Full-body interaction in young children's modelling of counting-based addition

MORTEN BJØRNEBYE

This study explores characteristics of kindergarteners' talk, use of tools, and full-body interaction in the modelling of counting-based addition from the perspective of embodied cognition. Ten 4- and 5-year-olds participated in a 5-week outdoor embodied intervention for learning the min strategy (e.g. count on from the largest addend "four, five, six" in 2 + 4). Video analyses of individual testing showed that strategy efficiency was associated with fluency in the embodied interaction, use of mental representations and a structural awareness of numerical relations. Inefficiency was shown to be related to extensive offloading of the additive thinking to fingers and/or dice to keep track of counted units. An unexpected finding was the inclusion of expressive body movements (e.g. rotation, rhythm, force, and tempo) while modelling their counting strategies. The study contributes to educational research on body-based learning in mathematics by revealing patterns of young children's physical modelling of arithmetic.

Early proficiency in addition is a key predictor for later mathematical achievement (Carr & Alexeev, 2011; Ostad, 1997), and mastery of counting-based addition is considered a prerequisite competence for later development of fluency in arithmetic (Butterworth, 2005; Clements & Sarama, 2013). Children between ages five and eight often refine informal strategies involving fingers or physical objects to count-all items (e.g. both addends in 2 + 4) into a counting-on strategy (Siegler & Braithwaite, 2017). This strategy can either start from the smallest addend (i.e. "two, three, four, five, six"), which is called the max strategy, or from the largest addend (i.e. "four, five, six"), which is called the min strategy (Groen & Parkman, 1972). Refinement of strategy is considered an early sign of mental calculation because it challenges the child to visualise one

Morten Bjørnebye
Inland Norway University of Applied Sciences

Bjørnebye, M. (2022). Full-body interaction in young children's modelling of counting-based addition. *Nordic Studies in Mathematics Education*, 27 (2), 21–41.

of the addends (e.g. to conceptualise "4" and count on "four, five, six" using the min strategy for 2 + 4). However, it is not clear which factors might enhance this refinement, or whether there are individual differences in how this conceptual leap is experienced (Carr & Alexeev, 2011). A growing body of experimental evidence highlights the body's role in the transition between external and internal representations (e.g. Crollen & Noël, 2015), with finger gestures being considered a prime example of embodied numerical cognition (Fischer & Brugger, 2011). Yet there is no consensus about whether finger strategies promote fluency in arithmetic processing (Moeller et al., 2011). In addition, except for a few studies (e.g. Link et al., 2013), inquiries addressing the integration of full-body movement in the early learning of mathematics are sparse (Malinverni et al., 2014). To address this gap, ten kindergarteners' participated in an outdoor embodied intervention targeting counting-based addition as a content domain. Our research question was "What characterises 4- and 5-year-olds' talk, use of tools, and full-body interaction in the modelling of the min strategy?", which was evaluated based on individual testing after the intervention. We hypothesised that individual differences in young children's motor skills (Malina, 2004) and mathematical knowledge (Sarnecka & Lee, 2009) would influence the bodily fluency of the interaction and the degree of congruence with the min strategy. Our aim was to better understand the characteristics of young children's body-based modelling of arithmetic thinking from the perspective of embodied cognition.

The ways that young children use, discover and develop strategies have been a focus of research in arithmetic over the past five decades (e.g. Ashcraft & Fierman, 1982; Fuson, 1988; Groen & Parkman, 1972). Research has shown that children use a range of strategies for solving addition problems (for reviews, see Fuson, 1992; Verschaffel et al., 2007) and that several of these strategies include different ways of combining counting with the use of gestures and tools for offloading the cognitive work and lightening the load on working memory. Carpenter and Moser (1984) describe three levels of proficiency in single-digit addition strategies that reflect different levels of mental fluency. These are: (1) using fingers or physical objects to count-all concrete objects; (2) counting-on from either first or largest addend; (3) retrieval or decomposition strategies based on direct recalling of facts (e.g. 2 + 3 = 5) or derived fact strategies (e.g. 3 + 3 = 3 + 2 + 1 = 5 + 1 = 6). Based on 33 sessions in solving addition problems, Siegler and Jenkins (1989) found that most 4- and 5-year-olds use a variety of addition strategies, that the training in their study led to a transition to the min strategy for most children, and that the counting-all strategy played a fundamental role in this transition. An earlier experimental study by Groen and Resnick (1977) had found that half of their sample of 4-year-olds moved from counting-all (which they had been taught) to a counting-on strategy without instruction. These results are in line with other research showing that pre-schoolers can learn counting-on strategies with or without instruction (Clements & Sarama, 2007; Secada et al., 1983).

