

The Problem of Recontextualisation

Part 2: A mode of re-instating mathematics

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At one aircraft company they engaged a team of four mathematicians, all of PhD level, to attempt to define in a programme a method of drawing the afterburner of a large jet engine. This was an extremely complex shape, which they attempted to define by using Coon's Patch Surface Definitions. They spent some two years dealing with this problem and could not find a satisfactory solution. When, however, they went to the experimental workshop of the aircraft factory, they found that a skilled sheet metal worker, together with a draughtsman had actually succeeded in drawing and making one of these. One of the mathematicians observed: 'They may have succeeded in making it but they didn't understand how they did it.' (Cooley, 1985; p. 171)

As I have indicated in Part 1, I've never thought that this anecdote from Mike Cooley's research in an aerospace company needs much by way of commentary; I'll provide one anyway. We have an intellectual/manual division of labour, here. Quite possibly, the task had been set independently to the mathematicians and to the experimental workshop. The mathematician, however, *fetches* the manual practice into her/his own sphere of interpretation, constituting it as incomplete, lacking 'understanding'. Presumably, 'understanding' would be achieved in the discursive definition of the problem in a form that would enable an effective procedure for the production of the drawing. This would fill the hiatus between the original formulation of the problem and the manual workers' construction of the afterburner, which, without 'understanding', stand in metaphorical relationship to each other, reduced by mere tacit knowledge, perhaps. Interpreted in this way, the mathematician's game might be said to entail the rendering of tacit knowledge as explicit within language, effecting a translation between what I refer to as low *discursive saturation* (DS^-)—the manual practice—and high *discursive saturation* (DS^+), the intellectual practice.¹ Mathematics abhors discursive aporias.

The movement between the experimental workshop and mathematics involves recontextualisation (we might imagine the response to the mathematician's comment from the workshop). In Part 1 I described school mathematical recontextualisation as *fetch* and *push* strategies between the *esoteric domain* of school mathematical schemes and principles, on the one hand, and non-mathematical—including domestic—practices. The result is the construction of a *public domain* as a *collection* of recontextualised—transformed by mathematics—sites and activities that is partially *integrated* by this esoteric domain *gaze*. This integration will include the kind of DS^- to DS^+ rationalisation described—or, rather, wished for—above and also concomitant constitution of different practices as essentially—which is to say, mathematically—the same. The public domain DS^- collection is thus (partially) integrated within and between elements of the collection by the esoteric domain *gaze*. I described the nature of the public domain in Part 1. In Part 2, I shall move to explore the nature of the esoteric domain of school mathematics.

¹ See Dowling (1994, 1998, 2009) for fuller exploration of the category, discursive saturation.

To begin with, I want to take a diversion in introducing a schema relating to the contexts of pedagogic transmission.² Firstly, transmission may be institutionalised within the context of the production and/or elaboration of the practice. This mode characterises the ‘legitimate peripheral participation’ of Lave and Wenger (1991) (though not necessarily all of their examples) and also what might be described as traditional apprenticeships (see Coy, 1989a). The craft apprentice—the apprentice Tugen blacksmith, for example (Coy, 1989b), learns his (sic) craft in the forge, alongside the master. Alternatively, transmission strategies may be mediated by relayers of the practice. In this mode, pedagogic theory will tend to take the foreground and the practice to be transmitted will be constituted as a curriculum. This is clearly the mode that is prevalent in schooling, where the official emphasis is on the transmission of the mathematical expertise, but not the teaching expertise, of the teacher.

The second dimension of transmission strategy can be introduced by reflecting on two different examples of craft apprenticeship. The first is the apprenticeship of Japanese *mingei* folk potters, described by Singleton (1989). This looks very much like legitimate peripheral participation. The initial part of the apprenticeship involves minimum risk labouring work and observation around the factory. When the apprentice is permitted to work at the wheel, they are told that they must first make ten thousand sake cups. For the most part, the apprentice’s products are thrown, unfired, into the clay bin for recycling until, eventually, the cups are rated as satisfactory and are sold in the shop—without the potter’s mark—as seconds. Here, the apprentice as acquirer is relatively untheorised; their competence will (or may not) develop in time. Rather, the emphasis in this mode is the production of adequate products, which is to say, on *performance*, rather than on *competence*.

