kommentarer.

Lycka till med årets Känguru!

e-post: kanguru@ncm.gu.se

För administrativa frågor, vänd dig till Ann-Charlotte Forslund:
Ann-Charlotte.Forslund@ncm.gu.se
031–786 69 85

För innehållsfrågor, vänd dig till Ulrica Dahlberg eller Peter Nyström:
Ulrica.Dahlberg@ncm.gu.se
Peter.Nystrom@ncm.gu.se

Svarsblankett

Markera ditt svar i rätt ruta

Uppgift	A	B	C	D	E	Poäng
1						
2						
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					SUMMA	

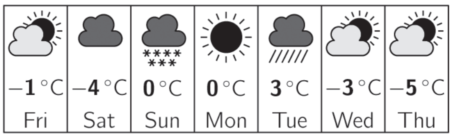
Namn:..

Klass:...

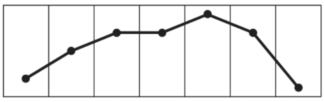
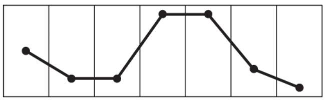
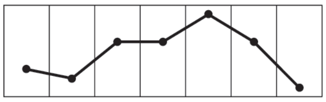
Kängurutävlingen – Matematikens hopp 2021
Student

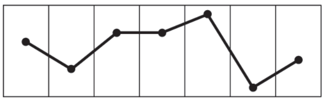
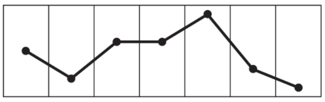
Three points problems

1 Paula's weather app shows a diagram of the predicted weather and maximum temperatures for the next seven days, as shown



Which of the following represents the corresponding graph of maximum temperatures?

A  B  C 

D  E 

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2 How many integers are in the interval $(20 - \sqrt{21}, 20 + \sqrt{21})$?

A 9 B 10 C 11 D 12 E 13

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3 A cube with edge 1 is cut into two identical cuboids.
What is the surface area of one of these cuboids?

A $\frac{3}{2}$ B 2 C 3 D 4 E 5

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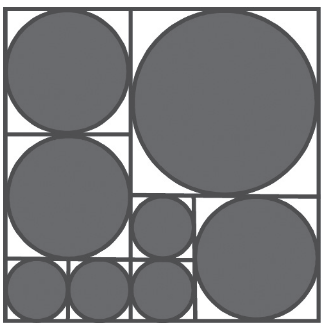
4 A large square is divided into smaller squares, as shown. A shaded circle is inscribed inside each of the smaller squares.
What proportion of the area of the large square is shaded?



A $\frac{8\pi}{9}$ B $\frac{13\pi}{16}$ C $\frac{3}{\pi}$ D $\frac{3}{4}$ E $\frac{\pi}{4}$

5 A rectangular sheet of paper has length x and width y, where $x > y$. The rectangle may be folded to form the curved surface of a circular cylinder in two different ways. What is the ratio of the volume of the longer cylinder to the volume of the shorter cylinder?

A $\frac{y^2}{x^2}$ B $\frac{y}{x}$ C $\frac{1}{1}$ D $\frac{x}{y}$ E $\frac{x^2}{y^2}$

6 Let $x = \frac{\pi}{4}$. Which of the following numbers is the largest?

A x^4 B x^2 C x D $\sqrt{x}$ E $\sqrt[4]{x}$

7 How many 3-digit-numbers formed using only the digits 1, 3 and 5 are divisible by 3? You may use digits more than once.

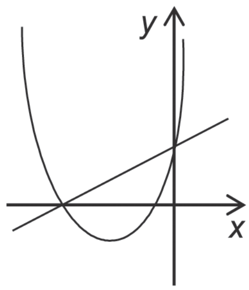
A 3 B 6 C 9 D 18 E 27

8 What is the area of the triangle whose vertices are at (p, q), $(3p, q)$ and $(2p, 3q)$, where $p, q > 0$?

A $\frac{pq}{2}$ B pq C $2pq$ D $3pq$ E $4pq$

Four points problems

9 The parabola in the figure has an equation of the form $y = ax^2 + bx + c$ for some distinct real numbers a, b and c.
Which of the following equations could be an equation of the line in the figure?



A $y = bx + c$ B $y = cx + b$ C $y = ax + b$
D $y = ax + c$ E $y = cx + a$

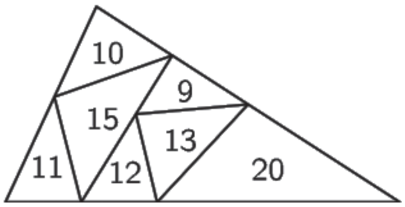
10 What proportion of all the divisors of 7! is odd?

A $\frac{1}{2}$ B $\frac{1}{3}$ C $\frac{1}{4}$ D $\frac{1}{5}$ E $\frac{1}{6}$

11 How many three-digit natural numbers have the property that when their digits are written in reverse order, the result is a three-digit number which is 99 more than the original number?

A 8 B 64 C 72 D 80 E 81

12 A large triangle is divided into smaller triangles as shown. The number inside each small triangle indicates its perimeter. What is the perimeter of the large triangle?



A 31 B 34 C 41 D 62 E none of the previous

13 For a positive integer N, we denote by $p(N)$ the product of the digits of N when written in decimal form. For example, $p(23) = 2 \cdot 3 = 6$.
What is the value of the sum $p(10) + p(11) + p(12) + \ldots + p(99) + p(100)$?