A growing body of research shows that children are reluctant to replace strategies involving counting single units once those strategies have been internalised (Gray et al., 2000; Ostad, 1998), and so there is good reason to consider introducing counting-on and decomposition strategies as early as kindergarten age. Support for early strategy flexibility comes from the study by Kullberg et al. (2020), which showed that 5-year-olds, who enrolled in an 8-month program focusing on using finger patterns to experience parts-whole relations, were able to use finger patterns to solve novel arithmetic tasks. Further evidence that reliance on counting to solve addition problems can be reduced by early intervention comes from an experimental study by Cheng (2012) that showed that 5- to 6-year-olds can learn decomposition strategies. A shared feature for several of these promising interventions is the use of concrete manipulatives, dot configurations, or fingers that allow the children to explore ways to structure arithmetic tasks, to see numbers as parts-wholes relations and to connect cardinal and ordinal aspects of numbers [1] (e.g. Björklund et al., 2018). These results suggest that more focus should be placed on structural-based learning activities that promote strategy flexibility. This draws our attention to the underlying proficiencies of counting-based addition as a basis for developing embodied interventions as complementary approaches to early strategy development.

Proficiency in the min strategy is based on a synthesis of skills in counting, subitising, and addition (Clements & Sarama, 2013). A procedure-oriented subskill mastered by most pre-schoolers is the ability to verbally count-on from a given number (Fuson et al., 1982), whereas conceptual knowledge is required to visualise the largest addend and subsequently count on the number of times equal to the smallest addend (cf. respectively the abstraction and stable-order principle of Gelman & Gallistel, 1978). The next trajectory, subitising, refers to the immediate perception of the numerosity of small sets of items, and the complementary term "conceptual subitising" refers to the use of the structure of a learnt configuration (e.g. dice-pattern) to determine the cardinality (Clements, 1999). The final tenet, addition, involves conceptual understanding of parts-whole relations of numbers involving the ability to see cardinality as an invariant property across any partitioning of a set (e.g. part-part or part-ordinal-relations; Clements & Sarama, 2013).

Theoretical framework of the embodied intervention

Embodied cognition posits that abstract concepts and reasoning are rooted in sensory-motor experiences and that humans facilitate working memory by offloading thinking to bodily and environmental resources (Barsalou, 2008; Wilson, 2002). Embodied numerical cognition is a research domain that focuses on the body's role in mappings from mental magnitudes to spatial extensions and locations (Moeller et al., 2012). Under this line of research, Lakoff and Núñez (2000) theorised that everyday experiences follow basic numerical properties and arithmetic laws (e.g. distributive, commutative and associative laws). Above all, object construction/collection and gait were highlighted as physical experiences with particularly rich inferential structures which may be mapped onto abstract mathematical concepts (Lakoff & Núñez, 2000). For example, the min strategy applied to $2+4$ may be congruent to (a verbal expression of) a mental visualisation of a squared array or its physical tagging (in this study, by using feet and hands to touch dots painted on the ground) followed by taking two steps forward. In this way, arithmetic can be understood in terms of embodied experiences and mental simulations of these, reflecting cardinal and ordinal properties of numbers (Lakoff & Núñez, 2000). Empirical support comes from a growing body of evidence suggesting that gestures might ground abstract numerical concepts in spatial extensions (Hostetter & Alibali, 2008; Núñez, 2006), and that conceptually congruent gestures matching abstract mathematical ideas and relations might promote performance (e.g. Jamalian, 2014; Segal et al., 2014). Educational research has demonstrated a more pronounced training effect for young children when full-body movement was an integrated part of spatially structured numerical tasks (e.g. Link et al., 2013). Accordingly, the embodied intervention in our study needs to foster correspondence between the kinaesthetic and visuospatial modalities of part-ordinal relations of numbers in a manner that matches the min strategy (e.g. using a foot to the tag the 4-dice ”four” as a part, followed by stepping onto two dots ”five, six” in $4+2$). In addition, the intervention must also allow offloading the additive thinking to tools and physical interaction on an individual basis. The conjecture of the present study is that an embodied approach to numbers will enable the young children to treat conceptual and procedural knowledge as complementary, thereby facilitating congruence in their full-body modelling of the min strategy (Rittle-Johnson et al., 2001).