The apprenticeship of the mediaeval scribe seems to operate differently. In a ‘school for scribes’ described by Aliza Cohen-Mushlin (2008), the master (sic) scribe would pen a few lines as an exemplar and then the pupil would take over. When the pupil’s performance was inadequate, the master would produce another exemplar. If the pupil progressed, they would be permitted to advance to more challenging tasks, such as rubrication and eventually take on the role of master. Here, there is clearly a sense of what is an adequate performance. However, a work completed in this mode will contain instances of both adequate and inadequate performances as the pupils’ work would not be scrapped; parchment would have been too costly for this and, presumably, such a procedure would have introduced too great a delay into book production. This leaves the emphasis of the apprenticeship far more on the competence of the apprentice than on the quality of the final product, which is always going to be imperfect. The distinction between this mode and that of the pottery apprentice is, as is generally the case, one of emphasis, almost nuance, perhaps, but nevertheless discernible. Pedagogic theory—or what we can know of it—is light, in the scriptorium, seemingly confined to the provision of exemplars, imitation, and correction, nevertheless it is there.

The cartesian product of the two dimensions of transmission strategy gives rise to the space shown in Figure 1. Two of these strategy modes, *teaching* and *apprenticing*, are quite familiar and commonly opposed as, indeed, they are here, though in what I think is an original way. The commodity outputs of schooling might be said to be various forms of credentials that attest to competence. We might say, then, that the tools of the school are the curriculum and assessment protocols and its raw materials are its students. The performances produced by the students are generally of little importance once they have been assessed. The

² I make no apologies for the use of the term, transmission, here. Social and cultural transmission or reproduction may not (certainly does not) exhaust the intentions or consequences of pedagogy, but it is a central aspect of formal schooling and crucial to the concerns of this paper.

commodity outputs of factories are the performances of their staff, so the situation is the reverse of that of the school and it is unsurprising that we find novices confined to low-risk (and probably low paid), peripheral activities until their performances are judged to be satisfactory. It would also be unsurprising if the employers of the manual and intellectual workers in Cooley’s anecdote awarded bonuses to the former, but not to the latter.

	Transmitter Focus	
Mediation	Competence	Performance
Unmediated	<i>delegating</i>	<i>apprenticing</i>
Mediated	<i>teaching</i>	<i>instructing</i>

Figure 1. Transmission Strategies

The leading diagonal of Figure 1 opposes *delegating* and *instructing*. The mode exhibited in Cohen-Mushlin’s scriptorium has been labelled, delegation, which is here being understood as a strategy of transmission rather than a strategy of management (though one might presume that the latter generally entails the former). Here, unlike the situation in teaching, master and pupil performances are the principal products of the activity, yet the emphasis is on the development of a community of competent practitioners, rather than or, at least, as well as, on the quality of any these products. I have no other empirical instances of this mode, though one might look to amateur, hobbyist activities. I have also encountered the sharing of repertoires of skills within informal (ie based in the pub) communities of jobbing builders and delegation might be an appropriate description of transmission strategy here. Consultancy work (the consultant being in the position of the transmitter) might be explored for evidence of this mode as might activities around succession planning in institutions.

Instruction is also frequently opposed to teaching and it constitutes the form of mediated transmission strategy that does not involve any developed pedagogic theory. I suppose sets of instructions accompanying consumer goods would frequently be described in this way. Not all instruction books are exhaustively described like this, however; the users’ manuals accompanying the professional grade cameras that I use tend to attempt to cater for incompetent users by including some teaching on the basic principles of photography and, in this respect the manuals differ from some of the reviews on websites concerned with photography.

I want to propose that both transmission in the context of production—delegation or apprenticeship—and transmission focusing on performance—apprenticeship or instruction—will tend to privilege the objects of production as the principle (which is not necessarily to say exclusive) referential objects for evaluation. This is because all three strategies must emphasise production as the main task in hand, closing down on opportunities for non-productive tasks. Uniquely, mediated, competence-oriented transmission—teaching—opens up a space for dedicated pedagogic action, because performances *per se* may be arbitrary and ephemeral. Competence is postulated rather than directly visible, so pedagogic action directed at its evaluation may be described as *forensic*, which is to say, directed at revealing things as they are in themselves, rather than providing interpretations (Dowling, 2009). In this case, ‘things’ refer to competence as a putatively objective (though potentially changeable) property of the one being evaluated. In the field of educational studies an assessment industry has developed. Forensic evaluation is theorised and standardised tests are

constructed and deployed nationally and internationally. Performances on such tests are recruited as metrics in demonstrating or challenging the success of government education policy and so forth so that performance as such is important. However, these are not production competences: in and of themselves, they do not matter other than insofar as they are interpreted as forensic indicators of underlying competences.

