Validation of adults´ proficiency – fairness in focus

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Content

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Terms

The Swedish term *validering*, which has it´s origins in the French term *validation des acquis de l'expérience* (VAE), has different translations in different contexts and countries. In Great Britain the term *accreditation of prior (experimental) learning* (APL/APEL) seems to be the most frequently used. In Australia and South Africa *recognition of prior learning* (RPL) is used. In USA and Canada the terms *prior learning assessment* (PLA) respectively *prior learning assessment and recognition* (PLAR) is in use. In this report we consistently use the term *validation*. This term is also used in EU co-operation.

Task and Aim

In January 2007, the National center for mathematics education (NCM), at the University of Gothenburg, was commissioned by the Validation Delegation to initiate work on the validation of adult´s proficiency in mathematics.

Given the complexity of validation, and specifically its application to adult mathematical proficiency, and a great need for analyses and knowledge reviews, the work in the first instance focused on documenting the problem areas which would then provide the foundation for more operationally oriented initiatives in the future. This report is the result of our analysis.

In the report, which should be regarded as a position paper, we have analysed and reviewed the problems connected to validation from a number of different perspectives. We have also tried to achieve a synthesis of the ideas and aspects which we believe are essential for developing more concrete methods for organising validation. Our analysis builds on the findings of research from different theoretical approaches, disciplines and traditions, and on policy documents concerning adult learning, as well as many years of personal experience from adult education (Gustafsson & Mouwitz, 2004). The synthesis also reflects an approach to the individual and to knowledge, and thus has an ethical as well as an epistemological dimension.

Special thanks to our excellent translator Brian R. Turner who has helped to make our reflections accessible to a wider audience.

It is our hope that this report will provide stimulus for the further development of knowledge and methods.

Lars Gustafsson & Lars Mouwitz

Vocational proficiency – two examples

The carpenters and the educationalist

Laying a sill-plate[1] for a house, is a small part in the overall scheme of building a house. This does not mean it is unimportant – quite the opposite! A correctly laid sill-plate shows that the tradesman has put in the care and attention necessary for work of good quality. It is also a prerequisite for the later stages carried out by other tradesmen that this work be carried out effectively for the final overall result to be perfectly satisfactory. The

[1] A sill-plate is essentially a horizontal plank, on which the vertical columns ("pillars") of a house rest. A carefully laid sill-plate is a prerequisite to having correct lengths and angles for the framework, and this makes the work process described a critical part of building a house.

following story is an eyewitness account from a building site, and shows how this work can be carried out. It was a pure coincidence that one of the authors just happened to witness the process the story describes. It is anecdotal. The aim of the description is that it should provide a context and serve as a starting point for examining the problems concerning the validation of adult mathematical proficiency.

Background and conditions

Two carpenters, let's call them Rolf and Stig, are going to lay sill-plates for a house on a masonry foundation wall. The bricklayers had been there earlier and laid a foundation where the corners were not perfect right angles. The method they used was to measure and compare the lengths of the diagonals. On a foundation with an approximate external dimension of 5.5 × 15 meters there was a difference of around 15 mm. The bricklayers considered this to be an acceptable deviation and thought that the carpenters would easily be able to adjust for this.

Rolf, the building site supervisor, is the person responsible and manages the process which take around 10 –15 minutes. LG, one of the authors, who describes the process, works at the National center for mathematics education (NCM) at the University of Gothenburg, but has no pretensions to being an expert in the building sector.

Description of the work process

Rolf first runs a quick check using a small set square which he always carries in his carpenter's belt and says he thinks it looks quite good.

The next step is to draw a line with chalk[2] on the top side of the foundation a few centimetres in from the external edge of the long side. The aim of this line is that it should mark where the sill-plate should be laid (this position is in turn determined by the thickness of the finished wall and depends on insulation, studs, plates and boarding). Another reason for this is to check that the foundation is straight. The work, as described below, assumes that it is straight and that the chalk line is correct in relation to the foundation.

After this, he measures up a length of 1 meter along the long and short sides. The measurements are made from one of the corners of the foundation

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[2] A chalk line reel is an essential instrument in the carpenter's toolkit. It consists of a container with coloured chalks and an extendable cord. The cord is drawn out between two points, tightened and then released with a flick. The result is a coloured straight line. The instrument has been known from Egypt since 5000 years ago, and also from other cultures.

along the chalk line, and a corresponding preliminary line along the short side. The sill-plate along the long side has been fixed with nails, but is loose on the short side. It is obvious that this is the one that will be adjusted. With the help of a carpenter's rule, Rolf states that the diagonal deviates from the 1.414 meters which is the rule-of-thumb he uses.

Using this, he scales up the measurements. Now the carpenters take a 5 meter measurement along the short and long side, and the process is repeated.

Rolf turns to me and asks: "You're the professor of mathematics. What should it be now?"

Somewhat uncertainly, I started doing the calculations in my head: "5^2+5^2 and the square root of this, a little more than 7"[3]. Rolf is not satisfied with this. He wants a more exact measurement, and I decide to go in and get a calculator to get an exact figure, and just in case any more calculations are necessary. Then Rolf, who wants an exact and fast answer, says: "What's 1.414 times 5?" and begins to work this out by setting up a multiplication algorithm on the sill-plate. He has the answer ready before I have even got as far as the house!

Now a long process starts with lot's of measurements and thinking. During the process some of the comments were: "Somewhere, there's something wrong, but what the hell is it?" "Why isn't it working?" "It should be right, if we pull the sill-plate outwards". It was not clear to me what the problem was, but obviously the corner was not at right angles and for some reason repeated measurements and resulting chalk lines gave different results. Carpenters "think" in an ongoing dialogue with each other, at the same time measuring, drawing and adjusting the loose sill-plate in an iterative process of thinking, responding, discussing and measuring based on new results. For an outside observer, it looks as if thinking, dialogue and action are inseparable in this process.

Suddenly Stig says: "Now, this is good enough. This tiny difference won't be noticed". Whereupon Rolf, quick as lightning, replies: "Hell no, it should be right".

The story ends here, but in real life it continues because the sill-plate is now in place and fulfilling its intended function.

[3] The correct measure using the accuracy Rolf needs is 7.07.

Interpretation perspective

If you have a background, as the author in this case, in the field of mathematics education it is easy to see the connections to, and interpret the process described from a "school-mathematics" perspective. This is based on a more or less explicit view of knowledge and on assumptions as to what mathematics is and what typifies the application of mathematics to concrete situations. Our conviction is that there is a risk in using such assumptions, and that is because interpretations will be excessively narrow and essential parts of adult proficiency will be overlooked. In reality, we see that the process of investigating, interpreting and describing the "translation" of vocational proficiency or informal proficiency into a more formal description is the greatest challenge we face in attempting to describe the complex proficiency which adults possess. To "identify" mathematics from a narrow school perspective in a validation process is thus both a risky and doubtful process if in some deeper sense, we are to understand, describe and document what is taking place in a way that adequately represents the individual's knowledge. The section on *Propositional knowledge and praxis knowledge* contains a more exhaustive analysis of this problem complex.

Below, despite the risks mentioned above, we will make an attempt to describe the carpenter's proficiency from a "school perspective". The aim of this is to show that carpenters actually possess substantial "mathematical" proficiency covering a range of mathematical objects, methods and strategies which they use to solve the problems they are confronting in an effective and flexible way. One aspect worth giving prominence to is the fact that carpenters in their praxis de facto reveal a strong belief in "mathematics" as an effective tool for solving problems. Whether they then view this as "mathematics" is an open question.

A school mathematics perspective

In the story about the carpenters, we can identify not only specific knowledge areas and general competencies/skills, but we can also see the connections to the syllabuses that steer adult education. We can also find, possibly less obviously, phenomena in the story which we recognize from research studies into the use of mathematics and mathematical proficiency in vocational life. The following provides some examples to illustrate some of these relationships.

Objects – physical and mathematical

For anyone who has ever visited a building site, it is obvious that it is a highly "physical" environment. Those working there are working with physical objects such as planks, tools, measuring instruments etc. In the story we can also bring into our interpretation the use of mathematical objects (numbers, geometrical forms, angles, lengths, coefficients, coordinates) of a more abstract nature. We see examples of numbers from different areas (whole numbers, rational numbers) and also in symbolic and geometrical representations (decimals, fractions, concrete and abstract figures). We also see how the carpenters carry out their work in an interesting interaction between tools and thinking where the tool "communicates" with the user and vice versa. There are grounds for reflecting over where proficiency is localised, the importance of the tools and how the body participates[4] in the praxis of the carpenter and what consequences this has when we choose validation methods.