Method

The first part of this section provides information about the participating children and the embodied intervention conducted outdoors in a kindergarten. The second part describes the procedures for collecting the data that allowed us to explore characteristics of the children's full-body interactions in the modelling of counting-based addition. The final section outlines issues related to assumptions, data reduction, and the method used to analyse the data.

Participants and the embodied intervention

Ten children (4 girls and 6 boys) between 4 and 5 years of age (mean age at the point of post-testing 4 years, 9 months) were strategically chosen to participate, and, upon receiving the informed written consent of their parents, were engaged in a 5-week embodied intervention consisting of pedagogue and researcher-guided one-hour sessions (mean participation 6.4 sessions) outdoors near their kindergarten. After the intervention, Give-N tests ("Can you give the puppy *N* items?") were conducted, assessing a child's level of cardinality (Lee & Sarnecka, 2010). Nine children were *cardinal principle*-knowers (hereafter abbreviated as CP-knower) as they had master the use of verbal counting for exact enumeration, while one child was classified as a C4-knower, showing consistency in the production of the maximum four objects on request.

Each session of the embodied intervention started with 20 minutes of joint activities in and around a circle ($d = 4$ m) drawn on the ground with 100 dots in (figure 1) to target aspects of the min strategy (for a detailed description of the embodied intervention, see Bjørnebye, 2021). The preliminary activities could involve rhythmic gait counting and touching dots to the beat of drums, or tossing a die and counting on two more, and keeping balance on a dot (as the child is doing in figure 5) while shouting the cardinal value. Additional physical activities could be the "Tag" game, in which a chaser verbally assigned another child they had tagged with a number (e.g. "six"). To re-join the game, the tagged child had to verbally express the number (e.g. "six") before entering the 100-dotted circle and using their feet to touch dots for counting-on to ten (e.g. "seven, eight, nine, ten"). The next 40 minutes involved a construction activity, in which the children had to complete the following stages (referred to as *Min task* below) to get a reward (e.g. a toy or a construction component to play with): (1) Roll two dice (e.g. 2 and 4), compare the values and pick up the one with the smallest addend; (2) Articulate and physically tag the largest addend as a whole (e.g. use the feet to tag the die and say "four"); (3) Enter into the 100-dotted circle and, informed by the value of the handheld die, use their feet to tag dots and verbally count-on the number of times equal to the smallest addend (e.g. "five, six"); (4) In the final physical tagging, they were encouraged to hold their balance while articulating the sum.

The analysis in this research is based on data collected after the embodied intervention, and the procedure for data collection is described below.

The procedure of Min task

Individual testing in the circle with 100 dots was recorded on video. First, the child was introduced to *Min task*: "You are to do as we did in the game earlier. Can you toss the die? Which die should you pick up?" If necessary, the child was guided via a practice trial. Next, to ensure variation in numbers to add, at least three tasks were given at three levels: Level 1 – two dice with the values 1 to 4; Level 2 – two dice 1 to 6; Level 3 – one die with the range 1 to 6, the other die set to the values 3, 5, and 6. During task solution, the researcher could ask: "What did you get?", "What did you say/do?" and "How many did you get?"

Assumptions, data reduction and method of analysis

Characteristics considered in the analysis include coherence, efficiency, and flow in strategy modelling and qualities in offloading the additive thinking onto dice, body movements, gestures and spatially-structured affordances within the circle with 100 dots. The analysis was based on the assumption that coherence in task behaviour is represented in connected multi-modal mappings of numerosity (i.e. verbal, visuospatial, kinaesthetic and tactile) matching the rules of the min strategy. Rich descriptors of physical and verbal attributes (e.g. tempo, rotation, stiff/fluent bodily movement, precise/imprecise body-spatial coupling, or monotone/modulated speech) were included in the data in order to elaborate the degree of fluency, coherence, and efficiency in strategy usage.