In mathematics education—and in other subjects in the curriculum collection—competence is often described as ‘understanding’, as in the Cooley episode. Here is Jeff Vass—a sociologist and former researcher in mathematics education who now works in social theory:

On leaving ed research I had got to the point where I thought it doesn't matter what is taught in maths. maths ed seemed to me to be full of spurious, hybrid psychological speculation justifying this or that teaching method. things that i thought might be useful, or have been useful to me, (like knowing by rote ones multiplication tables) were regarded with horror by people i met in education. They said children need to 'understand number' - i said this had never occurred to me when learning tables by rote, and while contemplating 'what is number' might be something i could see myself doing at some point i could get along without understanding anything at all when converting fahrenheit to centigrade. didn't go down well. (Personal email)

I want to refer to this emphasis on competence or understanding as *conceptualisation* and I want to claim—and I suspect I will find few challenges in the field of mathematics education here—that this is a central strategy in the teaching mode of transmission. Another realisation of the conceptualisation strategy is the contention that learning mathematics is primarily concerned with the cognitive acquisition of mathematical objects, which, Raymond Duval (2006) argues, are rather different from the objects relating to the ‘other domains of scientific knowledge’ (p. 107). Elsewhere (Dowling, 2010) I have quoted Duval extensively in order to establish that he makes the following five claims:

1. There is no perceptual contact with mathematical objects, but there is in other activities, in particular in science;
2. Mathematics consists of a complexity of semiotic systems for the representation of its objects;
3. There is, in general, no unambiguous transduction between representations in different semiotic systems;
4. This generates difficulties for students of mathematics.
5. Even within a single semiotic system, there are (what I would call) fundamental discursive differences between mathematics and other activities, in particular, in respect of the form of argumentation.³

Duval’s realist methodology is problematic, for me, in two respects. Firstly, I do not want to ontologise the objects of mathematics nor, indeed, those of the sciences. Let me put it this way. The natural sciences—say, endocrinology—might be interpreted as coordinating theory and method, where the latter is, in part, constituted by the instrumentation that Latour and Woolgar (1979) referred to as inscription devices and the principles of deployment of this instrumentation. The objects of endocrinological activity, then, are, like those of any other activity, including mathematics, to be taken to be constructed within the activity rather

³ I intend that my argument in this paper is—in terms of its broad structure, at least—mathematical in form, though clearly not in content; its syllogisms are established rhetorically rather than logically and by reference to the empirical as well as the analytic.

than noumenal objects sending transcendental messages to the endocrinologist. This, incidentally, is not an anti-realist claim, in a naïve sense, merely a form of a-realism that places its interest in the specific constructions of human activity rather than on faith in the metaphysical.

Secondly, for my purposes I do not find it helpful to essentialise semiotic systems—language, visuals, ...—or even registers—algebraic and other forms of discursive representation—nor to consider the phases or aspects of signification; given a mathematical context, I cannot hear the word ‘circle’ or see a visual representation of a circle without the one calling up the other (although the specific visual may depend upon the context). Rather, I want to focus my attention on strategies that structure the esoteric domain, here of school mathematics. So, instead of taking mathematics to comprise a range of semiotic systems, I suggest that it is more appropriate to say that it is characterised by a collection of strategies that includes: i) discursive definitions, principles, theorems and so forth; ii) visual exemplars, most obviously in the area of geometry; iii) formal nomenclatures (the decimal representation of number, for example) and heuristics; and iv) instrumentation (calculators, computers, geometric instruments, and so forth). Now this empirically based list can be reconceptualised as a complex apparatus that exhibits variation in semiotic mode—discursive (available within language)/non-discursive (not available within language)—and action—interpretive/procedural. This gives rise to the schema in Figure 2.

