

# Kängurutävlingen – Matematikens hopp 2020

## Student

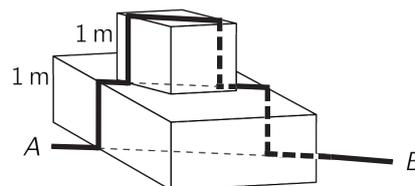


### Three points problems

1 The sum of the last two digits of the product  $1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$  is

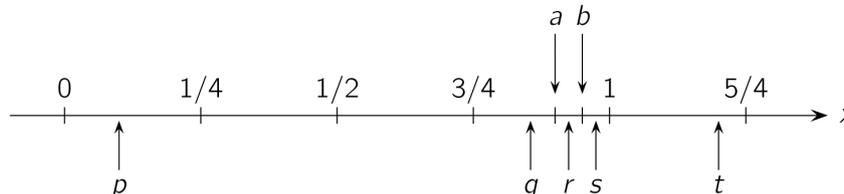
- A: 2      B: 4      C: 6      D: 8      E: 16

2 An ant walked every day on a straight horizontal line path from A to B, which are 5 m apart. One day humans placed on its path two strange obstacles of height 1 m each. Now the ant walks along or above the same straight line except that it now has to climb up and down vertically over both the two obstacles, as in the picture. How long is its path now?



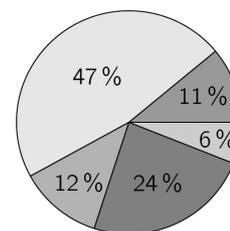
- A: 7 m      B:  $5 + 4\sqrt{2}$  m      C: 9 m      D:  $9 - 2\sqrt{2}$  m  
E: the length depends on the angles the obstacles are situated along the path

3 Rene marked as accurately as possible two points  $a$  and  $b$  on the number line. Which of the points  $p, q, r, s, t$  on the number line best represents their product  $ab$ ?



- A:  $p$       B:  $q$       C:  $r$       D:  $s$       E:  $t$

4 The pie chart shows how the students of my school get to school. Approximately twice as many go by bike as use public transport and roughly the same number come by car as walk. The rest use a moped. What percentage use a moped?



- A: 6%      B: 11%      C: 12%      D: 24%      E: 47%

5 The sum of five three-digit numbers is 2664 as shown on the board. What is the value of  $A + B + C + D + E$ ?

A	B	C
B	C	D
C	D	E
D	E	A
+ E	A	B
<hr/>		
2	6	6
<hr/>		
4		

- A: 4      B: 14      C: 24      D: 34      E: 44



6 What is the value of  $\frac{1010^2 + 2020^2 + 3030^2}{2020}$ ?

- A: 2020    B: 3030    C: 4040    D: 6060    E: 7070
- 

7 Let  $a, b$  and  $c$  be integers satisfying  $1 \leq a \leq b \leq c$  and  $abc = 1000000$ .  
What is the largest possible value of  $b$ ?

- A: 100    B: 250    C: 500    D: 1000    E: 2000
- 

8 There are two dice. Each one has two red faces, two blue faces and two white faces.  
If we roll both dice together, what is the probability that both show the same colour?

- A:  $\frac{1}{12}$     B:  $\frac{1}{9}$     C:  $\frac{1}{6}$     D:  $\frac{2}{9}$     E:  $\frac{1}{3}$
- 

#### Four points problems

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9 Which of the following numbers is not divisible by 3 for any integer  $n$ ?

- A:  $5n+1$     B:  $n^2$     C:  $n(n+1)$     D:  $6n-1$     E:  $n^3-2$
- 

10 Five coins are lying on a table with the "heads" side up. At each step you must turn over exactly three of the coins. What is the least number of steps required to have all the coins have their "tails" side up?

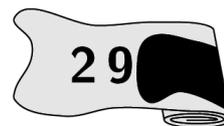
- A: 2    B: 3    C: 4    D: 5    E: It's not possible to have all "tails" up.
- 

11 Let  $a, b$  and  $c$  be integers. Which of the following is certainly *not* equal to  $(a-b)^2 + (b-c)^2 + (c-a)^2$ ?

- A: 0    B: 1    C: 2    D: 6    E: 8
- 

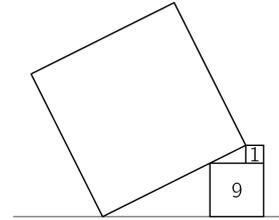
12 The first two digits of a 100-digit integer are 2 and 9.  
How many digits does the square of this number have?

- A: 101    B: 199    C: 200    D: 201    E: It cannot be determined.





- 13 A square is put on two other squares as shown in the picture. The numbers on the small squares show the area of the squares. What is the area of the big square?