To facilitate the organisation of the raw data, video linked to individuals was segmented into coded clips (Jacobs et al., 1999). The coding structure reflected the relation between simultaneous (e.g. a foot tagging a die or a dot was represented in a single clip) and connected mappings of numerosity (i.e. series of simultaneous mappings of numerosity). Along with rich descriptions from transcripts of the video-recorded material, the detailed coding aimed to capture characteristics of fluency, (in)efficiency and (in)coherence in strategy modelling (cf. Ekdahl et al., 2016). Through this micro-analytic approach (Siegler, 2006), patterns of the task behaviours of individuals were examined and compared with the min strategy. Next, to identify categories of task behaviour, a cross-case analysis (Yin, 2009) was conducted. Finally, characteristics across and within each identified category of strategy modelling were discussed from the perspective of embodied cognition, thus providing (dis)confirmatory evidence shaping principles of the embodied framework (Dooley, 2002). To support this theoretical generalisation, the selection criteria for the multi-case analysis were based on the aim of highlighting recurring patterns and variations within each identified category of strategy modelling. This procedure is consistent with Eisenhart (2009), who argued that the cases that one chooses to generalise from should always be selected because they are likely to establish, refine, or disprove a theory.

Results

The first part of this section presents the general results and characteristics of the children's performances on *Min task*, while the second part provides rich descriptions, illustrative clips, and case-based examples of each of the identified categories of modelling the min strategy.

General results

The empirical material consists of 106 tasks (see table 1; all names are pseudonyms). Mark (the C4-knower) and Kevin were unable to solve any tasks; five children solved all the tasks; one child, Kelly, developed solving abilities after three errors on the first two levels. In addition, failures on the stable order principle (e.g. counted "five, seven" on the 5 + 1 task) and on the one-to-one correspondence during the counting-on part made by Jack and Liam add up to a solution rate of over 90 % for the eight children, showing proficiency in coherent strategy modelling. An elaborated outline of the general characteristics of the children's task behaviour follows, structured according to the four stages that engagement with *Min task* could involve (cf. figures 1–5).

Table 1. *Solution rate and preference in representational mode in the counting-on part (cf. stage 3 below)*

Name (knower-level)	Susan (CP)	Kelly (CP)	Jack (CP)	Kevin (CP)	Eric (CP)	Mark (C4)	Lisa (CP)	Noah (CP)	Anna (CP)	Liam (CP)	Sum
Solution rate	9/9	9/12	10/12	0/10	10/10	0/9	10/10	12/12	9/9	12/13	81/106
Representational mode in Stage 3	Visuo-tactile	Visuo-tactile	Visuo-tactile	Visuo-tactile	Visual	Visual	Mental	Mental	Mix	Mix	

Stage 1: Most CP-knowers used pattern recognition (subitising) to determine the value of the dice (figure 1), while some used touch counting for dice numbers 4, 5, and 6. Some children provided quick answers of the addition after tossing the dice, suggesting retrieval of additive facts. All

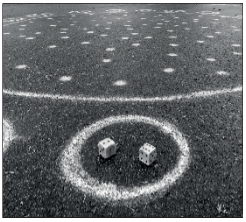

Figure 1. *Two dice and the 100-dotted circle*

Figure 2. *Tagging of the largest integer (stage 2)*

Figure 3. *Touch counting (from stage 2 to 3)*

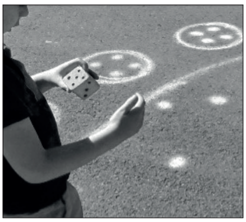

Figure 4. *Visual retrieval from die (stage 3)*

Figure 5. *Physical expression of the sum (stage 4)*

children showed the ability to distinguish between the quantities represented by the two dice (i.e. the more-and-less relation), and all except C4-knower Mark picked up the die with the smallest addend.

Stage 2: The CP-knowers coordinated the bodily touching of the largest addend with the articulation of its cardinal value, while C4-knower Mark ignored the die on the ground. Physically, some children tagged the largest die with both feet, while others integrated the tagging in gait or performed a distinct and forceful one-legged body pose (see figure 2) simultaneously with an exaggerated articulation.