Mathematical operations

From the story, we can see that actions are carried out – physical and cognitive/mental – of different kinds, and with the help of mathematical tools. Examples of such actions are counting (use of algorithms), measurements, and comparisons. Taking a wider perspective of the concept of mathematical operations, we can include such generic competences as judgement, assessment, evaluation, estimates of reasonability etc. The latter are probably critical components in the tradesman's proficiency and identity. What methods can be used to make these competencies transparent?

Mathematical relationships

In the story about the carpenters, we see that the tradesman and the mathematically trained observer "see" reality in different ways, and they use different strategies to solve the practical problems that occur. One example is the situation when placing the sill-plate with great precision, and when Rolf scales up the measurements to 5 metres.

The strategy for the mathematically trained is to relate the problem to Pythagoras' theorem, using the relationship between the sides of a right angled triangle and its hypotenuse. This well-known theorem has properties which are highly valued in school mathematics. This is a *model* with *general* applicability, i.e. it is independent of the concrete context in which a task is to be solved[5]. A price that has to be paid for these qualities is that the model becomes abstract and may be regarded by the uninitiated as

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[4] One person who has reflected over "the lived body's" importance and the unity between objects and the body is the French philosopher, Maurice Merleau-Ponty.

[5] In the section on *Knowledge and proficiency*, we look at the problems connected with this ideal.

unnecessarily cumbersome and difficult to apply, or perhaps in the extreme case somewhat terrifying. The special *symbolic notation* used in the language of mathematics undoubtedly strengthens this feeling.

The carpenter's repertoire of strategies includes what are usually called *rules-of-thumb* e.g. knowledge that the lengths measured along the short and long sides of the foundation, the diagonal in this case, is obtained by multiplying the length of 5 metres, by the number 1.414[6]. As shown in the story, this method provides an accuracy which is sufficient.

In this example, we see that the praxis of the carpenters can be related to a mathematical model of which they are unaware, and which they do not need any knowledge of. Their focus is on solving a practical problem and manufacturing a product as efficiently and safely as possible, not to understanding the underlying mathematical theory.

A view that is not unusual amongst those with a background in mathematics education is that the informal proficiency we have observed has limited validity in the sense that it is not transferable to other contexts. In order to clarify this objection for discussion, we can supplement this by looking at an event that happened at an earlier stage in the building project when a concrete mould was being made for the foundation. To align the angles in the corners, measurements were made of 3 and 4 metres, along the long and short sides respectively. The length between the end points of these lines was adjusted to 5 meters, so that the intervening angle was a right angle. The method has been known for thousands of years as the *Egyptian triangle* and it is a part of the standard repertoire of carpenters. Based on a school-mathematical perspective, it is once again easy to relate this method to Pythagoras' theorem, or more concretely expressed as a consequence of this theorem. When pointing out to the carpenter that his method is related to Pythagoras' theorem, he answers: "God knows. The main thing is it works."

A critical question is whether the carpenter's praxis in the two cases described constitutes an application of a mathematical model of a propositional nature? If it isn't, then what is it? Can it be the case that the tradesman has a repertoire of strategies such as this, and in a precise and flexible way, using analogical thinking, is able to adapt and vary the strategy in terms of the conditions and requirements dictated by the context?

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6 Mathematically, this means that the carpenter uses a linear relationship which is not trivial, and where it is not absolutely evident that it is related to Pythagoras' theorem. Pythagoras' theorem gives us:

$$5^2 + 5^2 = x^2 \rightarrow x^2 = 2 \cdot 5^2 \rightarrow x = \sqrt{2 \cdot 5^2} = \sqrt{2} \cdot 5 \approx 1,414 \cdot 5 = 7,07$$

A quotation (Kent, Hoyles, Noss, & Guile, 2004) from one of the leading research groups today in the area highlights some of the aspects we have touched on above:

> *It has been evident since the 1980s from studies of mathematical practices in workplaces that most workers use mathematics to make sense of situations in ways which differ quite radically from those of the formal mathematics of school and college curricula. Rather than striving towards consistency and generality – the hallmarks of "mathematical thinking" as conventionally conceived – what emerges from studies in workplaces is that people develop mathematical techniques to carry out their work which are generally strongly "situated" in their knowledge and experience and which exploit features of the context and its local regularities. These techniques are preferred because they are often quicker and more efficient than general mathematical techniques. Yet it is evident from looking at work experienced employees that a "generalised" mathematical ability which operates across contexts can emerge through experience in particular contexts.*

The educationalist in the sheet metal workshop

The mathematics used by carpenters does not constitute a single isolated proficiency, but is often combined, as we have seen, with a cluster of *generic competencies*. According to the Hoyles research group (Hoyles, Wolf, Molyneux-Hodgson, & Kent, 2002) the use of mathematics in vocational life is integrated with other competencies such as communication/ language competencies, use of ICT, judgement etc. They refer to these as "hybrid skills". Many professions, as a result of developments in technology, have undergone rapid change in recent years. This is not obvious in the story about the carpenters. An occupation in which this process has come further is that of the sheet metal worker. In this we see examples of traditional proficiency surviving at the same time as the industry changes with the introduction of new technology. Both the old and the new are necessary, and complement rather than compete with each other. The article *Gäst hos verkligheten. Nämnaren besöker plåtverkstaden* (*Guest of reality: Nämnaren*[7] *visits the sheet metal workshop*) (Gustafsson, 2005) deals with this theme.

[7] Nämnaren is a Swedish journal in mathematics education.

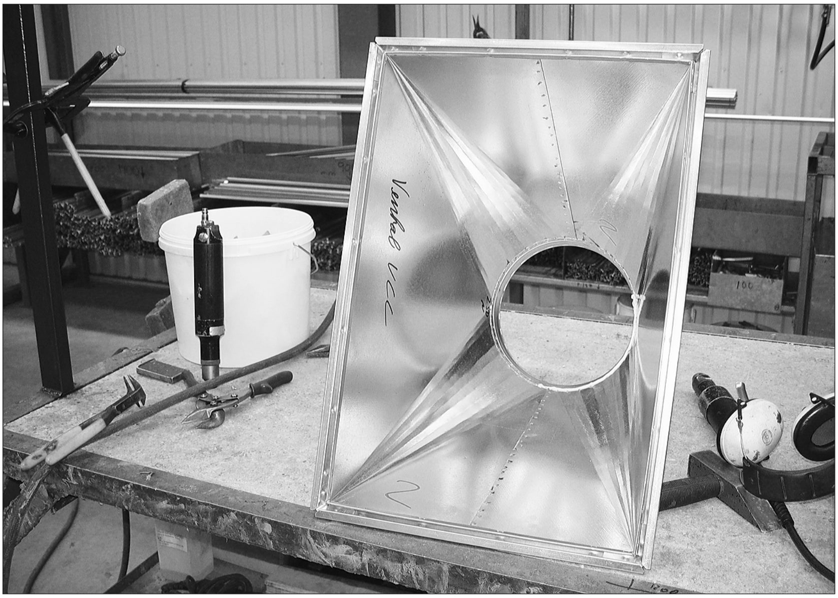
The visit to the metal workshop had two main aims:

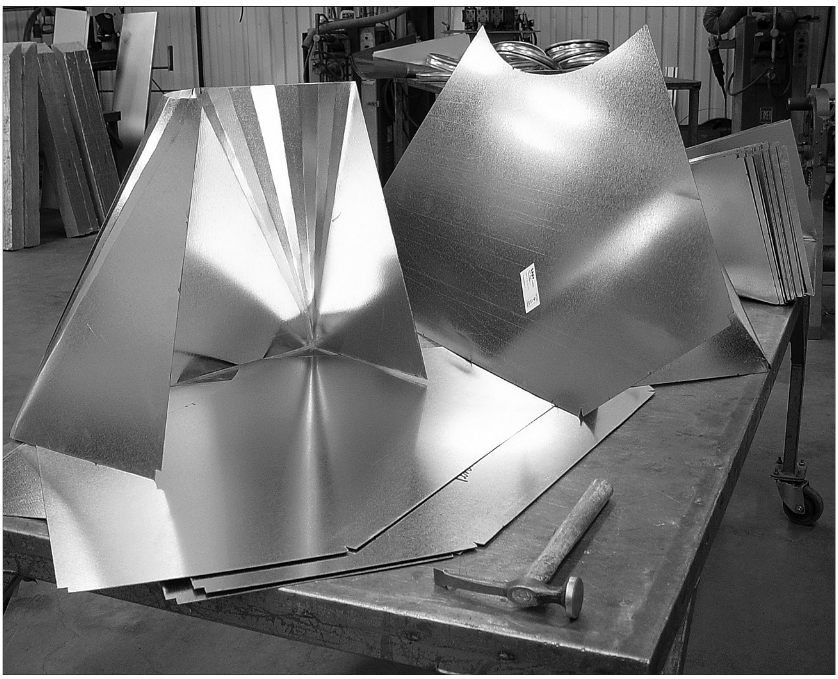
– an attempt to determine in a dialogue with the vocationally active what mathematics they think they need, what mathematics they actually use, and also how they look at the value of mathematics in carrying out their work,
– the ability from an external perspective and with "mathematical eyes" to try to identify the use of mathematics at the workplace.