Mode of Action	Semiotic Mode	
	Discursive	Non-Discursive
Interpretive	<i>theorem</i>	<i>template</i>
Procedural	<i>procedure</i>	<i>operational matrix</i>

Figure 2. Modality of Esoteric Domain Strategy

Now, quite clearly, the categories in Figure 2 refer to general/generalisable aspects of the esoteric domain; a template would be of little use if it constituted a unique instance. These general modes are then repeated as local instances, giving rise to a three-dimensional schema, represented in Figure 3.

Mode of General Action	Semiotic Mode	
	Discursive	Non-Discursive
Interpretive	<i>theorem</i>	<i>template</i>
Procedural	<i>procedure</i>	<i>operational matrix</i>
Mode of Local Action		
Interpretive	<i>enunciation</i>	<i>graph</i>
Procedural	<i>protocol</i>	<i>operation</i>

Figure 3. Modality of General and Local Esoteric Domain Apparatus

I’ll take a mathematical problem that I’ve modified from Duval’s paper to illustrate the schema:

What is the relationship between the perimeter of triangle ABC and the lengths, AE and AF in Figure 4.

The verbal statement of the problem is a localising of theorem, that is, an *enunciation*.⁴ Figure 4 itself is a *graph*, so a relevant procedure would be, identify a suitable *template*. This has been done in Figure 5.