Stage 3: Most CP-knowers showed the ability to map one unit of the smallest integer to a unique physical coupling of a dot on the ground and one verbal expression of a number word (i.e. the one-to-one principle; Gelman & Gallistel, 1978). This transformation of units from the hand-held die was based on mental representation, or tactile-visual (i.e. touch counting, figure 3) or visual (figure 4) retrieval of numerosity. C4-knower Mark started to count from "one" and, except for Kevin who verbally treated the two addends as separate parts, the CP-knowers showed verbal skills in counting on from the largest addend. Regarding the spatial and temporal features of the bodily interaction, some children walked at a slow pace while others combined jumping and rhythmic moves in a rapid and flexible appropriation of the spatial layout of the dots.

Stage 4: Regarding the final action that stated the sum of the addition, a bodily signature for most CP-knowers was to halt in a one-legged body pose, while some also attributed expressive body movements (e.g. rotation, original body posture, figure 5). Some also contributed an auditory dimension as the final step was expressed more distinctly and forcefully, which could be accompanied by an accentuated and prolonged articulation. In contrast, C4-knower Mark continued counting past the cardinal value of the handheld die without assigning the final tag to any physical or verbal significance.

Categories of embodied simulation of the min strategy

The comparison of the children's task behaviours revealed four categories of modelling the min strategy (see table 1), and the characteristics associated with these categories are elaborated below.

Figure 6. *Visual retrieval and gentle movement*

Figure 7. *Touch counting and stiff gait*

Figure 8. *Mental retrieval and action*

Figure 9. *Incongruent modelling in the form of tagging error*

The analysis shows that the major differences in strategy modelling were reflected in the degree of offloading of the cognitive work in the counting-on part of the min strategy (see "Representational mode in stage 3" in table 1). In particular, two children preferred visual retrieval of numerical information from the handheld die (figure 6), four opted for visuo-tactile support (i.e. touch counting, figure 7), and two relied on mental representations (figure 8), while two showed flexibility across representational modes (labelled *Mix* in table 1). To provide a thorough description of similarities and deviant features across and within these categories, the following sections present illustrative examples in addition to a separate outline of Kevin and Mark, who showed incongruence in strategy modelling (figure 9).

Preference for visuo-tactile support

A shared feature of this group of four children is the coordination of touch counting, articulation of number words, and feet touching dots in the ordinal part (cf. stage 3) of the min strategy. This complex multimodal synchronisation is associated with a slow and stiff gait (see figures 3 and 7), occasionally imprecise tagging of dots, and for everyone except Susan, a monotone speech. An additional deviation was Susan's use of direct retrieval of additive facts, as demonstrated below.

Susan: Six and four. It is ten. [picks up the 4-die and tags the 6-die using her right foot] Six. [walks slowly into the circle and touch-counts the dots on the handheld die holding a steady tempo but a stiff gait while tagging dots on the ground] Seven, eight, nine, ten. [in a one-legged body-pose, she articulates the sum in an extensive manner]

After tossing the dice 3 and 3, Susan quickly said, "Three and three are six." On one-more-tasks (e.g. 4 + 1), Susan represented the sum in a physically and verbally distinct manner without support from dice.

Kelly used pattern recognition of dice faces 1–5 and counted die face 6, and used visual support from the handheld die only once during the first six tasks. In three of these tasks, she continued counting past the value of the handheld die, while she produced correct sums on the others tasks (i.e. 4 + 2, 4 + 3, 1 + 1), suggesting mental retrieval. However, Kelly showed strategy adaption using touch counting to solve the final six tasks. Although this change involved monotonous articulation and a rigid movement pattern, the use of visuo-tactile sensory information from the handheld die supports coherence in strategy modelling.

Preference for retrieving visual information from die

Eric and Mark showed a preference for visual perception of numerical information from the handheld die to guide the ordinal part of the min strategy (Mark's behaviour is outlined later). A recurring pattern for Eric was to physically tag the largest addend, then walk into the circle and, supported by visual examination of the die in-between each tagging (see figure 6), continue in a slow and steady gait to produce the sum. However, in the following excerpt, Eric deviated from this pattern.

