Conservative and transformative changes in algebra in Swedish lower secondary textbooks 1995–2015

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The present study examines textbook algebra tasks in an attempt to understand how textbooks change in a reform of lower secondary school algebra. Changes in 1557 textbook tasks for year 8 are described in terms of algebraic activities and school algebra discourses. The tasks were taken from textbooks published before and after a new syllabus was introduced in Sweden in 2011. The results show that the new syllabus' focus on mathematical competences was not stressed in the textbooks and that the greatest change was an increase in word problems connected to everyday situations. It is suggested that, in this reform, textbooks have been conservative and transformative in relation to the syllabus.

In the 20th century, textbooks have been used to change Swedish school mathematics (Prytz, 2017). In a more decentralized system, before 1950, innovations were introduced via textbooks rather than the syllabi. Textbooks were also important during the 1960s, by which time governance had become highly centralized. The implementation of *New math* in the early 1970s was largely a question of replacing traditional textbooks and directing publishing companies to follow the examples set by a major state-driven development project (Prytz, 2018). A key instrument was the mandatory state textbook review. Since the *New math*, policy governance has changed, from highly centralized to highly decentralized (Lindensjö & Lundgren, 2014). A key idea underlying today's policy is to govern by goals: The state establishes the goals, and local actors – such as municipalities, schools and teachers – are given great freedom to find methods to achieve them. This idea is clearly reflected in the mathematics syllabi of 1994 and 2011. They contain goals articulating what pupils should learn, but no guidelines on how to teach. As to textbooks, they are

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not controlled by the state. Another key idea underlying today's policy of governance is to evaluate goal fulfilment and use evaluation results to develop methods and goals. Evaluation takes place through national and international tests (Sundberg & Wahlström, 2012). However, little is known about how textbooks have changed during Swedish reforms of school mathematics since the *New math* and how they are changing today.

The purpose of our study is to shed light on textbooks' relation to the syllabus in contemporary Swedish school mathematics reforms. Here we are referring to the phenomenon of textbooks either mediating, modifying or ignoring ideas expressed in the syllabus or even introduce ideas other than those expressed in the syllabus.

Our study focusses on lower secondary school algebra and the 2011 curriculum reform. It is a relevant focus because the state operates according to the prevailing policy of governance – revising goals on the basis of evaluations. In connection with the latest Swedish curriculum reform, launched in 2011, there were clear changes with respect to algebra. One of the reasons for these changes was Swedish pupils' results on large-scale international assessments (Skolverket, 2011a); the National Agency for Education indicated that pupils needed better proficiency in algebra in particular. The research questions are:

> What characterizes algebra in lower secondary school textbooks?
>
> To what extent do algebra characteristics in the textbooks change during the period 1995–2015?
>
> In what respect can these textbook changes be linked to ideas in the syllabus? [1]

The disposition of the paper is as follows. After an overview of research on reforms, textbooks and algebra, we describe the major innovations in the 2011 syllabus, especially those related to algebra. We then compare algebra chapters from three textbook series published from 1995 to 2015 in an attempt to understand changes in their algebra characteristics. Through this comparison, we hope to grasp the support that textbooks possibly provide for implementing ideas about algebra which are expressed in the 2011 syllabus.

Previous research

The importance of textbooks has been underscored in different ways. Researchers in mathematics education have pointed out the important function textbooks have in teaching (cf. Hill & Charalambous, 2012; Stein et al., 2007). We also know that Swedish pupils do a great deal of work with textbooks (Johansson, 2006; Neumann et al., 2015). Researchers looking at school reforms (e.g. Apple, 2004; Apple & Aasen, 2003), on the other hand, have underscored the importance of textbooks in governance of teaching; textbooks work as a link between the syllabus and the teachers.

Only a few studies have focussed on Swedish contemporary curriculum reforms, mathematics and textbooks. Johansson (2003) argued that power-coercive strategies have been used in Sweden to implement new curricula. This means that educational authorities introduce new ideas. In her study of a textbook series for year 7 published between 1979 and 2001, not all goals in the syllabus are implemented in the books. Still, textbooks do not necessarily change just because the syllabus changes. Boesen et al. (2014) considered textbooks in relation to a reform process, but their main focus was on the impact of the 1994 curriculum reform on the practice of about 200 mathematics teachers (year 1–12) and how the concept of competence was introduced in that reform. They studied textbooks by analysing what tasks pupils work with, not how textbooks change in a reform process. In their case, teachers did not adjust their teaching to the competence message stressed in the reform. None of the studies mentioned have concerned the 2011 curriculum reform.