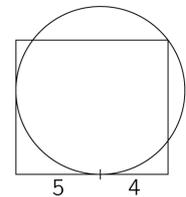


A: 49      B: 80      C: 81      D: 82      E: 100

- 14 The sequence  $f_n$  is given by  $f_1=1, f_2=3$  and  $f_{(n+2)}=f_n+f_{(n+1)}$  for  $n \geq 1$ . How many of the first 2020 elements of the sequence are even?

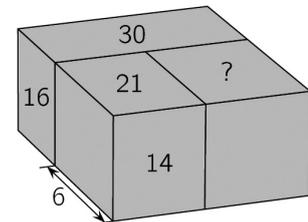
A: 673      B: 674      C: 1010      D: 1011      E: 1347

- 15 We are given a rectangle and a circle that touches two of the sides of the rectangle and passes through a vertex, as shown. One of the touching points is at a distance 5 and, respectively, 4 from the adjacent vertices of the rectangle. What is the area of the rectangle?



A:  $27\pi$       B:  $25\pi$       C: 72      D: 63      E: none of the previous

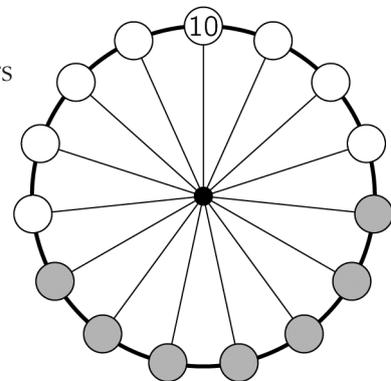
- 16 Three cuboids are arranged to make a larger cuboid as in the figure. The width of one of them is 6 and the areas of some of their faces are 14, 21, 16, 30, as shown. What is the area of the face with the question mark?



A: 18      B: 24      C: 28      D: 30      E: cannot be determined

Five points problems

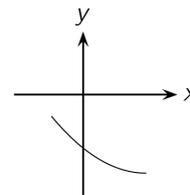
- 17 We have 15 numbers placed on a wheel. Only one of the numbers is visible (a 10 at the top). The sum of the numbers in any 7 consecutive positions on the wheel (such as the grey ones) is the same in all cases. If we add all 15 numbers, exactly how many of the numbers 75, 216, 365, or 2020 are possible totals?



A: 0      B: 1      C: 2      D: 3      E: 4

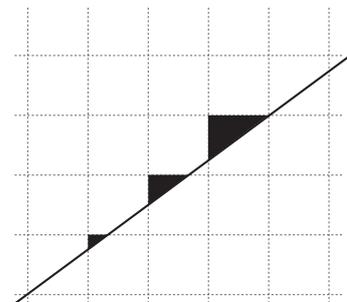


- 18 The figure shows a section of the parabola with equation  $y = ax^2 + bx + c$ . Which of the following numbers is positive?



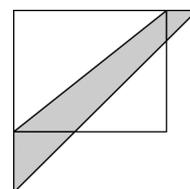
A:  $c$       B:  $b+c$       C:  $ac$       D:  $bc$       E:  $ab$

- 19 A little kangaroo draws a line on a square grid paper and colours in three triangles as shown. Which of the following could be the ratio of the areas of the triangles?



A 1:2:3    B 1:2:4    C 1:3:9    D 1:4:8  
E none of the previous is correct

- 20 The length of one of the sides of a rectangular garden is enlarged by 20% and the other one is enlarged by 50% so that it has become a square garden as shown in the picture. If the shaded area between the two diagonals is  $30 \text{ m}^2$ , what was the area of the original garden?



A:  $60 \text{ m}^2$     B:  $65 \text{ m}^2$     C:  $70 \text{ m}^2$     D:  $75 \text{ m}^2$     E:  $80 \text{ m}^2$

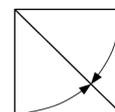
- 21 A large number  $N$  is divisible by all integers from 2 to 11 except for two. Which of the following couples of integers could be these exceptions?

A: 2 and 3    B: 4 and 5    C: 6 and 7    D: 7 and 8    E: 10 and 11

- 22 Tony has 71 marbles at his disposal in a box. He is allowed to take out exactly 30 marbles from the box or to return exactly 18 marbles to it. If Tony is allowed to apply each operation as many times as he wishes, what is the smallest number of marbles than can be in the box?

A: 1      B: 3      C: 5      D: 7      E: 11

- 23 Wajda took a square piece of paper of side 1 and folded two of its sides to the diagonal (see image), obtaining a quadrilateral. What is the area of this quadrilateral?



A:  $2 - \sqrt{2}$     B:  $\frac{\sqrt{2}}{2}$     C:  $\sqrt{2} - 1$     D:  $\frac{7}{10}$     E:  $\frac{3}{5}$



- 24 An iceberg has the shape of a cube. Exactly 90% of its volume is hidden below the surface of the water. Three edges of the cube are partially visible over the water. The visible parts of these edges are 24 m, 25 m and 27 m. How long is an edge of the cube?

A: 30 m    B: 33 m    C: 34 m    D: 35 m    E: 39 m