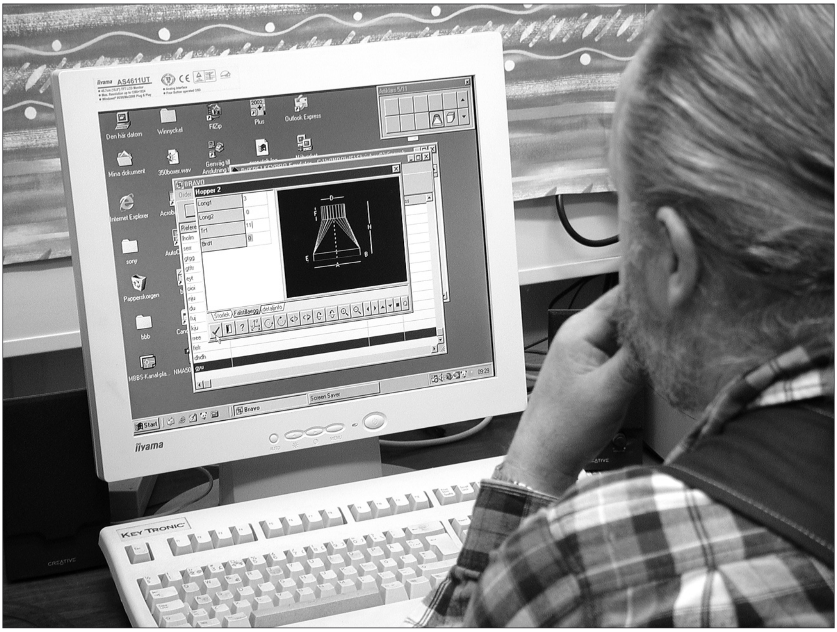
We return to the story about the visit to the sheet metal workshop, albeit in a somewhat modified form:

When I (Lars Gustafsson) go into the workshop, I am met by Anders who will be my guide during the visit. The first thing I notice is a number of metal parts at the entry to the workshop ready for delivery. I was fascinated by all the mathematical forms represented there.

Tage is standing at a bench and making a connector between a rectangular ventilation duct and a circular pipe. This connector is made out of two metal plates. [See photos on page 11.]



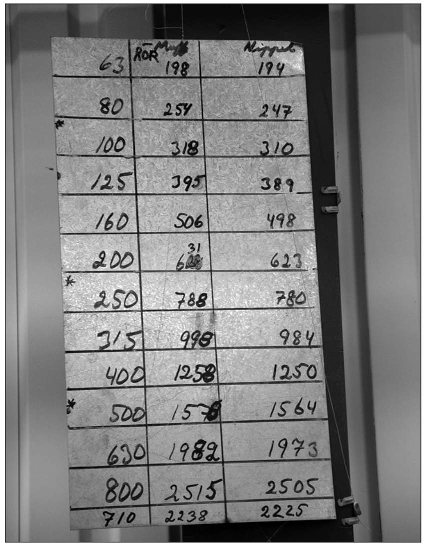




In response to my question of how these pieces are cut and what mathematical knowledge and competence is required, the answer I get is: "The computer does it". At the computer the design is created using a special computer programme where he enters the required values.

After this he takes a diskette to the sheet metal cutter who cuts and optimises the plates. Anders thinks that this doesn't require any mathematics. The competence needed is correct use of the tools, machines and computer programmes. This does not mean that the mathematics is unimportant. From having been earlier a part of vocational proficiency, it is now built into the computer software. Parts of vocational competence are thus transferred from the tradesman to the computer programmer. In this process, a change in vocational proficiency takes place. Some things disappear, others arrive. This could possibly be regarded as a loss of knowledge and skills.

In the next stage, the two metal plates are bent and joined together, an act of pure craftsmanship. The metal sheets should be repeatedly bent in the corners. The number of degrees, in this case 16, is related to the diameter of the pipe it should fit into. "You do it almost by instinct and then check to see that it fits, otherwise you adjust the number of degrees so that it becomes correct", says Anders. In other words, this is a moment of judgement and experience, where practical vocational proficiency is used. In this



phase, we can't see any signs of traditional school mathematics i.e. where you make calculations and measurements. This is one example of experiential practical proficiency which the work requires and which I see a number of examples of during my visit.

Above the plasma cutter, there is a sheet of metal hanging with a handwritten table.

The measurements in the left column represent the diameter, and the two other circumferences for the circular pipes which should fit externally and internally. We see that the values do not strictly correspond to those we would expect from the formula for calculating the circumference of a circle. Some values have also changed. Anders explains that the measures are based on those of the spiral pipes when they are delivered to the company, and that sometimes these measures can be changed somewhat. These types of tables, templates or "rules-of-thumb" are often used and are known from other studies of vocational mathematics and they have been developed for a number of very good reasons. One is to save time and simplify the work, making it unnecessary to measure and calculate every time a part is to be manufactured. Another reason is that it can be part of a quality assurance process. If the instructions are followed, then it will be correct.

When I say I think I can see in this work evidence of the use of substantial mathematical proficiency, the response I get from Anders is that the mathematics he needs or uses is fairly superficial. "The formula for the circumference and simple addition is all that is needed" he says. Apart from measuring the diagonals in order to make sure we have right angles.

In the concluding discussion, I get to know from Sven how the work was carried out before the advent of computers. Sven has worked with sheet metal the whole of his vocational life. He has witnessed many changes and he possesses great proficiency. Not without pride, he shows me his "bible" called, *Marking up and preparation*. This handbook has over 117 diagrams and descriptions on how to mark up different shapes that are to be produced. To an outsider, the complexity of these descriptions is striking. Sven tells the story about his younger days when sitting at home by the kitchen table on paper he measured, drew, folded and later at the workshop produced most of the shapes in the handbook. Parallel with his vocational pride, I think I could detect an element of nostalgia over the lack of traditional vocational proficiency which today is on the point of disappearing. The example of the role of the computer illustrates and possibly provides an explanation for this change.

My conclusion from the visit is that mathematics takes a prominent place in sheet metal working. I see algorithms, formulae, calculations, geometrical forms, tools and mathematical activities wherever I turn. In contrast to this is the picture the vocationally active give us that there is not very much mathematics involved, and that a sheet metal worker does not need to have any special mastery of mathematics. This paradox can perhaps be explained, not only by the fact that the mathematics and competence is not visible – or a lack of awareness, but also by what one defines as mathematics and its use. It also occurs to me that there are different forms of knowledge that come into play at a workplace such as this, and in a school context. In the text I have described two examples of this. One is the procedure for measuring and comparing diagonals in order to determine right angles. In school mathematics, this is presented in the form of abstract symbols in formulae, in this case Pythagoras' theorem, which is then applied to carry out the calculations. The second example is the transition from a rectangular to a circular form. Calculating this by using a formal mathematical method is far from trivial. The methods described are based on experience and practical implementation, almost subconsciously. This is not trivial, and it does represent a different type of knowledge. Based on these two examples, one can ask what methods are most effective and whether school mathematics has anything to learn from this.

I left the workshop with great respect for the vocational competence I had just witnessed, and with more questions than when I originally arrived. A visit to a workplace such as this raises a number of questions concerning the content of school mathematics and what it means to be proficient in mathematics.

Knowledge and proficiency

Learning environments

Adults are active in many different arenas: as citizens in a democratic society where the demand on individuals is not just that they should be able to receive information, but also that they are involved in shaping the society of tomorrow; as consumers/users in an increasingly deregulated and globalised market; in working life scenarios that impose a set of demands different from those required earlier, and where the demands themselves are changing at an increasingly rapid rate; in family, organisational and recreational life, and not least in studies, competence development and voluntary educational activities of different kinds. It is these arenas which are described in policy documents as formal, non-formal, and informal learning environments.

The increasingly higher demands imposed on the individual not only provides opportunities, but also entails risks. In the society described above, one can look at the individual's situation from different perspectives. One perspective is that individuals have to confront demands and challenges which means that they must always be prepared to learn, re-learn and expand their learning. On a policy level, these demands on the individual have been launched under the term *lifelong learning*. Those who do not manage to fulfil the requirements this imposes run the risk of being marginalised.