As to Nordic research on textbooks and algebra, a number of studies – for instance, Kongelf (2015), Lundberg (2011), Bråting et al. (2019), Häggström (2008), and Jakobsson-Åhl (2006). Kongelf (2015) and Lundberg (2011) –have focussed on the introduction of symbolic algebra and proportionality, respectively, and compared these areas to the syllabus. However, they have not looked at how the textbooks' algebra characteristics have changed over time. Bråting et al. (2019) studied the progression through primary school, focussing on opportunities to engage in algebraic thinking practices. Häggström (2008) compared Swedish and Chinese algebra tasks, and Jakobsson-Åhl (2006) examined the development of algebra in Swedish upper secondary school between 1960 and 2000. None of the above-mentioned researchers have discussed the relation between textbooks and the syllabus in mathematics curriculum reform.

Main ideas on algebra in the 2011 syllabus

In the two latest Swedish mathematics syllabi (1994 and 2011), aims and goals have been expressed as competences (Boesen et al., 2014; Sundberg & Wahlström, 2012). This change followed an international trend including the NCTM Standards, where mathematics is envisioned as processes rather than products. In this trend, the focus shifted from mathematical objects and constructs to mathematical practices, such as problem-solving, reasoning, and communicating. This is hereafter described as taking a *competence perspective* on mathematics. In relation to our study, four reform ideas were raised in the 2011 syllabus compared to the 1994 syllabus [2].

1. In the 2011 reform, the competence perspective was further explicated (Skolverket, 2011a). According to the 2011 syllabus (Skolverket, 2011b), teaching is supposed to help pupils to develop knowledge in, e.g. formulating and solving problems; understanding and using concepts; reflecting over and evaluating their strategies, methods, models and results; applying mathematical reasoning and arguing logically; and exploring problems. Hence, mathematics is much more than using procedures and practising skills. Unlike the 1994 syllabus, transformations were not mentioned.

2. Another innovation of the 2011 syllabus concerned the role of problem-solving in everyday situations. The syllabus aimed at preparing pupils to take everyday decisions and to increase their confidence in using mathematics in different situations. Problem-solving was understood both as a competence and as core content; it was captured by the description of algebra in relation to algebraic expressions, formulae and equations in situations relevant to the pupil (Skolverket, 2011b). In the 1994 syllabus, problem-solving was promoted only as a competence, not as a content area (Skolverket, 1994).

3. The formulations concerning algebra content in the 2011 syllabus had a clearer progression than in the 1994 syllabus. Working with number sequences and geometrical patterns in the early school years was explicitly seen as a basis for year 7–9 and as ways of expressing and describing generally (Skolverket, 2011a). These generalizations involve understanding and using the concept of variable in algebraic expressions, formulae and equations (Skolverket, 2011b). General descriptions were exemplified as the areas of geometrical figures or the cost of a mobile phone contract (Skolverket, 2011a). Further, R*elations and changes* became a specific topic, including functions (Skolverket, 2011b). In the 1994 syllabus, the concept of variable was not mentioned, and the contents in *Relations and changes* were integrated into the other topics.

4. The 2011 syllabus (Skolverket, 2011a) was more specific than the previous one concerning its content in relation to competences. A recurring theme was the use of algebra to achieve aspects of generalization, e.g. to use algebra for general descriptions and methods, and for general reasoning in problem-solving.

Theoretical considerations

Textbook authoring involves a process of interpreting the curriculum and giving it a concrete and more detailed form. Through our analysis, we wish to discern what changes in the textbooks can be linked to the curriculum reform, but also what changes can be considered independent of the reform. Thus, it is reasonable to employ an analytical tool that can highlight details and aspects of algebra that are not expressed explicitly in the curriculum. To achieve this, we combine an analysis based on an activity framework with the results from an analysis based on a social-semiotic framework. First, we present the activity framework.

For many years, algebra was characterized in three ways, by focussing: on manipulating equations and expressions, on the concept of function and modelling, or on structural aspects and set theory (Kieran, 2007; Ponte & Guimarães, 2014). More recently it has been suggested that algebra concerns processes rather than different objects and content areas (e.g. Kieran, 2007). Others (e.g. Kaput, 2008) have instead suggested that algebra is both about objects to acquire and actions to perform. These newer perspectives resonate with how the curriculum is constructed based on competences, as they focus on ”doing” mathematics.

