



Till läraren

## Välkommen till Kängurutävlingen – Matematikens hopp 2021 *Junior, för elever i gymnasiekurs 2 och 3*

- Tävlingen genomförs under perioden 18 mars – 15 maj. *Uppgifterna får inte användas tidigare.*
- När du redovisar antalet deltagare får du tillgång till facit och ett kalkylblad där du matar in elevernas svar. Du får då en sammanställning av klassens resultat. Sista dag för redovisning av antalet deltagare är den *15 maj*.
- Redovisa resultatet senast *20 maj*.
- *Tävlingen är individuell* och eleverna får arbeta i 60 minuter. De tre delarna ska genomföras vid *ett och samma tillfälle*.
- Eleverna behöver ha tillgång till papper för att kunna göra anteckningar och figurer. Linjal behövs inte.
- *Miniräknare eller sax får inte användas. Observera att telefoner, datorplattor och datorer inte heller får användas.*
- Läs igenom problemen själv i förväg så att eventuella oklarheter kan redas ut.
- Kontrollera att kopiorna blir tillräckligt tydliga så att nödvändiga detaljer syns.
- Besök *Kängurusidan* på [ncm.gu.se/kanguru](http://ncm.gu.se/kanguru) där vi publicerar eventuella rättelser och ytterligare information. Där finns också information om hur kalkylbladet fungerar.
- Samla in problemformulären efter tävlingen. Problemen får inte spridas utanför klassrummet förrän efter 20 maj, men ni får gärna arbeta med problemen i klassen.

### *Mikael Passares stipendium*

Mikael Passare (1959–2011) var professor i matematik vid Stockholms universitet. Han hade ett stort intresse för matematikundervisning på alla nivåer och var den som tog initiativ till Kängurutävlingen i Sverige. Mikael Passares minnesfond har instiftat ett stipendium för att uppmärksamma elevers goda matematikprestationer. Information om hur du nominerar elever kommer tillsammans med facit och kommentarer.

### *Lycka till med årets Känguru!*

e-post: [kanguru@ncm.gu.se](mailto:kanguru@ncm.gu.se)

För administrativa frågor, vänd dig till Ann-Charlotte Forslund:

[Ann-Charlotte.Forslund@ncm.gu.se](mailto:Ann-Charlotte.Forslund@ncm.gu.se)

031–786 69 85

För innehållsfrågor, vänd dig till Ulrica Dahlberg eller Peter Nyström:

[Ulrica.Dahlberg@ncm.gu.se](mailto:Ulrica.Dahlberg@ncm.gu.se)

[Peter.Nystrom@ncm.gu.se](mailto:Peter.Nystrom@ncm.gu.se)



# Svarsblankett

Markera ditt svar i rätt ruta

Uppgift	A	B	C	D	E	Poäng
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# Kängurutävlingen – Matematikens hopp 2021

## Junior



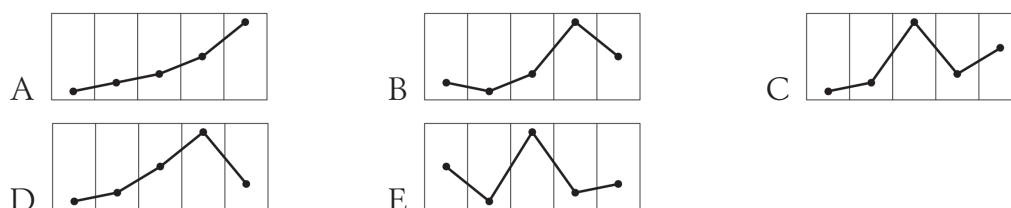

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### Three points problems

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- 1 Jenny looks at her weather app that shows the predicted maximum temperatures for the next five days. What does its corresponding graph look like?

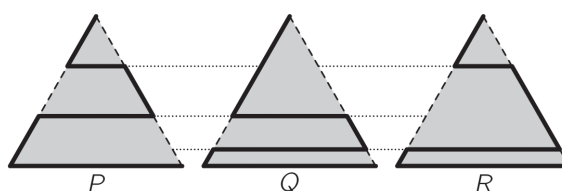
-1 °C	-2 °C	0 °C	6 °C	2 °C
Fri	Sat	Sun	Mon	Tue



- 2 Each year, the third Thursday of March is named the Kangaroo day. The Kangaroo days for the following years are scheduled as follows. There is made one error. Which entry contains the error?

- A 2022, 17 march      B 2023, 16 march      C 2024, 14 march  
D 2025, 20 march      E 2026, 19 march

- 3 A park has the shape of an equilateral triangle. Elsa often walks through the park from the upper corner to the bottom right corner. She walks along one of the three indicated paths (thicker lines). The lengths of the paths are  $P$ ,  $Q$  and  $R$ , as shown. Which of the following statements is true?