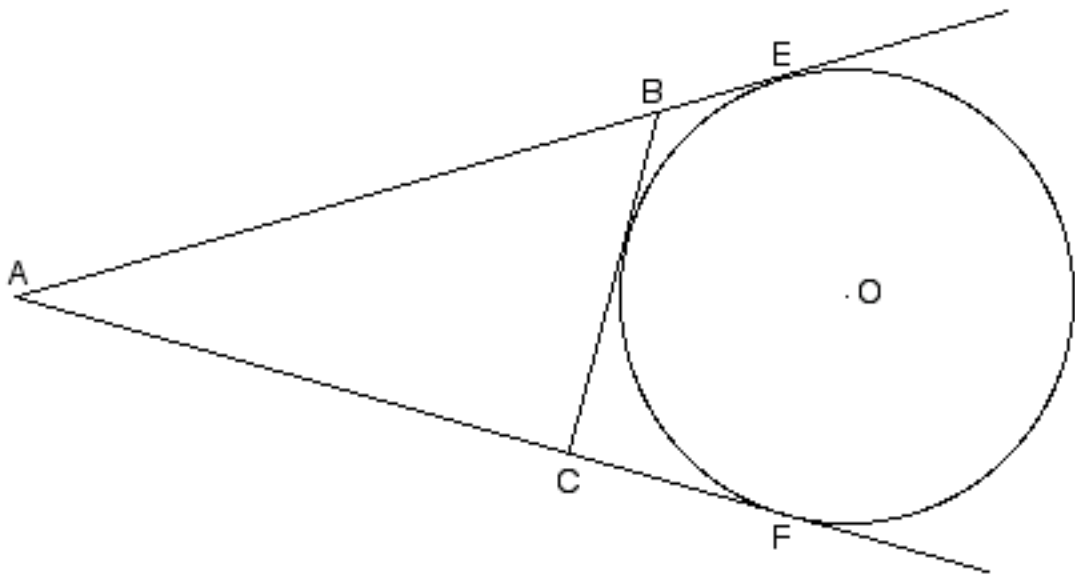


Figure 4. A Geometrical Graph

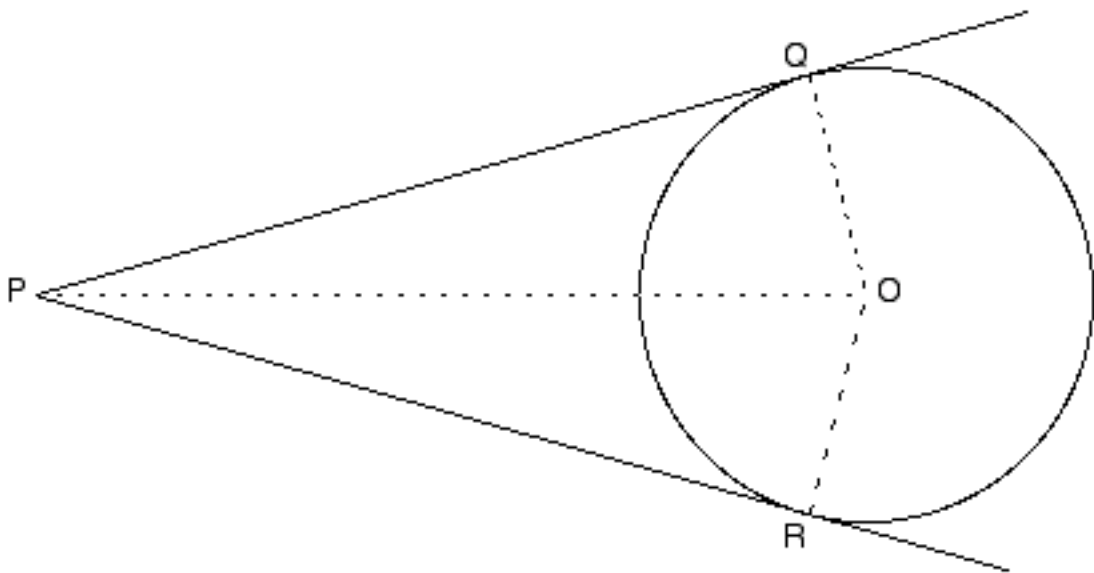


Figure 5. A Visual Template

⁴ The grammatical issue that it is in the form of a question rather than a statement is not relevant here.

The template serves (in my reading) to articulate theorem and operational matrix as follows (I'll restrict myself to plane geometry):

Theorem

1. A circle is the set of points that are equidistant from a fixed point that is its centre;
2. There are two tangents to a circle from any point outside the circle;
3. These tangents are of equal length;
4. The tangents are perpendicular to the radius of the circle at the point at which they touch the circle;
5. The two tangents to a circle from a point outside of the circle, the line joining this point with the centre of the circle, and the radii of the circle at the points at which the tangents touch the circle form two congruent, right-angled triangles

Operational matrix

6. Circles and line segments may be constructed by straight edge and compasses or using draw software on a computer.

A simple solution to the problem lies in recognising the template, Figure 5, in Figure 4; it can be seen to occur three times, as shown in Figure 6.

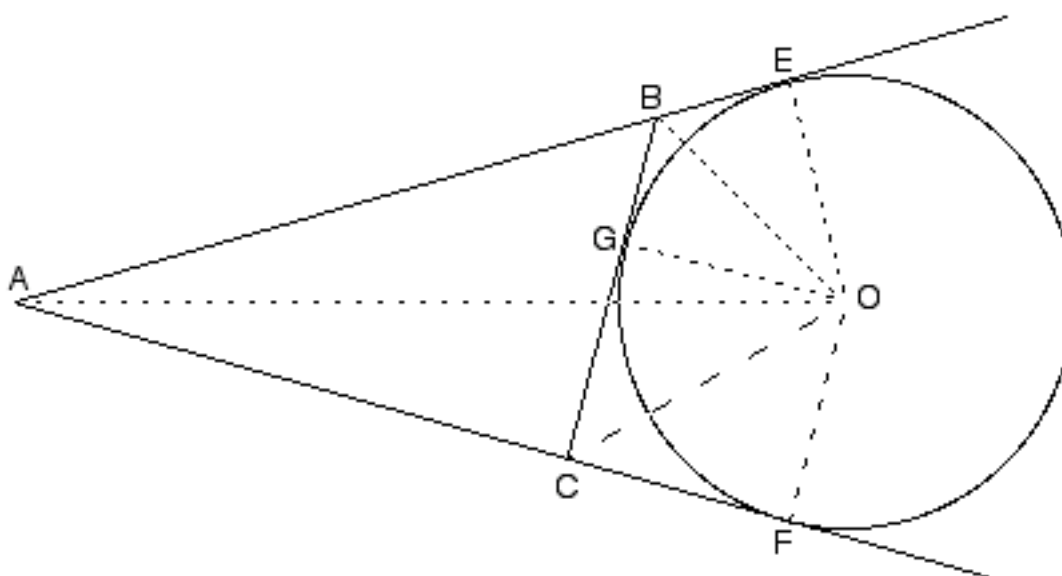


Figure 6. Realisations of the Template in the Original Graph

Figure 6 reveals equivalents to the kite shape, PQOR (Figure 5) as AEOF, BEOG and CGOF, so we know that $BE = BG$ and $CG = CF$ and so we can conclude with the solution: the perimeter $ABC = AE + AF$. This has arisen out of the selective articulation of the original enunciation and graph with the procedure and template and its associated theorem. Presumably, the problem might have been produced entirely as an enunciation; I will not attempt this here, but I suspect that the result would be quite tortuous; the visual texts enable