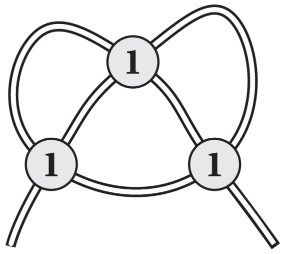
A 2025 B 4500 C 5005 D 5050 E none of the previous

14 In the 5 x 5 square shown the sum of the numbers in each row and in each column is the same. There is a number in every cell, but some of the numbers are not shown. What is the number in the cell marked with a question mark?

	16		22	
20		21		2
	25		1	
24		5		6
	4		?	

A 8 B 10 C 12 D 18 E 23

15 A piece of string is lying on the table. It is partially covered by three coins as seen in the figure.



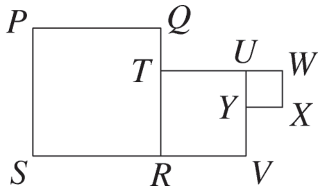
Under each coin the string is equally likely to pass over itself like this:

or like this:

What is the probability that the string is knotted after pulling its ends?

A $\frac{1}{2}$ B $\frac{1}{4}$ C $\frac{1}{8}$ D $\frac{3}{4}$ E $\frac{3}{8}$

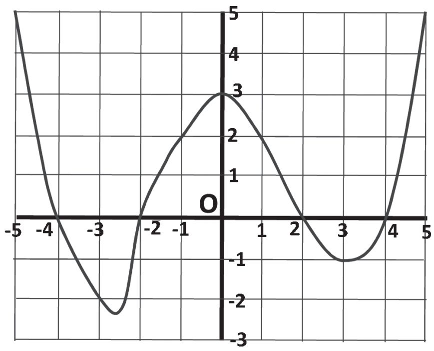
16 The diagram shows three squares, $PQRS$, $TUVR$ and $UWXY$. They are placed together, edge to edge. Points P, T och X lie on the same same straight line. The area of $PQRS$ is 36 and the area of $TUVR$ is 16.
What is the area of triangle PXV?



A $14\frac{2}{3}$ B $15\frac{1}{3}$ C 16 D $17\frac{2}{3}$ E 18

Five points problems

17 The figure shows the graph of a function f.
How many distinct solutions does the equation $f(f(x)) = 0$ have for $-5 \leq x \leq 5$?



A 2 B 4 C 6 D 7 E 8

18 The numbers 1, 2, 7, 9, 10, 15 and 19 are written down on a blackboard. Two players alternately delete one number each until only one number remains on the blackboard. The sum of the numbers deleted by one of the players is twice the sum of the numbers deleted by the other player. What is the number that remains?

A 7 B 9 C 10 D 15 E19

19 Each of the numbers a and b is a square of an integer. The difference $a - b$ is a prime number. Which of the following could be neither a nor b?

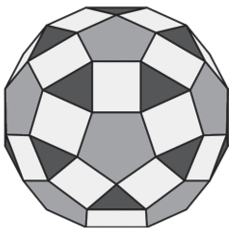
A 100 B 144 C 400 D 625 E 2500

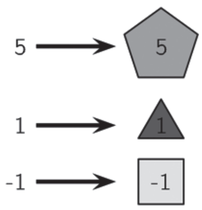
20 The function $f(x)$ is such that $f(x+y) = f(x) \cdot f(y)$ and $f(1) = 2$.

What is the value of $\frac{f(2)}{f(1)} + \frac{f(3)}{f(2)} + \ldots + \frac{f(2021)}{f(2020)}$?

A 0 B $\frac{1}{2}$ C 2 D 2020 E none of the previous

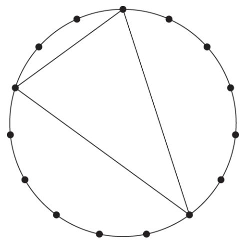
21 The solid shown in the diagram has 12 regular pentagonal faces, the other faces being either equilateral triangles or squares. Each pentagonal face is surrounded by 5 square faces. Each triangular face is surrounded by 3 square faces. Each square is surranded by 2 pentagonal and 2 triangular faces. John writes 1 on each triangular face, 5 on each pentagonal face and -1 on each square. What is the total of the numbers written on the solid?





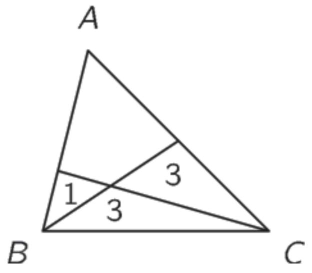
A 20 B 50 C 60 D 80 E 120

22 On a circle 15 points are equally spaced. We can form triangles by joining any three of these. We count two triangles as being the same if they are congruent i.e. one is a rotation and/or a reflection of the other.
How many different triangles can be drawn?



A 19 B 91 C 46 D 455 E 23

23 A triangle ABC is divided into four parts by two straight lines, as shown. The areas of the smaller triangles are 1, 3 and 3.
What is the area of the original triangle?



A 12 B 12,5 C 13 D 13,5 E 14

24 A certain game is won when one player gets 3 points ahead. Two players A and B are playing the game and at a particular point, A is 1 point ahead. Each player has an equal probability of winning each point.
What is the probability that A wins the game?

A $\frac{1}{2}$ B $\frac{2}{3}$ C $\frac{3}{4}$ D $\frac{4}{5}$ E $\frac{5}{6}$