Our framework of *algebraic activities* builds on Blanton et al. (2018, 2015), who in turn draw on Kaput. Blanton et al. characterized algebra as different ”big ideas” that give opportunities to participate in the algebraic thinking practices of generalizing, representing, justifying, and reasoning. The ”big ideas” connected to these thinking practices are: *Equivalence*, *expressions*, *equations and inequalities*, *Generalized arithmetic*, *Functional thinking*, *Variable*, *and Proportional reasoning* (Blanton et al., 2015). From this point of view, procedures and manipulating expressions are not algebra, just as playing the scales is not music (Kaput et al., 2008). However, we include changing the symbolic form of expression, e.g. simplifying expressions or solving equations, as an algebraic activity. Here we draw on Kieran (2007), who argued that transforming symbols is not merely a question of learning skills, but also of understanding concepts and theory by doing manipulations. Our analytical activity framework is detailed, because it connects activities with the mathematical concepts and content areas that pertain to algebra. It is further explained in the Methods section.

Through our textbook analysis, we also wish to discern changes related to social aspects, e.g. pupils' relation to algebra. For instance, Herbel-Eisenmann's (2007) discourse analysis showed how a reform message aimed at enhancing pupils' opportunities for participation, reasoning and justification is concretized in an ambiguous manner: In one textbook, mathematics is portrayed as absolutist and pupils are positioned as ”scribblers”, at the same time as human actors engage in and do mathematics.

We therefore apply a framework based on social-semiotics (cf. Halliday & Hasan, 1989). The social aspect of language enables us to study actions in language so that we can say something about interaction between people and the nature of this action, as well as about how these social actions are formed by and simultaneously form culturally, historically and institutionally acknowledged ways of acting (Luke, 1995; Wetherell, 2001). This further means that language is not neutral; it does not simply reflect the world, but constructs versions of it (Luke, 1995; Morgan, 2016a).

As a consequence, textbook tasks are texts that say something about the pupil's interaction with, or possible responses to, the author and the text. Texts also build up versions of the world, which are institutionally formed and acknowledged ways of acting and describing what algebra is. Features such as these can be studied through choices in language (Halliday & Hasan, 1989; Luke, 1995). Recurring choices in language construct patterns across texts: discourses that both are formed by cultural, historical and institutional ways of acting and form these ways of acting (Luke, 1995). In the present study, *school algebra discourses* are such discourses. The school algebra discourses were identified in Palm Kaplan (2018) by means of Systemic functional linguistics (SFL).[3] They were the main results in that article.

Methods

The textbook data of the study are presented first in this section. The analysis that follows focusses on tasks and entails several steps. To understand algebraic activities in the textbooks, a qualitative framework adapted from Blanton et al. (2015) is used to code the tasks. Algebraic activities are tracked based on their appearance in textbooks published before and after 2011. School algebra discourses, which are identified in a previous study (Palm Kaplan, 2018), are also tracked based on their appearance in the textbooks. The point of using two types of algebra characteristics is to capture different kinds of changes in the textbooks. The analysis is developed after presentation of the textbooks. Influences from the syllabus may also play out in magnitude, so to understand the changes in the textbooks, descriptive statistics are calculated in SPSS. Finally, the results of these steps are compared to the above-described changes in the syllabus.

The textbooks

Owing to the depth of the analysis, textbooks for year 8 were selected. We expected less algebra in year 7 and a certain amount of repetition in the last year of compulsory school: year 9. We studied textbooks from all series published both before and after the 2011 syllabus. For year 8 and during the period 1994–2015, this amounts to a total of six textbooks in three series. One of these was published in 2010, but written to comply with the 2011 syllabus (S. Carlsson, personal communication, September 9 2015). Five more series were published in Sweden between 1994 and 2011. After introduction of the new syllabus, three other series were published.

The unit of analysis is a task. Here we draw on Sidenvall et al. (2015), who claimed that Swedish pupils (year 10–12) primarily use textbooks for working with the tasks. We believe this is generalizable to year 8.

A task is the unit marked with a separate task number, or for some special themes with a separate letter or other symbol. When a unit has one task number but several subtasks – a), b) and c) – the subtasks are counted as one task. For the analysis, algebra chapters in the textbooks were selected (see table 1). These chapters are named, e.g. *Equations*, *Algebra*, *Algebra and patterns*. Initially, all tasks in these chapters were chosen for analysis. However, for a task to count as algebra, it has to be explicit that the task should be solved using algebra instead of, e.g. arithmetic or statistics. Otherwise no algebraic activity is possible to identify. Out of a total of 1557 tasks, 80 are deemed *not algebra*[4]. Examples from the textbooks are denoted by their respective labels (see table 1).