us to recollect and organise the esoteric domain apparatus rather more efficiently, at least, than would be the case without them. They work very effectively here, and in school mathematics more generally, because the template, Figure 5, is strongly institutionalised within and by the practice as are other geometrical templates (a pair of parallel lines traversed by a third line being another example).

Now I've referred to the discursive elements of problem and solution as enunciation, which is to say, local rather than general discourse (theorem). However, the lack of specific measurements (for the lengths of line segments or the magnitude of angles) does tend to generalise the figure, so there is a sense in which the solution is constituted by both enunciation and theorem strategies. Alternative solutions to that offered by the argument presented here might involve operational matrix/operation strategies. For example, it is clearly possible to measure the lengths directly from the graph and draw inferences from the results. This may be deterred in practice by another enunciation to the effect that the graph is not drawn to scale, but the operational solution would clearly eliminate theorem from its strategy.

In the consideration of number, we might consider that it is more helpful to think of, say, 351 as a text rather than as a sign.⁵ As a spoken text, we would say 'three-hundred-and-fifty-one', marking out the place value system by the addition of words ('hundred') and inflections (five becomes fifty). This simple enunciation is associated with procedures that have to do with arithmetic. However, 351 is also a graph in the sense that the signification of each digit (digits are signs) is given only by its spatial position relative to the others.⁶ A relevant operational matrix might be a spike abacus (there is no other obvious (and certainly no other obviously simple) way to move from the local instance, 351, to a more general text. Setting up the abacus as 351 is an operation. The abacus provides a simple technology for addition and subtraction within the positive integers, but is less successful (which is to say, rather complicated) for multiplication and division. We are left, then, without a non-discursive strategy for these latter operations; all we have are procedures. Clearly, we can produce alternative graphs for multiplication, rectangular arrays of dots, for example, transducing one graph, 352×792 , into another—a rectangular array of 792, rows each containing 352 dots. Alternatively, we can produce a procedure for multiplication in columns (a procedure for manipulating graphs) or we might attempt to use a number line for all four operations. As Duval points out, there is clearly scope for considerable confusion (see, also, Dowling 1998, c. 2).

Duval also emphasises that there are, in general, no unambiguous transductions between semiotic registers. This translates into my schema as the principle that, whilst there may be legitimate and illegitimate specific articulations between modes, the possible legitimate articulations may outnumber those that are necessary for the particular purpose at hand. In the geometry case above, for example, we do not need to know that the angles AEO and AFO are right angles, nor how the diagram was constructed. This entails that there is a problem of selection from the possible articulations, but the localisation that the template achieves clearly reduces this. For a discursive example, we might note that $2x + 3 = 0$, cannot be unambiguously transformed, but its recognition as a particular category of mathematical problem is likely to narrow things down via the recognition of relevant procedures/protocols.

⁵ See Dowling (2009) for a discussion of the important distinction between text and sign. This numerical example is also recontextualised from Duval's paper.

⁶ This is not the case in spoken or written language. Although the position of a word in a sentence (say) is sententious, it is not definitive of meaning as is the case with written numbers.

We would, for example, expect most students making this recognition to calculate $2x = -3$ and then $x = -1.5$ and this is the key: becoming a successful school mathematician entails the acquisition of a complex apparatus of interpretation and procedure. The solution of a problem is then a matter of selecting one or more suitable interpretation or interpretations and one or more procedure or procedures. The mathematical apparatus also constitutes the basis for the mathematical gaze that constructs the public domain.