Eric: Six and six. [touch-counts both dice] One, two ... eleven, twelve. [picks up a die and tags the other die using his right foot] Six. [jumps into the circle and, without looking at the die, continues in a coherent and fluent physical tagging] Seven, eight ... eleven, twelve. [standing in a one-legged pose, he articulates the sum in an extended manner]

Preference in mental-based representations

Shared features for this group comprise pattern recognition (subitising) of dice, articulated one-legged body postures in mapping the largest addend (see figure 10), and mentally maintaining and retrieving numerical information to support the counting-on part of the min strategy. The bodily action in the circle was usually performed nonstop when the smallest integer was within subitising range, while the handheld die could be looked at one time for larger integers. Additional recurring patterns came in the qualities of the movement in terms of force, rhythm, fluency and tempo in the modelling of the ordinal structure (figures 11 and 12). Moreover, unique body postures and bodily rotations (figure 13) could be attributed to prolonged physical and verbal expressions of the sum. Below, a rich description of Lisa's task behaviour illustrates this.

Lisa showed mental and physical fluency with tasks where the smallest addend was less than and sometimes equal to four (e.g. 5 + 2, 5 + 3, 6 + 3 and 6 + 4) and she did not look at the die after entering the circle (figures 11–13). The following excerpt exemplifies the strategy pattern when the smallest addend was larger, starting after the dice showed 6 and 6.

Figure 10. *Tagging the largest addend*

Figure 11. *Rhythmic mapping of the ordinal structure*

Figure 12. *Rapid and fluent bodily coupling of dots*

Figure 13. *Bodily rotation in expressing the sum*

Lisa: [picks up one die and touches the other with her left foot] Six. [holds the posture briefly, jumps into the circle and tags dots in a rapid motion synchronised with articulating] Seven, eight, nine, ten. [halts and quickly looks at the die before continuing in rhythmic tagging] Eleven, twelve. [the articulation of the sum is extensive and synchronised with a one-legged bodily rotation]

Variations in representational modes

Two children showed flexibility across tasks in terms of a varied use of mental representation and tactile and visual retrieval of numerical information from the handheld die to support coherence in the counting-on part of the min strategy (labelled Mix in table 1). Anna, who used mental retrieval for addends one to four and varied between touch counting and pure visual support to solve other tasks (e.g. 6 + 5 and 5 + 5), exemplifies this category of strategy modelling.

From a physical perspective, Anna's modelling of the min strategy may be characterised as a goal-directed and fluent expression of the largest addend and the ordinal structure (figure 14) followed by a bodily rotation in expressing the cardinal value (figures 15–17). The combination of motor fluency and verbal modulation was especially evident when the smallest integer was within subitising range, as the final tagging included extensive articulations (e.g. shouting "nine") in synchronisation with bodily rotation.

Figure 14. *Fluency in physical coupling*

Figure 15. *Initial phase in mapping "nine" on the 6+3 task*

Figure 16. *Swing-phase in mapping "nine"*

Figure 17. *Completion of rotation in mapping "nine"*

Incongruence strategy modelling

The two children in this category of task behaviour mastered the more-and-less relation (i.e. comparing the value of both dice) and their embodied interaction reflected several non-verbal aspects of the min strategy. However, they were unable to link the action verbally in a manner consistent with the logic of counting-based addition.

Figure 18. *Unique touches mapped onto multiple number words and dots on the ground*

Figure 19. *Same finger-tagging two steps and number words later, cf. figure 18*

Figure 20. *Ignoring the handheld die*

Figure 21. *Tagging error*

The task behaviour of Kevin is characterised by subitising and pattern recognition of dice faces 1 to 6 and verbalised physical tagging of the larger die. Inside the circle with 100 dots and guided by the other die, he started counting from "one", using visual retrieval for numbers one and two while opting for touch counting for larger numbers. However, the use of visuo-tactile support is error-prone as unique finger-tags frequently map onto multiple number words and feet touching dots (e.g. on 3 + 3, 3 + 4, 6 + 5, and 5 + 5; figures 18 and 19). Moreover, the bodily movement was stiff and the speech was monotone.