Another perspective is based on the fact that adults, in the arenas in which they act, continuously develop strategies and acquire knowledge, proficiency and competence. It is this proficiency which enables most of us to manage the challenges and requirements we confront. In connection, for instance, with international comparative studies[8], we hear that some

[8] Examples of this are the Second international adult literacy survey (SIALS) and the Adult literacy and lifeskills survey (ALL).

segments of the adult population are not able to manage their lives and the challenges they confront in the post-modern society. Sometimes middle-aged men in sparsely populated areas in the middle and northern parts of the country are referred to and identified as a "problem". Another, and possibly more respectful and fair way of viewing these individuals, is based on the notion that a fundamental prerequisite for managing in an often demanding, complex and changing environment, is that individuals in a flexible and competent way can use the proficiency they possess. The picture of a *competent adult* possibly provides a more accurate and constructive view of adults' skills. Perhaps adults also have other life and identity projects that do not involve being part of the construction of the new Europe where the ambition is to "become the world's most competitive and dynamic knowledge-based economy" (European Commission, 2005)? Society's interest in adults' overall proficiency from this perspective may possibly be interpreted in a narrow instrumental way where the focus is on the need to make visible and operationalise this proficiency for the purpose of achieving the goals set up? Perhaps it is also the case that the questions we put determine the answer we get. The tools used to assess the competence of the adult population can be based on notions of knowledge and proficiency which do not adequately recognise and document the individuals' competence.

Learning environments and lifelong learning

Learning environments have, as has already been mentioned, come to be regarded as a generic term for the arenas described above. A basic idea is that both from an individual and a societal perspective, it is valuable to make transparent and effectively use the overall proficiency the individual possesses. An instrument for achieving this is *validation*.

Lifelong learning has been described as consisting of two dimensions: the *lifelong* and the *lifewide*. The underlying idea of lifelong learning is that learning is not just limited to the years spent in school. In a rapidly changing society, there is a need for ongoing learning and re-learning. The term "lifewide" learning is used to refer to learning that takes place in different learning environments. This learning has been categorised in terms of *formal*, *non-formal* and *informal* learning. Descriptions of these categories can be made, for instance, on the basis of the extent to which learning is structured and organised, as well as in terms of the learner's intentions. The following provides a short résumé of the meaning of these concepts[9].

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[9] These definitions are partly based on definitions from the European centre for the development of vocational training (Cedefop).

Formal learning

Formal learning covers learning that takes place within the framework of the formal educational system. In the first instance, this is what we think of when we speak about education and it covers: school-age child care, preschooling, compulsory school, upper secondary school, adult education and higher education.

Formal learning takes place in an environment that is organised and structured for learning. It also takes place in a context which is specifically created for learning. The learner takes part in an activity for the express purpose of learning.

Non-formal learning

Included in non-formal learning is what takes place in organised education, e.g. labour market training, in-service training and competence development in private and public sectors. Popular adult education is a part of this, i.e. folk high schools and adult education associations, and other types of courses.

Learning takes place in activities that are not necessarily and explicitly created for learning specific contents, but which nevertheless, more or less explicitly, contain an essential element of learning. Similar to formal learning, there is in this learning environment an intention to learn on the part of those participating in the activities.

Informal learning

Informal learning is learning that takes place through individuals participating in a broad range of situations covering family life, everyday life, recreational and organisational life. Learning takes place as a "secondary effect" of participating in these contexts. It is not organised and structured for the express purpose of learning, nor is the focus on the "learner's" perspective. Nevertheless, research shows that a large part of adult learning takes place in informal learning environments.

Reviewing the problem complex

Categorisation of learning as *formal*, *non-formal* and *informal* takes place, as shown in the descriptions above, mainly from an organisational framework. These are concepts which, without being defined, are widely used in national and international policy documents where they are transformed into *policy concepts* that are charged with different meanings and, more or less explicit, ideological assumptions. In an epistemological discussion, they then become somewhat problematic since they don't reveal anything about the nature of knowledge.

One way of characterising informal learning which says a little more about its nature and which also gives a clue as to its complexity and the challenges involved in assessing adults' informal proficiency has been put forward by the French researcher, Michel Feutrie (Feutrie, 2007):

The main problem is that this learning is:

- *not formalised, less codified than traditional knowledge*
- *not organised as traditional knowledge in disciplines, domains, ...*
- *rather unconscious*
- *hidden in action*
- *contextualised, attached to a specific environment*
- *built of elements more or less coherent, specific to an individual*

We will return to some of these aspects later on in the report.

Theory and practice

Introduction

A common view is that practice is applied theory. This, however, is only one special type of practice, typical of science and technical applications. At least equally common is that practice lives its own life, without the support of scientific models and formalised language. Scientific application is, in addition, dependent on the adaptation and implementation of "silent" practice which is not formulated in the model itself.

Models and applications

The knowledge ideal of our era is *theoretical* and intimately related to the concept of a *model*. A model is an *abstraction* formulated with the support of one or more examples. The modelling phase involves removing the concrete which is considered to be unnecessarily complex and also the distracting deviations. A model thus always represents a *loss of reality* and implicitly a *view* since it is based on a view of what exists, its structures and connections, how these interact, and what is considered to be essential. A model is perceived as *general*, i.e. it claims applicability to a variety of new situations, it should be able to approximate reality, and be applicable to the complexity of specific cases in the future, as well as able to explain or forecast concrete future events. A model is explicitly formulated, e.g. mathematically formalised, to enable its meaning to be communicated through education.

Since the model should be able to explain the complexity it has been extracted from, some practical complications arise. In many cases, successive adaptation between the model and the individual case is required for its application to be possible. Sometimes a real situation creates such intensive "resistance" that the model must be revised or rejected. If the problem has to be resolved quickly, the model must be replaced by the hands-on knowledge and skills of the labour force, as when an unanticipated error suddenly occurs, for example, with a nuclear power facility.

Practice and analogy

Practical proficiency developed through more or less practical actions does not fit comfortably into the model domain. The boat builder, the carpenter and the sheet metal worker do not develop theoretical models in the same way as a physicist, chemist or economist does. The proficiency that a tradesman possesses is of a more *analogical* type. The analogies are not abstract, but rather consist of a number of *concrete* examples, analogically connected with each other from the practitioner's repertoire of past experience. In this way knowledge becomes highly personal and often unformulated. The examples may retain their complexity. Each new concrete situation is compared and related to earlier concrete examples and these examples have more or less *general applicability* without claiming the generality of a model. Analogical proficiency is related to the person, situation and complexity, and is not easily transferred through formal education. Instead it must be *demonstrated* rather than formulated, and the classical method of conveying such proficiency is through a master-apprentice system.

Theoretical knowledge is largely dependent on practical proficiency, formalised knowledge comes to life and becomes meaningful in the confrontation with the concrete. On the other hand, practical proficiency can to some extent be transformed into theoretical models or even be replaced by them.

Use of mathematics in the school world

In school situations, attempts are often made to solve mathematical problems of a purely theoretical nature, i.e. they deal with a world of mathematical concepts. Sometimes attempts are made to link these to reality, but these become "cosmetic" since not only the starting point, but also the aim, is to solve what is essentially a mathematical problem. Practical reality is thus used to illustrate a *theoretical* problem, as opposed to formulating a practical one. It is also common that focus is put on pupils demonstrating a particular theoretical *method* of solving a problem, a method which in the problem context is perhaps unnecessarily advanced and cumbersome.

It is also worth considering that this practical reality does not exist as such, but only as an artificial interpretation, a virtual and theoretical "school reality". The pupils' *real* practice is to sit at their desks, and satisfy the specific requirements on how theoretical knowledge should be presented orally and in writing.

Sometimes concrete aids such as plastic or wooden cubes are used. The aim of this, however, is also *abstract*, and you only "touch down" in the concrete world for a brief landing. Concrete "tools" are not used to process a concrete reality, but instead to represent a theoretical activity. The actual arrangement of the cubes has no practical relevance, and they are discarded as soon as the theoretical problem is solved.

Other types of aids such as calculators and computers are different in kind, as they are not used to illustrate theoretical reasoning, but as tools to *replace* the person making the calculations. Computers can with greater speed and precision in their calculations carry out theoretical work that earlier required significant brainpower. In such cases, however, the theoretical school domain also determines the nature and purpose of the activity.

Despite claims for generality, it can be argued that theoretical mathematical education is just as context dependent as other mathematical activities in vocational life. A strong indication that this is the case is the relative helplessness that people with purely theoretical backgrounds initially demonstrate at a workplace. New aims, methods, strategies and evaluations of results must be identified and internalised, much that was valued and encouraged in the school environment lacks to varying degrees immediate relevance at the workplace.