It may be possible to reduce the whole of school mathematics to theorem/enunciation strategies. It may further be the case that the resulting discourse would be characterised as DS^+ and highly coherent, which is to say that it would construct its objects in a consistent way. But it would not be teachable; we can hardly introduce the Peano axioms before the number lines that adorn the walls of elementary school classrooms (can we?). To render it teachable, we, in effect, pedagogise it. In order to achieve the discourse, the practice has to diversify its apparatus as in Figure 3 and weaken the institutionalisation of its significations in constructing a public domain. The latter also serves as a marketing device in enabling school mathematics to be presented as something other than a closed mystery.

Now my interpretation of Duval's argument is that it seeks to establish that the work of signification in school mathematics is rendered complex because the objects of mathematics are signified in diverse semiotic registers between which there are no unambiguous transductions, so an essential aspect of the work of mathematics education lies in the coordination of these registers. This entails coordination around the objects that they supposedly signify—conceptualisation. However, it may be that the basis of the problem lies not in a lack of coordination, but in the perceived necessity of coordination that itself derives from the ontologising of mathematical objects, conceptualisation. Such ontologising is facilitated in the sciences, according to Duval, by the fact that we have extra-semiotic, perceptual access to the objects of the sciences albeit often mediated by technology. But, of course, we do not have access to anything at all that is not semiotically mediated, though the successfully marketed articulation of discursive and non-discursive apparatus in the natural sciences may have misled us into believing that we do.

On the other hand, school science does often seem to be very 'thingy'. We arrange 'experiments' to find out or confirm or sharpen up our knowledge about 'things'. We can separate the components of a mixture of sand, wood, iron, and salt by taking advantage of what we know about them (sand sinks in water, wood floats, salt dissolves and iron is attracted to a magnet); we can make graphs to predict the extension of a spring under different loads (and this is how a spring balance works); we can cut up a mouse to see where the various bits are; and so forth. The objects of science are, of course, not, generally, these things, but the objects constructed in science discourse: specific gravity; solubility; magnetisability; elasticity; anatomy (and its components); and so forth. This language is available to describe these things and other things like them. We do not—at least not below high school—very often go much beyond describing.

We often do the same kind of exercise in mathematics, but the contexts are often rather different. Firstly, the things in the mathematics class are often presented, at least initially, in public domain terms; they are presented as if they are the way they are in our daily lives; let's think about shopping, for the time being. This, of course, is part of the point of mathematics as it is sold in the mathematics classroom: to enable us to engage more effectively in everyday situations (what I refer to as the myth of participation (Dowling, 1998)). However, because we are to constitute the things in mathematical discourse and not within the everyday practices from which they are fetched, what students see is a contortion of their own lives.

Science, on the other hand, is not generally claiming to assist in managing one's daily lives. The things in science are, right from the beginning, placed in an unusual context and often in an unusual room—the laboratory. 'Suppose you (we) have a mixture of sand, wood shavings, salt and iron filings (not bits of a bicycle or car engine) and we want to separate out the various components; what can we do?' As with mathematics, there is no unambiguous transduction from the enunciation and its associate graph (the beaker containing the mixture) and no necessary separation between scientific discourse and everyday practice: we're not interested in the molecular structure of the salt nor its taste. School maths seems to be telling you lies about your own life; school science seems to be telling you new stuff about things that you're kind of familiar with, but perhaps have not really thought about.

School mathematics also has its 'laboratories', of course (generally not specially designed rooms, though). Here, we construct geometrical figures, play with abacuses and blocks of wood and hoops and chalk circles and so on. This must seem an arcane game. It's not immediately obvious why we should be interested in such things, but, even if we are, mathematics is not really about them at all; they seem to be pointing at something else. In formal terms, both school mathematics and schools science technologies construct their objects; school science defers entry into its discourse, perhaps, whilst school mathematics will not.⁷ This is not a philosophical or perhaps even a psychological problem, it's a pedagogic problem.