C4-knower Mark picked up the larger die without tagging the smaller one and started counting inside the circle, sometimes ignoring (figure 20) or only arbitrarily using visual information from the die to guide the bodily interaction. For example, for the 6 + 2, 5 + 3 and 6 + 5 tasks, Mark counted to 15, 10 and 22, respectively, and the random sequences of looking at the die were especially prone to violating the multi-modal one-to-one correspondence (figure 21).

Discussion

This study investigated the characteristics of 4- and 5-year-olds' talk, use of tools, and full-body interaction in the modelling of the min strategy. The results showed that eight of the ten children showed proficiency in coherent strategy modelling, while preference in mental retrieval, or off-loading the additive thinking in visual- or visuo-tactile interactions with the handheld die in the ordinal part, reflected main modes of simulating the min strategy.

A signature for children opting for mental representations is a fluent interaction with the verbal and bodily domains. Anna exemplifies this description in the 6 + 3 task where pattern recognition of the two addends was projected onto a two-staged coherent motion in parallel with the articulation "six" and then the fluent rhythm "seven, eight, nine" (cf. figures 14–17). Hence, the largest addend was physically and verbally expressed as a part both rapidly and distinctly, followed by the transformation of the smallest addend into a composite combination of three steps. Lisa's strategy modelling for the 6 + 6 task further exemplifies the mapping of structural properties of numbers onto compound body movements. Based on pattern recognition of dice, Lisa's interaction suggests decomposition of the addition in three parts (i.e. 6, 4, 2), perceived and retrieved one at a time and holistically expressed as a three-staged movement trajectory reflecting a part-part-part structure of the whole (i.e. 12). Put together, the effectivity of the use of the structural properties of the addends in the embodied modelling of the min strategy is consistent with the study by Kullberg et al. (2020), which demonstrated the beneficial effects of a structural approach to part-part-whole relations of numbers in the early learning of addition. Moreover, the use of mental representations seemed to support fluency in movement and a flexible modulation of speech across the four stages of task solution. This was expressed in the form of louder articulation and more forceful physical tagging of the largest addend and the sum, through rhythmic expressions of the smallest addend, and in the integration of forceful rotations in expressing the sum (figures 10–13). In a related study, Paliwal and Baroody (2018) found that 2 to 5-year-olds' emphasising of the last word in a verbal count promoted cardinal understanding substantively more than two other training conditions (i.e. the counting only and the labelling the cardinal value first and then counting). Their results suggest that the verbal modulation observed in the current study might reinforce arithmetic achievements. In addition, the observation of complex body movements (figures 10–17) underlines the potential of integrating existing motor skills to the modelling of the min strategy.

Characteristics of children perceiving visuo-tactile information from the handheld die include monotone speech and a stiff gait on dots (see figure 7). These actions were also observed in children relying on visual information from the handheld die to guide the tagging of single units inside the circle (figure 6). An exception is that a pre-count on the 6 + 6 task enabled Eric to deviate from this pattern of monotone speech and a stiff gait as the mental representation of the sum 12 supported a fluent physical modelling of the min strategy. These observations suggest that Eric treated the counting-all and min strategy as equivalent approaches (cf. Siegler & Jenkins, 1989).

The findings of this study (at least in part) challenge two results of the experimental work of Gelman and Meck (1983) investigating children's counting skills. First, Gelman and Meck showed that 3- and 4- year-olds could accurately detect one-to-one errors (e.g. double counting and items skipped) made by a teddy bear in verbal object counting, and they also found significantly better performances in counting accuracy when the young children could touch the items (compared a 2D context with items behind plexiglass). For children mastering the min strategy in our study, errors in one-to-one correspondence were mainly made by children with a preference in visuo-tactile support from the handheld die (i.e. touch counting), suggesting that the resource-demanding multimodal integration made it easier to make errors and harder to identify those errors.

Counting-on strategies synthesize the cardinal and ordinal properties of numbers, and immature conceptions of cardinality might therefore explain difficulties in strategy modelling. C4-knower Mark's inability to follow rules or to recognise the multimodal correspondence between the spatial information of the die, the dots in the circle, and the bodily and verbal domains elaborates this picture. Although Kevin's task behaviour also expressed separateness in his inability to use verbal skills in combining the two addends into a whole, he demonstrated proficiency in coherent body-based modelling of small numbers. Fuson (1991) had previously found that the mapping of multiple words onto unique finger tags was among the most frequent errors made by 3- to 5-year-olds in counting objects, and this type of error was especially evident for Kevin in the error-prone use of tactile support in expressing numbers 4 to 6. All these results suggest that mastery for the two children showing incongruence in strategy modelling rests on the synthesis of cardinal understanding, flexible use of the stable list of number words, and the coordination of cross-modal mappings of numerosity.