The role of mathematics in practical proficiency

The aim of mathematics in practical proficiency is more instrumental. Now mathematics is the intermediary: not just the problem, but also its solution is of a *practical* nature. From this, it follows that the mathematical reasoning most often carried out is in the form of simple rules and approximations. The heights of mathematical theory, advanced use of methods or extreme precision is of little relevance or value in solving practical problems. Here there is a *rationality of practice* which is just as effective as the theoretical rationality used for theoretical problem solving. In many cases, problems are solved on the spot immediately, in physical interaction with the surroundings. Withdrawing to a different setting to carry out calculations becomes both cumbersome, time consuming, costly and unnecessary. In the practical application of mathematics, there is no need for mathematical proof or internal theoretical consistency, practical usability is the criterion for "truth" and relevance of the methods used.

In addition to mathematics in the form of rules and methods in practical proficiency, there are also mathematical models incorporated in e.g. computer programs. One example mentioned is how a successful sheet metal worker today must both master a long established trade tradition, and at the same time understand how to handle a computer and various drawing and spreadsheet programs. Usability in this context is also the point, not the underlying mathematical theory in the software.

Both as regards validation and instruction of adults with vocational experience, it is important to take into account different types of practical mathematical proficiency. Here we provide some examples:

- Knowledge of the rule of thumb type, which from a mathematical viewpoint can be regarded as a special case or an approximate application of theories or methods.
- A general belief in the usability of mathematics for practical problem solving.
- The ability to make judgements and realistic requirements for precision in the use of numbers and forms.
- Qualitative competencies, e.g. the ability to reason using scales and proportions, or the capacity to represent figures in three dimensional form.
- Knowledge of calculations and algorithms, i.e. knowledge of how to handle formulae and carry out calculations.
- Knowledge of how mathematical models can be applied, and the calculations for using these.
- Knowledge of different software with mathematical content, and how the programmes can be used e.g. CAD and calculation programmes.
- Knowledge of different forms of representation and how they are to be interpreted, e.g. linear and pie-charts, tables and formulae.

In many cases instruction can be linked to the proficiency the adult already possesses; for instance working as a carpenter with the number 1.414 "opens the window" to Pythagoras' theorem, linear models and irrational numbers in a possible theoretical education process.

Another form of "window opening" is when a dilemma occurs, which ordinary rules of thumb cannot resolve, but where a more general method could provide a solution. Practical proficiency, however, is most often intimately related to its practical application and traditional theoretical validation takes the adult back into a "school context" which can be both confusing and somewhat humiliating.

Ethics, aesthetics, dialogue and cooperation

Practical proficiency is person-related and often forms a part of the adult's identity. Knowing one's job is a source of self-esteem and vocational pride, and leads to the desire to do a "good job" which has both an aesthetic and ethical dimension: taking responsibility for ensuring that the result is good, and corresponds to the customer's or employer's quality expectations.

Loosing one's job can lead to an identity crisis, which is further aggravated if the adult's vocational proficiency is not identified, validated and taken advantage of in future educational or vocational situations.

Much practical work is carried out in teams where communication and the ability to co-operate is an important competence. Sometimes joint initiatives are taken putting high demands on discipline, planning and coordination. It is also possible to see that the tools used "speak to" the user and vice versa. The tool becomes an extension of the body in a continuous interplay with the work situation. In some industries, there is also a master-apprentice trainee period, or where a new employee merely functions as an observer, and the person with experience demonstrates and talks whilst the trainee imitates and puts questions.

The above are important aspects of practical proficiency, often involving some mathematical content, aspects which have very low priority in "school mathematics". In practical application in the real world, thought and action, tools and materials, quality and responsibility, identity and co-ordination together form an integral whole.

Propositional knowledge and praxis knowledge

Background

In Arbetslivscentrum (the Swedish centre for working life) during the 1970s, there was an intensive discussion on the meaning of vocational proficiency in relation to contemporary research into working life at that time. The latter basically focused on *research into qualifications*, i.e. research into the qualifications an individual needed to be able to carry out a specific work task. In the first instance, the findings showed that vocational proficiency appeared to be an *application* of specific advanced theoretical knowledge. Attempts to theoretically describe different work tasks produced, however, only marginal success, as they were often misleading or counterproductive.

The skill and familiarity typical of well-established vocational proficiency appeared to be quite different from what could be "caught" in theoretically formulated models and rule systems. This insight gradually led to the development of a completely new research area, *Yrkeskunnande och Teknologi* (*Skills and Technology*) under the supervision of professor Bo Göranzon at KTH (The Royal institute of technology). A number of philosophers, amongst others, Bengt Molander, Tore Nordenstam and Kjell S. Johannessen at the same time worked on trying to analyse the underlying theoretical knowledge complex. The latter in particular has had a major impact on this research and this section is primarily based on this analysis (see e.g. Johannessen, 1999). It may be worth mentioning that other research environments, which from completely different theoretical and methodological starting points, focusing on the relationship between theoretical and practical proficiency, have come to conclusions which essentially coincide with the analysis we present here. Examples of this are narrative research, activity theory, situated learning and socio-cultural theories.

Praxis knowledge and mode of articulation

Fundamental to the analysis of vocational proficiency is the concept of *praxis*, which was inspired by the later work of the philosopher Wittgenstein. Based on this there is, in this tradition a discussion on how concepts are formed, used and transferred. The primary means by which vocational proficiency is expressed is through its *practical application* and not through *description*. This is why it is sometimes tempting to claim that vocational proficiency is "tacit" and thus necessarily "owned" by a specific individual and not transferable to others. But some elements of vocational proficiency can be articulated through language, and others by *modes* other than the purely verbal.

To demonstrate an appropriate practice of a profession is the specific *mode of articulation* for vocational proficiency. Proficiency can thus be handed down from one generation to another, i.e. transmitted, but in the first instance this does not take place *theoretically*. Instead it occurs in concrete working situations where the expert through concrete examples *shows* how the work is to be carried out. A typical form for such generational transmission is the traditional master-apprentice relationship.

The more abstract and formalised a language is, the more inappropriate it becomes as the mode for articulating descriptions of practical vocational proficiency, which in the first instance are based on personal experiences and examples from concrete and complex working situations. Since the mathematics taught in the formal education system in school and university has this characteristic, validation thus faces a special set of problems. If the validation criteria are based on such a view of mathematics, mathematical praxis knowledge will remain invisible.

Tacit knowledge is thus not absolute, its scope is dependent on which mode of articulation is considered legitimate. In a society increasingly permeated by abstract formalised language, praxis knowledge tends to be marginalised, not only is it "silent", but it is also "silenced". There is an undoubted risk that mathematical proficiency embedded in praxis is not recognised. We thus raise the question of whether such mathematical proficiency could possibly be articulated and validated by means of modes other than traditional mathematics tests in school. Such a validation instrument would be of great value both from an economic perspective and an individual perspective, in terms of the individual's self-esteem and vocational pride.

Propositional knowledge, practical knowledge and knowledge by familiarity

In connection with the use of language in science and bureaucracy, and its application in the formal education system, knowledge has increasingly come to be identified as what is expressed through language. The requirement that knowledge can be formulated linguistically has become a necessary element in all assessments of knowledge. Showing that one "knows" has become equivalent to formulating *statements* that can *be verified*. This is in contrast to praxis knowledge where proficiency is demonstrated by *carrying out* a practical task leading to the desired *result*.

Propositional knowledge transmitted via verbal communication, in writing and orally, dominates the formal education system. As a contrast praxis knowledge requires other modes of articulation and is transmitted by other forms of human interaction.

Praxis knowledge itself can be viewed from two different perspectives: as *knowledge by familiarity* and as *practical knowledge*. The former is about the degree of familiarity the individual has with the nature of the environment, and the latter is about the individual's capacity to act successfully in this environment, e.g. a carpenter's familiarity with materials and tools[10] and skills in applying these in practice. Praxis knowledge has also aesthetic and moral elements, in addition to the more factual and functional, e.g. the carpenter's desire and ability to do a "good job" which should also look "nice". Praxis knowledge is closely related to self-esteem and vocational

[10] The relationship between knowledge/proficiency and cultural tools, artefacts, is an example of a recurring theme in studies taking different theoretical approaches.

pride, which further strengthens the humiliation that many vocationally experienced adults feel when tests are used to measure "impersonal" propositional knowledge.

Propositional knowledge in formal education

Typical of propositional knowledge is that terms are defined by using other terms. Ultimately, one must move beyond the language aspect, and language then represents a special form of *action* in a broader context of other actions, which provide the specific language utterance with meaning and relevance. Such knowledge statements are thus context dependent, the meaning of the statement is dependent on non-linguistic foundations. Trying to hand over, or transfer propositional knowledge is thus in practice much more difficult than the formal educational system is prepared to recognise: the result is that the newly qualified engineer is generally quite helpless when starting at a technology intensive workplace despite having a solid grounding in mathematical-technical education.