Table 1. *The textbook material (cf. Palm Kaplan, 2018)*

Textbook	Label	Curriculum	Number of tasks in the chapters	Tasks considered algebra
Matematikboken Y (Röd) (1996)	Y96**	1994	338	310
Matematikboken Y (2012)	Y12**	2011	455	417
Matte Direkt år 8 (2002)	MD02*	1994	160	145
Matte Direkt år 8 (2010)	MD10*	2011	154	154
Formula 8 Matematik (2007)	F07*	1994	196	196
Formula 8 Matematik (2013)	F13	2011	254	254

Notes. * These textbooks also have a chapter called Relations or Functions and graphs.
** This publisher has two chapters on algebra in each textbook. Hence, the larger number of tasks.

In the study, chapters named *Relations* or *Functions and graphs* are not included. Only three of the textbooks have such chapters (see table 1), one published after 2011, which makes comparison of them difficult. In these three chapters, functional relations are presented alongside coordinate systems and relations, e.g. percentages. A look at year 7 and 9 in the same textbook series shows that functions, graphs and coordinate systems are treated in year 9. In five out of six textbooks for year 7, patterns are treated.

A framework of algebraic activities

Blanton et al.'s (2018, 2015) view of algebra is in accordance with the practice-orientated competence perspective in the NCTM Standards and can be said to fit the competence perspective of the Swedish curricula quite well. In the 2011 syllabus, this is evident from the frequent mentioning of representing and reasoning. Justifying and generalizing are mentioned more indirectly through formulations such as to reason and argue, express and describe generally, or reason generally. The "big ideas" have been used to help children successfully engage with algebra in the early grades (Blanton et al., 2015). They have also been previously tested as analytical tools (Bråting et al., 2019). To account for the present study's focus on lower secondary school, the ideas have been compared to other conceptualizations (cf. Eddy et al., 2015; Kieran, 2007; Radford, 2011). The resulting framework of *algebraic activities* is described below. Examples are labelled as in table 1.

Equivalence, *expressions*, *equations and inequalities* (EEEI) are used in the algebraic activity aimed at developing a relational understanding of the equal sign. Equivalence, expressions, equations and inequalities are also used with generalized quantities in symbolic form to represent, justify and reason, as well as to model and interpret problems (Blanton et al., 2015).

> Example 1. Mira simplified $a(b + c)$ to $ab + c$. Albulena got the answer abc. Did either of them carry out the calculation correctly? Justify your answer.
>
> (Y12, p. 140)

To justify the answer above, the pupil has to reason and account for conclusions as well as be able to perform the calculation [5]. In the following EEEI example, the pupil is supposed to represent and model a problem.

> Example 2. The entrance fee to the public pool is 60 SEK more expensive for adults than for kids. One day, 255 adults and 347 children came. That day, the public pool collected 33520 SEK in entrance fees. What is the entrance fee for children? (MD10, p. 87)

The activity of *Manipulation* (M) does not explicitly maintain the thinking practices of generalizing, justifying, and reasoning that Blanton et al. (2015) draw upon, but stresses the transformation of symbolic expressions, such as the transformational activity in Kieran's (2007) model. It entails work with symbols without any modelling context, such as pure equation solving, using, simplifying and calculating values of algebraic expressions. Thus, it concerns the same content as EEEI, but is a different kind of activity. Hence, a task in our material cannot entail both EEEI and M. This can be exemplified as follows.

> Example 3. Write the expression without parenthesis.
> a) $3(a + b)$ b) $6(a - b)$ c) $7(x + 3)$ d) $2(8 - x)$ (MD10, p. 81)

Here, no context is provided for the simplifications required. Solving the task does not explicitly entail generalizing, justifying or reasoning. Although it concerns the same core content as examples 1 and 2 (i.e., equations and expressions), example 3 requires different actions or competences on the part of the pupil.

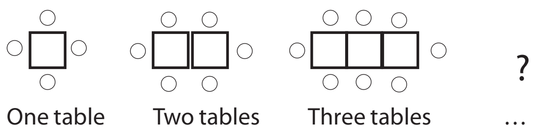
Generalized arithmetic (GA) is the area in which the algebraic activity is focussed on exploring properties that algebra and arithmetic have in common. It entails "generalising arithmetic relationships" and "reasoning about the structure of arithmetic expressions" (Blanton et al., 2015, p. 43) as well as making conjectures about arithmetic relationships, expressing them in generalized forms, identifying the range of these generalizations, and justifying arithmetic generalizations.

> Example 4. Write a two-digit number where the digits are not the same. Let the digits change places and calculate the difference between the largest and the smallest number. Try several different numbers. The answer is always a part of the same multiplication table. Which one? (MD02, p. 103)

In example 4, the pupil needs to investigate the differences between pairs of numbers in order to conjecture as to what multiplication table the differences are part of. This is followed up in the textbook by two more tasks aimed at further generalizing the arithmetic relationship in question. Not all GA tasks are as extensive, but they all address reasoning about the structural aspect of arithmetic expressions.