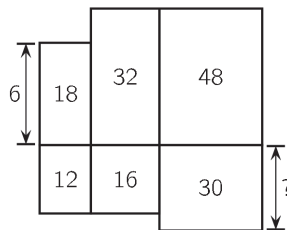
- A  $P < Q < R$       B  $P < R < Q$       C  $P < Q = R$   
D  $P = R < Q$       E  $P = Q = R$

- 4 At halftime of a handball match, the visiting team was leading. The score was 9–14. In the second half, the home team played much better and scored twice as many goals as the guests. Finally the home team won the match by one goal. What was the result of the match?

- A 20–19      B 21–20      C 22–21      D 23–22      E 24–23

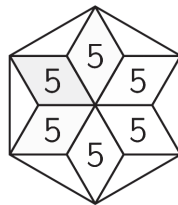


- 5 Six rectangles are positioned as shown. The left one at the top has the height 6 cm. The numbers within the rectangles indicate their areas in  $\text{cm}^2$ . What is the height of the last rectangle at the bottom right?



- A 4 cm      B 5 cm      C 6 cm      D 7,5 cm      E 10 cm

- 6 Six congruent rhombuses, each of area  $5 \text{ cm}^2$ , form a star. The tips of the star are joined to draw a regular hexagon, as shown. What is the area of the hexagon?

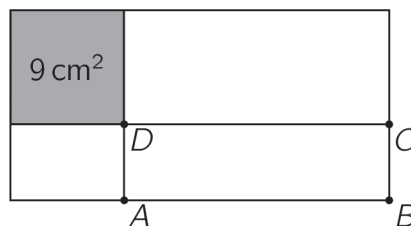


- A  $36 \text{ cm}^2$       B  $40 \text{ cm}^2$       C  $45 \text{ cm}^2$       D  $48 \text{ cm}^2$       E  $60 \text{ cm}^2$

- 7 In the jazz band, Göran plays the saxophone, Sara plays the trumpet, and Elisabeth sings. They are all of the same age. There are 3 more members in the jazz band. They are 19, 20, and 21 years old, respectively. How old is Elisabeth if the average age of the jazz band members is 21?

- A 20      B 21      C 22      D 23      E 24

- 8 A rectangle with perimeter 30 cm is divided by a vertical line and a horizontal line, forming within it a square of area  $9 \text{ cm}^2$ , as shown in the figure. What is the perimeter of rectangle  $ABCD$ ?

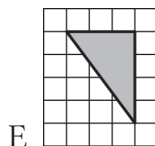
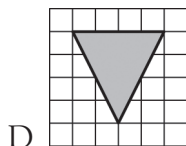
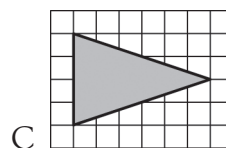
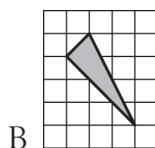
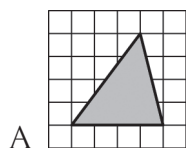
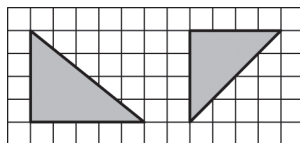


- A 14 cm      B 16 cm      C 18 cm      D 21 cm      E 24 cm



## Four points problems

- 9 Ally drew 3 triangles on a grid. Exactly 2 of them have the same area, exactly 2 of them are isosceles, and exactly 2 are right triangles. Two of the triangles are shown. Which can be the third one?



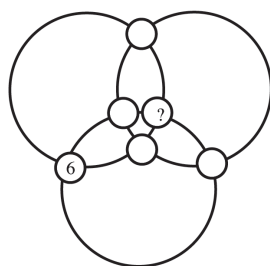
- 10 Lars has found a special number. If he subtracts  $\frac{1}{10}$  from the number, he gets the same result as if he multiplies the number by  $\frac{1}{10}$ . What number has Lars found?

A  $\frac{1}{100}$       B  $\frac{1}{11}$       C  $\frac{1}{10}$       D  $\frac{11}{100}$       E  $\frac{1}{9}$

- 11 Tom had ten sparklers of the same size. Sparklers burn at the same speed along their entire length. One sparkler will burn in 2 minutes. He lit one first. When only a tenth of it remained, he lit the second one, when only a tenth remained of it, he lit the third, and so on. How long did it take for all 10 sparklers to burn down?

A 18 min 20 s      B 18 min 12 s      C 18 min  
D 17 min      E 16 min 40 s

- 12 The numbers 1, 2, 3, 4, 5 and 6 should be placed at the intersections of three circles. The number 6 is already placed. The sum of the numbers on each circle is the same. Which number must be placed at the intersection with the question mark?