In addressing this pedagogic problem, one is led to ask, if scientific discourse is dispensable, at least in the earlier phases of school, might this not also be the case for mathematical discourse? Indeed, whether or not a fully principled, DS⁺ version of school mathematics is possible, the integrating discourse would have to be constructed by meta-activity. At any point within school mathematics—which incorporates templates, procedures and operational matrices as well as discourse—the impossibility of general, unambiguous translation or transduction between school mathematical texts constitutes hiatuses, aporias and I want to suggest that it is the resolution of these aporias that motivates conceptualisation as is illustrated—though not in a *school* mathematics context—by Cooley's anecdote. I want to speculate that this is because mathematical integrating discourse cannot abide a vacuum. This is why Gödel's inconsistency theorem is so profoundly disturbing; this is why Foucault is justified in describing mathematics as 'the only discursive practice to have crossed at one and the same time the thresholds of positivity, epistemologisation, scientificity, and formalisation' (1972; p. 188). Duval's distinction between mathematics and 'other domains of scientific knowledge' has some validity after all. However, the difference is to be sought, not in ontology, but in the opposing strategies relating to the empirical that are prevalent within the respective activities: science must confront an empirical universe and must, therefore, be perpetually incomplete; mathematics must quickly discard the potentially corrosive empirical world even though its temporary introduction may be necessary for pedagogic reasons.

Conceptualisation, then, is a strategy that produces mythical mathematical objects—mythical in the sense that they are postulated to plug the hiatuses, resolve the aporias that characterise the esoteric domain of school mathematics; mythical until and unless school mathematics is resolved into a coherent formal system—and directs pedagogic action to their transmission. But conceptualisation is the promise of a state that can arrive—if it ever does—only at the end of a curriculum, a curriculum that may be helical and so never end. To put this

⁷ In terms of the fully developed domains of action schema (see Dowling, 1998, 2009, 2010), whilst the entry to school science seems to be via the descriptive domain, the entry to school mathematics is often via the expressive.

another way, integration is possible only from outside an activity, so the emphasis on conceptualisation within the curriculum is misplaced. An alternative might be to consider rolling back on the conceptual approach to mathematics education that has held sway for the past sixty years or so and focus, instead, on the study of ‘things’. I am suggesting that we refrain from erecting an arcane and only putatively integrating structure as mathematical discourse and concentrate, instead, on the fostering of, shall we say, *petits re cits*: theorems (but not to claim integration), procedures, templates and operational matrix strategies that are recruited within the study of ‘things’. What’s the best way to learn the functional use of a foreign language: to attempt to acquire integrating grammatical discourse; or to acquire a collection of useable chunks? To the extent that grammar is never generative, but only at best interrogative, then the answer would seem to be the second alternative; grammar can, if we absolutely must have it, come later; Minerva’s owl is nocturnal.

My argument in Part 1 sought to establish that the fetch and push strategies of school mathematics must transform the practices that it recontextualises that constitute its public domain collection. This is not to say that mathematics can be of no use whatsoever. Now and again I find myself considering a restaurant bill, perhaps because I want to be sure that I am not being over or under charged, perhaps because the bill is to be shared, perhaps because I want to add an appropriate gratuity. In engaging these problems, I am quite prepared to fetch whatever resources I may have to hand and, on occasion, these include school mathematical resources. The point, however, is that the setting always determines which resources are fetched and how they are recontextualised. It is, of course, legitimate for schooling to expand the range of resources that are available, but do we need ten years of compulsory school mathematics to achieve this? In any event, the development of strategies appropriate to any given activity—whether it be political action relating to policing, an academic activity, such as science, or a domestic activity, such as dining out—is effectively achieved only within that activity, because they must reproduce the practices that make them recognisably distinct. Clearly, the school curriculum cannot incorporate the totality of non-school activities. However, earnest research, in the school, into these activities might be more effectively achieved by abandoning the mathematics curriculum imperative and allowing them to speak for themselves, as it were.

In Part 2, I have paid attention to the esoteric domain of school mathematics. I have introduced a modality of transmission strategies and argued that, only in ‘teaching’—mediated transmission emphasising competence rather than performance—is a space opened up for dedicated pedagogic action and, in particular, for forensic assessment. In this mode, pedagogic action will concentrate on the presumed conditions for assessment performances, which is to say, it presumes underlying competences. These integrating competences, however, are more appropriately understood as the retrospective construction of mathematical discourse; essentially, a curriculum—and this must include both esoteric and public domains of mathematics—can be presented only as collections both of and within school subjects. The acquisition of mathematics will involve the acquisition of a larger or smaller—but not integrated—apparatus as a collection of theorems, procedures, templates and operational matrices; the success of its fetching and deployment is to be measured in the context of its use, not in the mathematics classroom.

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