In sum, the children's embodiment of the min strategy reflects variations spanning from effective to less effective use of the structural properties of numbers. In particular, the results suggest that mental retrieval of numbers as parts and wholes supports fluency and coherence with the verbal and bodily domain. In contrast, children perceiving information from the die in-between each step forward in the counting-on part seemed to use their cognitive and motor resources in the complex synchronisation of eye, dice, feet and spatial configuration on the ground. Overall, these observations suggest that children's strategy modelling is a complex issue reflected in the concurrent use of multimodal resources. Although this complexity suggests that there is no unique path for modelling the min strategy in embodied action, the children's autonomous choices of physical and mental representations can be seen as epistemological strengths. For kindergarten pedagogues, the identified characteristics connected to (in)effective and (in)congruent task behaviour may be used as cues for guiding children towards fluency in full-body modelling of counting-based addition.

Summary and concluding remarks

The transition from counting-all to counting-on strategies, and then the subsequent transition to the flexible use of retrieval and decomposition strategies, constitute two major conceptual leaps in the learning of addition (Carr & Alexeev, 2011). Based on at test conducted after 4 and 5-year-olds' engagement in an embodied intervention, this study investigated the first of these transitions by exploring characteristics of their body-based modelling of counting-based addition. The findings show that strategy efficiency was associated with fluency and accuracy in the embodied interaction, use of mental representations, and a structural awareness of numerical relations. In contrast, inefficient strategy usage was associated with extensive offloading of the additive thinking onto fingers and/or tools to keep track of counted units. Accordingly, the findings agree with the basic argument of educational research that advocates a move away from cumbersome finger strategies (e.g. Geary et al., 2004; Ostad, 1998; Ostad & Sorensen, 2007) and towards the cultivation of structural aspects of numbers (e.g. part-ordinal- and parts-whole-relations) to support fluency in arithmetic (e.g. Björklund et al., 2018; Kullberg et al., 2020). More importantly, the present study reveals the potential of integrating expressive body movements (e.g. force, tempo, rhythm, rotations and shifts in movement pattern; cf. figures 15–17) to the strategy modelling, thereby adding a subjective layer of meaning to the experience (cf. Radford, 2015). Related to this, Reikerås et al. (2017) found that toddlers with weak, medium, and strong motor life skills also exhibited respectively low, medium, and high skills in mathematics. Thus, to counter developmental differences in addition strategies (Ostad,

1997) and gender differences that favour boys' uses of mental strategies in addition (Carr & Alexeev, 2011; Sunde et al., 2020), future research should investigate the impact of integrating motor life skills and expressive body movements in early arithmetic learning.

To conclude, this study contributes to our understanding of strategy development (Siegler & Jenkins, 1989), and in particular, it contributes to educational research that focuses on how full-body interactions can help young children develop skills in arithmetic. The ecological trustworthiness is supported by situating *Min task* in an outdoor area frequently used by the participating kindergarten. Moreover, the composite body movements build on the children's natural movement skills, which are the basis for play in ecologically meaningful situations. However, the study is limited in light of its explorative nature and the small sample size. Despite this limitation, the study underlines the potential of modelling abstract mathematical thinking in the everyday movement behaviour of young children.

Acknowledgement

This paper is based on a paper presented at the CERME 11 conference (Bjørnebye, 2019), and it is part of my Ph.D. thesis (Bjørnebye, 2021).

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Notes

1 Cardinality refers to the quantity of a set (or a part), and answers the question "How many?" In counting, ordinality refers to the stable order of number words, and answers the question in "What position?"

Morten Bjørnebye

Morten Bjørnebye is an Associate Professor in mathematics education at Inland Norway University of Applied Sciences. His main research interests are design-based research and children's body-based learning of matehematics.

morten.bjornebye@inn.no