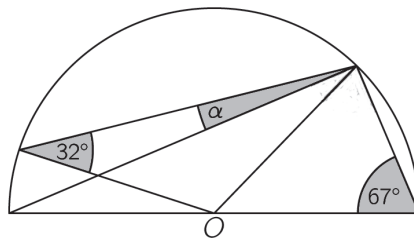
A 1      B 2      C 3      D 4      E 5



- 13 Five boats take part in a sailing competition. The crews are 9, 15, 17, 19 and 21 people and consist of either only boys or only girls. When the first boat has passed the goal-line, the number of girls on their way to the goal is three times as many as the number of boys who have not yet reached the goal.  
How large is the crew on the boat that has passed the goal-line??

A 9            B 15            C 17            D 19            E 21

- 14 The figure shows a semicircle with center  $O$ . Two of the angles are given.  
What is the size of the angle  $\alpha$ ?



A  $9^\circ$             B  $11^\circ$             C  $16^\circ$             D  $17,5^\circ$             E  $18^\circ$

- 15 Five cars participated in a race. They started in the order shown below, A first and E last.



Whenever a car overtook another car, a point was awarded. The cars reached the finish line in the following order.



What is the least number of points in total that could have been awarded?

A 10            B 9            C 8            D 7            E 6

- 16 A  $3 \times 3$  squares initially has the number 0 in each of its cells. We then pick any  $2 \times 2$  subsquare (as for example the shaded one) and add 1 to all four of its numbers. When we repeated such operations several times we got the arrangement on the right. Unfortunately some numbers are hidden. What number is in the square with the question mark?

0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

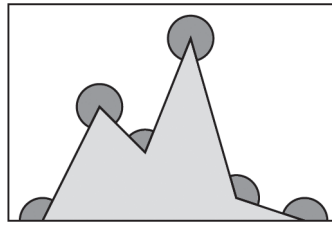
0	18	0
0	47	0
13	0	?

A 14            B 15            C 16            D 17            E 19



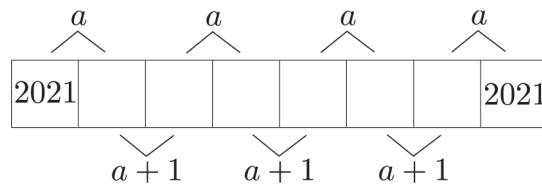
## Five points problems

17 What is the sum of the 6 marked angles in the picture?



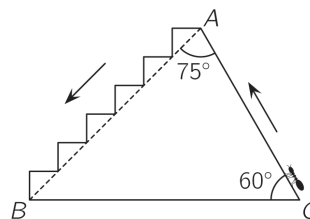
- A  $360^\circ$     B  $900^\circ$     C  $1080^\circ$     D  $1120^\circ$     E  $1440^\circ$

18 There are 8 boxes in the strip shown. There should be a number in each box. Numbers in adjacent boxes have sum  $a$  or  $a + 1$ , as shown. The numbers in the first box and the eighth box are both 2021. What is the value of  $a$ ?



- A 4041    B 4042    C 4043    D 4044    E 4045

19 An ant climbs the hill on the road  $CA$  and descends on the stairs  $AB$  (see the figure). What is the ratio between the lengths of the path  $CA$  and of the path  $AB$ ?



- A 1    B  $\frac{1}{2}$     C  $\frac{1}{3}$     D  $\frac{\sqrt{2}}{2}$     E  $\frac{\sqrt{3}}{3}$

20 If  $a + b + c = 0$  and  $abc = 78$ , what is the value of  $(a + b)(b + c)(c + a)$ ?

- A -156    B -78    C -39    D 78    E 156



- 21 Three girls played a "Word" game in which they each wrote down 10 words. A girl scored 3 points if neither of the other girls had the same word. A girl scored 1 point if only one of the other girls had the same word. No points were awarded for words which all three girls had. When they added up their scores, they found that they all had different scores. Sylvia had the smallest score (19 points), and Janet scored the most. How many points did Janet score?

A 20      B 21      C 23      D 24      E 25

- 22 In the  $4 \times 4$  table some cells must be painted black. The numbers next to and below the table show how many cells in that row or column must be black. In how many ways can this table be painted?

				2
				0
				2
				1
2	0	2	1	

A 1      B 2      C 3      D 5      E more than 5

- 23 How many 5 digit positive numbers have the product of their digits equal to 1000?

A 10      B 20      C 30      D 40      E 60

- 24 Krister has eight coins whose weights in grams are different positive integers. When Krister puts any two coins on one side of a balance scales and any two on the other side of the balance scales, the side containing the heaviest of the four coins is always the heavier side.

What is the smallest possible weight of the heaviest coin?

A 8      B 12      C 34      D 128      E 256