As has already been mentioned it is not the case that knowledge in the formal educational system is "without context" and thus applicable everywhere, instead this knowledge is permeated by a specific school context which gives the propositional knowledge meaning and relevance. Statements of a mathematical nature learnt in a school environment also derive their meaning from this environment, e.g. depending on how tests in mathematics are designed, what examples textbooks highlight, the particular interests of the teacher and how grading is carried out. Students who take these preconceptions of mathematics to their first workplace remain virtually helpless until their preconceptions take on a new meaning in the praxis of specific workplaces. Parts of the propositional knowledge acquired may remain meaningless in their new context[11], and there may also be substantial praxis knowledge which remains "tacit" since it is not covered by the propositional statements learned. Moving from a theoretical education to vocational praxis can thus be as precarious as taking the other route.

[11] This complex of problems subsumed under the discussion on "transfer" have been the subject of intense debate for a long time. An important conclusion from this discussion is that the question of the transferability of knowledge between different contexts or activity systems is far more complicated than is recognised in a school context.

Model thinking and analogical thinking

Modern vocational life often consists of a mixture of propositional knowledge in the form of models and praxis knowledge with a more traditional trade background. One example is that of sheet metal work, where on the one hand, work is carried out using software programs for designing different constructions, and on the other hand the use of praxis knowledge transferred over many generations. In the first case, the starting point is a mathematical-geometric *model*, and practice involves the *application* of this model. In the second case, the thinking is more *analogical*: in the absence of an explicitly formulated model, the work is instead guided by experience from the use of earlier *examples*, which are assumed to have analogue structures. Designing a bend in a ventilation duct for a specific building is a unique task, but earlier designs for other houses may be sufficiently similar to provide guidance in the new task.

Model thinking and analogical thinking represent two thinking styles which can very well come into conflict with each other. In many cases model thinking is the winner in such conflicts, and this can lead to a loss of praxis knowledge in e.g. a company. This applies particularly where there is a generational change and the importance of hidden praxis knowledge becomes evident. A new group of practitioners, even though highly educated, may not be able to replace the many years of praxis knowledge accumulated over time[12].

Model thinking represents a form of propositional knowledge which itself must be based on praxis. For example, we can take a software program that produces drawings of what the parts in a non-linear ventilation duct should look like. The results from using the programme must be interpreted, modified and supplemented in the light of the concrete situation, e.g. depending on the characteristics of the sheet metal to be used, access to usable tools and the conditions specific to the construction of the building.

Mathematical proficiency at the workplace

It is evident that the use of mathematical models in the form of programmes or formulae requires a degree of mathematical proficiency. This proficiency is similar to propositional knowledge, but also contains elements of praxis knowledge. Propositional knowledge is by definition formulated in abstract terms, and a special form of praxis knowledge is required for its

[12] Another aspect to this is that the employer when facing the choice between employing a new person who has a strong theoretical education or alternatively developing the competence of an existing employee who is familiar with the context and praxis of the work, often chooses the latter (Hoyles et al., 2002).

interpretation and application in specific practical situations. In such contexts, mathematics is usually viewed as *instrumental*, i.e. the focus is not on mathematics as a subject per se, but only as a tool for solving practical problems.

An interesting question is the extent to which mathematical proficiency is also embedded in the analogical thinking that typifies the more trade-like aspects of vocational proficiency. The ability of a sheet metal worker to recognise that a desired construction is similar to something he has done before requires some form of ability to understand similarities and differences between two geometrical structures in three dimensions. Such recognition gained through experience should be of great relevance, not just in the sheet metal trade, but also in many other occupations with similar demands.

Essentially the vocationally active person thus has, or needs, three types of mathematical proficiency:

- Propositional knowledge enabling formulae and software to be used correctly in mathematical terms, e.g. solving for a specific variable from a formula. This knowledge is similar in nature to "school knowledge", but often has a highly instrumental orientation.
- Praxis knowledge of how programmes with extensive mathematical content and formulae should be handled, interpreted and applied in relation to concrete situations particularly in one's own occupation. This praxis knowledge creates the necessary bridge between theory and practice, and gives propositional knowledge its meaning and relevance. This knowledge is developed in praxis and usually acquired at the workplace.
- Praxis knowledge representing analogical trade thinking. The knowledge is based on seeing analogies between different examples. This type of knowledge must also be developed in praxis at the workplace, and often contains hidden mathematical proficiency.

As regards validation and also the formulation of qualification requirements, focus is usually put on the first type of mathematical proficiency i.e. "school knowledge".

The two types of praxis knowledge – practical knowledge and knowledge by familiarity – are on the other hand often neglected, this is partly due to the fact that they are not formulated and are perceived as having lower value. If conceptual tools do not exist for making a deeper analysis of knowledge, then praxis knowledge will also remain invisible and unknown. Praxis knowledge is also in contrast to propositional knowledge *personal*, whilst instruments for validation and assessing qualifications are generally of a more abstract and *impersonal* nature. Vocational proficiency is closely connected with questions about identity and self-esteem, and an impersonal test that only recognizes school knowledge may have an overwhelmingly negative impact on a person's desire and ability to develop and advance.

Mathematical skills and big ideas in mathematics

Background

During the years 2005 and 2006, intensive preparations were carried out in Sweden prior to the introduction of the new upper secondary school in 2007, both at programme and syllabus levels. The National agency for education on this occasion, in contrast to earlier, adopted an open working approach, and many different players had the opportunity of participating in the work. Anette Jahnke and Lars Mouwitz, both working at NCM were enlisted as experts. In order to further broaden the material for developing the syllabuses in mathematics, NCM took the initiative in organising a conference in September 2005, where many of the participants were mathematicians, educationalists, teacher trainers and individual teachers. The proposal that gradually emerged received an unusually wide degree of support from the world of practitioners and scientific research. As is well known the reform programme was discontinued when the centre-right government after taking office announced a more wide-ranging school reform with a very different orientation. A number of comprehensive international research reports were considered, as well as Swedish development work carried out by e.g. the Department of educational measurement, Umeå university and by PRIM-gruppen, Stockholm. Despite pressure from, amongst others, Anette Jahnke and Lars Mouwitz, the National agency for education, however, did not take into account the special conditions concerning adult education and the final document was exclusively based on the regular school system and its environment. This was not unremarkable since the government in its instructions to the National agency for education emphasised that the special conditions concerning adult education must be investigated and incorporated.

Although adult education is usually only viewed as a reflection of youth schooling, it may also be of interest to analyse the contents of the proposal for the syllabus, where the requirement was that it should be modern, widely supported and "fit-for-purpose".

Mathematical readiness

A fundamental concept in the proposal for the syllabus is *mathematical readiness*, which refers to a general competence in handling different situations with a mathematical content in the future. The concept should be viewed as a part of *lifelong learning*, where also *confidence* in one's own ability to learn and use mathematics, as well as a feeling for the *relevance* of the subject in both studies, as well as in vocational life are essential components. Mathematical readiness is expressed in terms of five *skills*, or competencies, which may be said to cover the *spectrum* of mathematical proficiency. A pupil may have developed these skills to varying *degrees*, as expressed in the criteria for assessment and awarding of grades. In this way each pupil can be said to have a "profile" concerning the quality of their mathematical skills. In addition, mathematical proficiency is always related to specific *contents*, it is possible to be an excellent problem solver in relation to one's existing knowledge e.g. compulsory school mathematics level. Both instruction and assessment are considered in this way to be based on developing proficiency in three dimensions: breadth, quality and content. Similar approaches have existed in earlier mathematics syllabuses, both for the compulsory and the upper secondary school, but they have been implicit and less structured.

The five skills are of a fairly general character and are not connected to any specific course content. What makes them interesting in terms of validation is that they are not linked to any specific school context. One can well imagine the five skills being developed in a vocational or non-vocational situation. It is also conceivable that mathematical skills of this general character are valid not only in the specific vocational context in which they are developed, but that they can thus be transferred from one vocational context to another without major loss.

Mathematical readiness involves the development of skills in terms of the following categories:

- *Concepts and relationships*: The ability to use mathematical concepts and understand their relationships, both theoretically and in relation to different practical applications.
- *Problems and modelling*: The ability to handle problems, as well as understand, apply and create models for solving concrete problems. This includes the ability to critically evaluate values to be entered, methods and results. Problem-solving here has an advanced meaning: it involves being able to solve problems where there are no obvious methods that could be directly used without prior reflection or adaptation.
- *Procedures and routine tasks*: The ability to apply different mathematical procedures effectively and with confidence and precision. Solving routine

problems is relevant here, as is the skill of using relevant technical aids and assessing what procedures are appropriate in different contexts.
- *Communication and reasoning*: The ability to interpret and use mathematical expressions, symbols, graphs and diagrams of different kinds. This also involves communicating and being able to reason over mathematical content numerically and in writing. In addition, it involves being able to listen to and understand the mathematical explanations and instructions of others.
- *Context and relevance*: The ability to put mathematics into a wider context in professional and societal life, and history. The ability to see the relevance of the subject outside school and the importance of mathematical proficiency in vocational and societal life, as well as in everyday life.

As mentioned earlier, there is also the question of *quality* in proficiency i.e. how well-developed the skills are. Keywords in this context are movement from the elementary, superficial, vague, uncertain, dependency, and stereotypes to the more complex, greater depth, certainty, precision, enhanced judgement, creativity and the general in the different applications where the skills come into play.

As regards the content of mathematical theory, the new syllabuses do not contain very much that is new, partly because no real research has been carried out in the area. Different countries have partially different mathematical content in their courses, but this appears to be based more on tradition than a deeper analysis of future vocational, societal and personal needs in the respective countries. In any case, it is evident that vocational groups in different countries become equally successful, although their syllabuses are different. This could well be an argument to support the notion that overall mathematical skills (competencies) are more relevant than any specific mathematical content.

Big ideas in mathematics

In the proposals for the syllabus, already mentioned, a traditional approach was used for categorising mathematical content into arithmetic, geometry, algebra, statistics, the theory of functions, and also parts of discrete mathematics. These categories are related to a long tradition of school mathematics, and their meaning is fairly well established, especially in school teaching materials. In recent decades, however, other categorisation systems have been applied that are more related to the underlying ideas of mathematics than to groups of mathematical theories and theorems organised by chapter. Such a division has been made by the American mathematician, Keith Devlin (Devlin, 1997):

- *Calculating*: quantities, numbers, fractions, decimal figures, measurements, arithmetic, understanding magnitudes, approximate values etc.
- *Reasoning and communication*: logical thinking, mathematical language and symbolic language, abstraction, generalisation, analogies etc.
- *Movement and change*: functions, marginal and average change, proportions, linear and exponential relationships etc.
- *Diagrammatic representations*: geometrical forms in two and three dimensions, representations in a coordinate system, projections, maps, similarity, scale etc.
- *Symmetry and regularity*: symmetry in geometry, formulae and algebra, positioning mosaic patterns, maximising and minimising material for different forms, wallpaper patterns etc.
- *Location*: schematic maps, coordinate systems, logistics, networks, linear and structural overviews etc.

In this case one could also imagine that mathematical contents do not need to be tied to a specific context, but rather that the ideas which are developed, for instance, in one vocational situation could be used with advantage in another. Different people apply different mathematical ideas with varying degrees of quality and depth, and in validation contexts this should be given prominence.

What is lacking in Devlin´s listing are the traditional areas "statistics" and "probability" which in other approaches are sometimes subsumed under "chance". An overall understanding of the fundamental ideas in these areas is critically relevant in both vocational and societal life. A couple of examples to illustrate this is quality assurance work (e.g. Three/Six sigma) and reporting of accident statistics which are often presented in advanced mathematical form which the average member of the labour force is not only expected to understand, but also use as a basis for their actions.

Conclusions

By means of a more multi-dimensional and informed view of what mathematical proficiency as described above is, new opportunities have been created for validation that goes beyond the traditional school framework. Here there is also some overlap between school knowledge and vocational familiarity which can be interesting to explore. There may also be approaches both into and out of school mathematics which take as their starting points skills and the major ideas of mathematics. Validation material for adults that does not directly aim to measure formal schooling should be developed to assess adults' more general mathematical proficiency with respect to the following aspects:

- Breadth concerning overall mathematical skills.
- Quality in proficiency as regards skills.
- Breadth concerning major mathematical ideas.
- Quality of proficiency in terms of these ideas.

Such a validation instrument would be fairly sensitive to and take into account a whole range of relevant aspects that traditional "school tests" cannot identify and measure.

Validation

> *All adults should have the opportunity of expanding their knowledge and developing their competence in order to promote personal development, democracy, gender equality, economic growth and employment, as well as equitable distribution.*

These are the ambitious goals which the Swedish parliament have decided should apply to adult education (Proposition 2000/01:72). A part of the strategy for achieving these goals is:

> *... recognition of existing, already acquired knowledge.*

Validation has become the instrument and the process for making this operational.

The bill on adult learning takes its starting point in an analysis of the concept of *lifelong learning*. The discussion on lifelong learning and validation, of course, is not limited to our country. In the main international policy documents, not least those concerning EU co-operation, a highly intensive discussion is taking place concerning these questions. For instance, in the Lisbon strategy the work of recognizing the overall learning of adults – both formal, non-formal as well as informal – is a key issue.

There are many reasons for the increasing interest in the overall proficiency of adults. It is argued that we live in a knowledge society characterised by rapid technological development, globalisation and increasing competition, all of which put greater focus on the competence of citizens and the labour force. In the area of adult learning, there has been a shift from education and formal merits to putting greater focus on the overall proficiency an individual has acquired in different environments. In recent years, based often on philosophical starting points, knowledge and proficiency and how these are to be evaluated have been the subject of intensive discussion. This has been elaborated on in other areas in this report, but basically it can be stated that interest in other forms of knowledge – practical proficiency, praxis knowledge, tacit knowledge, proficiency in daily activities, vocational proficiency etc – as opposed to traditional academic forms have increased and led to the understanding that the relationship between praxis and theory is far more complex than we often recognise.

A further aspect worth mentioning is that adult mathematical learning has been established as a specific area of research by the international organisation *Adults learning mathematics* (ALM), and that today we have a more solid foundation of knowledge to draw on.

An extremely important aspect is the question of how we view adult citizens and employers. Are they incompetent and ill-equipped to handle life as is sometimes put forward when international comparative studies concerning adult proficiency are interpreted and discussed, or should we regard them as active and rational beings, reflecting and competent in their daily work and civic roles?

Historical background

Laches: What now then, Socrates? Have you ever noticed that in some cases people can be more competent without teachers than with?

Socrates: Oh, yes, Laches. But you would not be prepared to trust them when they say that they are good tradesmen if they couldn't show you at least one or more examples of good work in their own field.

(Platon, 2000, p. 98. Our translation.)

Validation as a *phenomenon* is, as apparent from the quotation above, nothing new. Making adult proficiency visible through evaluation and taking this as a starting point for new learning has been virtually axiomatic in adult and popular education in Sweden. On the other hand, it is relatively new as a term in an educational context, and in Sweden it can be traced back to the first interim report of the Commission for the adult education initiative 1996 (SOU 1996:27). Internationally, however, it first appeared at Princeton university in the USA at the end of the 1960s in connection with a research project[13] the aim of which was to study whether experience based knowledge could be the foundation for admission to higher education. The starting point was thus to provide people with different education backgrounds and experiences access to higher education. In the 1980s, this was established in the UK and for the same reasons as in the USA. In South Africa, validation became an instrument for remedying educational

[13] CAEL (Cooperative Assessment of Experiential Learning).

shortcomings of large groups in society inherited from the apartheid period. After this, there was a shift in the rationale for validation with the *needs of the labour market* and *vocational proficiency* increasingly coming to the forefront. This took place in Australia and France, but it is no exaggeration to say that today this is also a dominant perspective in Sweden as well as in EU co-operation.

In Sweden, as mentioned, the concept of validation was introduced 1996. A source of inspiration was an article from 1995 (Colardyn & Durand-Drouhin, 1995) in the OECD Observer. The starting point in this article is economic development and competitiveness i.e. a narrow perspective of economic rationality. The historical background to validation in Sweden is more closely connected to vocational proficiency and the needs of the labour market even though a recent policy document (Proposition 2000/01:72) broadened the perspective and made the needs of the individual the focal point.

Thus a *shift in the rationale* has taken place from focusing on fairness for the individual to a perspective which is dominated by efficiency thinking where employability, economic growth and competitiveness are the focal points. In the work of the EU, we can see this trend in e.g. the Lisbon strategy. In this way the rhetoric surrounding validation follows a trend which adult education has generally undergone in recent decades (Gustavsson, 2002; Larsson, 2006). It may, however, be worthwhile noting that some of the crucial documents (see e.g. Council of the european union, 2002) concerning the EU's common educational policy state the goals as self-realisation, personal development and active citizenship i.e. aspects which we connect to general education, are treated as being equivalent to narrow instrumental economic goals.

It is in this area of tension between the needs and requirements of society versus the needs and experiences of the individual that we can understand the validation problem complex.

What is validation?

Validation has been given different meanings – which is hardly surprising – given that its complexity is not easily represented by a simple definition. The definition which has become established in Sweden and which is used today by most players is the following:

> *Validation is a process which involves structured assessment, evaluation, documentation and recognition of knowledge and competence which a person possesses independently of how it has been acquired.* *(Ds 2003:23)*

How validation has been applied in practice shows it can mean just about anything. For this reason, there has been a need for clarification at the policy level. Examples of this are drawing a distinction between validation, testing and credit recognition. Regarding the distinction between validation and testing, the principal difference has been described as follows: the former involves *exploration* whilst the latter focuses on *checking* against criteria.

This distinction can be related to different forms of knowledge assessment used in connection with validation. *Divergent* knowledge assessment, which unconditionally attempts to determine what an individual knows and is able to do, is in terms of ideas close to the exploratory process which validation is intended to be. The checking function, on the other hand, is an example of *convergent* assessment applied in relation to predetermined criteria, e.g. the grading criteria.

Aim and context

> *Validation involves creating an overall view of the student's knowledge and experiences, irrespective of whether this is represented by facts, practical actions, skills or understanding of how knowledge is to be used.*
>
> *(Prop. 2000/01:72)*

In national and international steering and policy documents, there are high expectations as to what validation should achieve, as cited in the quotation above. The issue, both from a theoretical and practical starting point, is highly complex and covers many aspects and domains. There are some ambiguities and contradictions in steering and policy documents which provide scope for different interpretations of what is involved and what the aims of validation are. In addition, we lack both theoretical knowledge and practical experience on many critical points, and these represent major challenges to be resolved if we are to live up to expectations.

There is a direct link between the aim of validation and the *contexts* in which validation may be said to be relevant. The above-mentioned ministerial document (Ds 2003:23) gives three contexts in which validation is relevant:

- as *a stage in ongoing education* for the purpose of mapping the level of knowledge, adapting contents of and/or shortening study periods for the individual,
- in connection with *counselling* in order to *define target levels* for further studies and also
- to document actual knowledge and skills prior to applying for *employment* or in connection with *development* at the workplace.

Apart from these aims and contexts, validation also has the potential to strengthen self-esteem and self-confidence, and this is highlighted as a positive effect (Andersson & Fejes, 2005; Ds 2003:23; SOU 2001:78). This applies particularly to adults who have remained outside formal education for some time. In this respect, the subject of mathematics should be given special attention since no other school subject to such a high degree is related to feelings of failure.

The report *Validering av vuxnas kunskap och kompetens* (*Validation of adult knowledge and competence*) (SOU 2001:78) describes the value of validation for different players: the individual, education organisers, employers, the state and municipalities. From this description, we get the impression that the validation process is free of conflict and all are winners, and that this is a process that takes place in harmony and in a spirit of consensus between all the parties involved. The fact that validation can have highly positive effects, both from an individual and societal perspective, few would disagree with, but there are also difficulties and tension of both a practical and principal nature. Some of these are examined elsewhere in this report. There is a body of literature richly illuminating questions concerning validation (Andersson & Fejes, 2005; Andersson & Harris, 2006; Colardyn & Bjørnåvold, 2005; Harris, 1999, 2000). Examples of the dilemmas which are examined in these sources with a bearing on the validation of adults' mathematical proficiency are:

- What is the dominating discourse? Is economic rationalism taken as the starting point? What importance do these questions have concerning personal development, democracy and citizenship etc i.e. the balance between the individual and society?
- Validation for equity or efficiency? Is validation in the first instance a question of treating the individual equitably or the starting point for making existing structures more effective?
- What view of knowledge permeates the contents and methods of the validation process? This question is related e.g. to the question of the distribution of power between players.
- Is validation typified by a perspective with a focus on maintaining the system or changing the system?
- What value (in terms of exchange, personal and for the user) does validation have for different players, and how will these values be expressed in the process?

Fairness for the adult

The result of a validation process is strongly dependent on that those who are responsible for its organisation and implementation are aware of the complexity of the task and the risks of limiting the perspective to a purely formal school context. This relates not only to different forms of proficiency, and how these can be represented through different modes of expression, but also the affective needs of the individual for recognition and other reactions prior to the power situation that is implicit in validation. Society's needs for discipline and control do not always coincide with the individual's needs for freedom and self-determination. The adult has also in practice demonstrated both to himself and others competence in handling life and work, but in the validation situation can easily appear as incompetent, not least in those cases where validation focuses primarily on what the individual *does not know* and *cannot perform*. It should also be noted that the adult has also learnt a lot of mathematics in daily and informal situations, a proficiency which should also be covered in a validation situation.

The aim of validation in this context must be made explicit, it can e.g. involve:

- strengthening the individual's *self-esteem*, vocational pride and confidence in his/her own abilities,
- directly finding a *new job* where the individual's current proficiency provides conditions for a favourable start,
- finding intersecting points between the individual's proficiency such as *initial knowledge* to link up to a prospective formal education situation,
- carrying out validation as a part of a *learning process*, i.e. a formative assessment instead of a summative assessment,
- *translating* the individual's proficiency into formal school knowledge, e.g. in connection with formal eligibility requirements,

- giving the individual an overall picture in terms of a *knowledge profile*, which can then be used in different contexts,
- gathering data for *research and development* concerning e.g. vocational proficiency, eligibility requirements, and conditions under which dispensation can be granted.

The aim is intimately connected with the means by which validation is carried out. Validation carried out as a test using paper and pen in a "school situation" is far too non-specific and misleading, and for this reason can hardly fulfil any of the aims mentioned above. Essential parts of the adult's proficiency probably fall into the realm of *praxis knowledge* and must *be shown* in the right context, and not formulated in a school context. In many cases, proficiency may be intuitively physical and closely connected with the materials and tools used in practice.

As regards mathematical knowledge, it is also critical that *qualitative proficiency*, such as different mathematical skills and familiarity with the *big ideas* of mathematics can be expressed. A general belief in the usefulness of mathematics and its importance in vocational life can be of great value in the future. Giving due recognition to the forms of mathematical knowledge above involves a *democratisation* of the concept of knowledge. Knowledge also exists outside the academic world and school, and outside the domain of written documents and formulae.

A well-known fact is that recognition of the individual's strengths reinforce self-confidence and also have a positive impact on the individual's weaker sides in the learning process. This is in sharp contrast to the persistent focus school mathematics puts on identifying errors. On the other hand, the adult may also have strong revanchist motives and wish to demonstrate both to themselves and others that they can "manage" a traditional test in school mathematics.

In this context, it is also crucial to differentiate between the formal purposes of validation and its real-world consequences. The explicit aim may be inclusive, for example, to stimulate the individual to take part in further education and work, whilst the consequences may well be exclusive: the individual feels written off by society and ends up in emotional and societal exclusion.

Mathematics as a subject provides for many adults a highly emotive experience connected with different types of negative experiences from earlier school environments. At the same time mathematics functions as an instrument for society to determine access to many advanced vocational education programmes with initial knowledge requirements based on school mathematics. A more flexible and multifaceted validation instrument could in this context serve the dual purpose of not only recognising an individual's praxis knowledge in mathematics in terms other than school mathematics, but also serve as an instrument for society to formulate more realistic and specific initial knowledge requirements in many occupations and vocational education programmes.

In order to take further steps towards drawing up a fair and effective validation instrument for both society and the individual in mathematics, we consider that the following needs to be accomplished:

- A multifaceted development of methods related to different forms of mathematical proficiency and the modes of expression required to represent these, including assessment and reporting of results.
- Development of methods for formative validation, i.e. validation as a part of an education or personal development process.
- Research and development into the relationships and interlinkages between knowledge based on familiarity and propositional knowledge in mathematics, and relationships between the application of mathematics in trades and more technologically oriented applications in vocational life.
- An in-depth analysis concerning affective attitudes of the individual, and reactions in relation to the subject of mathematics, and validation in the subject.
- A critical analysis concerning the initial knowledge requirements in mathematics for different occupations and education programmes.
- An in-depth analysis of the perspectives concerning power, inclusion and exclusion in relation to validation, not least with regard to the subject of mathematics and the special status it has.
- An in-depth discussion concerning the different aims of validation and the tensions between the needs of society and the individual, and the real consequences of validation.
- Targeted competence development measures for persons responsible for administration and practical implementation of validation.

Ultimately the aim of validation is to provide fairness to the adult. A society, where citizens feel vocational pride and receive personal recognition for their proficiency is always, we believe, in the long run the most desirable and successful society.

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