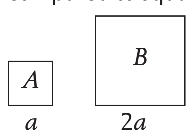
Functional thinking (FT) is the algebraic activity in which relationships between co-varying entities are examined, generalized, and represented by different semiotic resources, e.g. tables, graphs and symbols (Blanton et al., 2015). Aspects of this activity can be seen as investigations of number patterns, but could just as well concern variation and change, or how to determine the domain and range of functions (cf. Eddy et al., 2015). Example 5 shows chairs and tables in a pattern. The pupil is supposed to examine the relation between these and generalize it to determine the number of chairs for some larger number of tables.

Example 5. The pictures show how many persons can sit at 1, 2 and 3 tables. How many persons can sit if one, in the same way, puts together
a) four tables b) seven tables c) ten tables (Y12, p. 119)



Qualitative and proportional reasoning (QPR) is an algebraic activity that has been adapted from several sources. Blanton et al. (2015) described proportional reasoning as having to do with two generalized quantities of the same proportions. However, proportional reasoning is not as well defined as the other "big ideas", nor is it used by Blanton et al. (2018). Therefore, the activity is also précised as proportionality in making qualitative predictions and comparisons without numerical values (Cramer & Post, 1993; Lundberg, 2011). Example 6 demonstrates QPR, in that the pupil needs to make a comparison and reason about the proportions between both the circumferences and the areas.

Example 6. How many times as big is
a) the circumference of square *B* compared to square *A*
b) the area of square B compared to square *A* (F13, p. 234)



According to Blanton et al. (2015), *Variable* is integrated into the other ideas by playing different roles in different contexts. More specifically, "students ideally learn about variable as a varying, unknown quantity in the study of functional relationships and as a generalized number when examining the fundamental properties" (Blanton et al., 2015, p. 43). This big idea thus draws on a different set of algebra aspects than the others do. In an elaboration where the "big ideas" are developed as core content areas in algebraic thinking practices, *Variable* is not mentioned (Blanton et al., 2018). It is neither a core content area nor an algebraic thinking practice. As for this analysis of algebra characteristics as algebraic activities, *Variable* per se is not one activity. It is therefore deemed sufficient to discriminate the other "big ideas" in the analysis, which then ideally function as different contexts: where *Variable* may be interpreted as specific unknowns, generalized numbers, unknown and varying quantities, or as parameters (cf. Bloedy-Vinner, 2001).

A task is identified as including a specific algebraic activity if it is assessed as a reasonable and fair activity to engage in when solving the task in a given context. This means that explanations and written examples in textbooks are considered a context for the tasks, when a context is needed to identify the activities in the tasks. However, these parts of the textbooks are not analysed per se, because they do not require that the pupil engage in algebraic activity in the same way as a task does.

Some tasks are deemed *not algebra*, as they do not in any respect concern algebraic activity. Still, the framework is generously adopted, and early algebra is included. For each task, *all* algebraic activities the pupil is supposed to engage in are noted. Accordingly, several activities may be noted for one task. Thirty-four tasks of 1557 are considered to include combinations of two or more activities [6].

Two researchers, familiar with Blanton et al.'s work, have repeatedly given feedback on the analysis of unclear cases. At several seminars and workshops, colleagues have had the analytic framework and data, allowing them to check and comment on the reliability of the analysis. The distribution and frequency of algebraic activities in the different textbooks are presented in table 2.

Linking algebraic activities to school algebra discourses

In Palm Kaplan (2018), five school algebra discourses were identified: the *symbolic discourse*, the *geometrical discourse*, the *arithmetical discourse*, the *(un)realistic discourse* and the *scientific discourse*. These discourses construct meaning about the nature of algebra; what humans are doing and to what extent they are doing algebra; what kind of action the pupil is expected to engage in. The analysis is conducted using SFL (cf. Halliday & Hasan, 1989; Martin et al., 1997; Morgan, 2016b) to inductively identify linguistic patterns. Similar methods have previously been used in studies of Palestinian textbooks and British exam tasks (Alshwaikh, 2016; Morgan & Sfard, 2016). The resulting school algebra discourses are used in the present study. They can be described as follows:

- In the *symbolic discourse*, algebra is constructed as depersonalized. It involves symbols and subject-specific words. The pupil is mostly required to handle symbols when calculating and manipulating (see example 3).

- In the *geometrical discourse*, algebra is constructed as atemporal relations between geometrical objects, thus as a Platonic world. The pupil is mainly required to manipulate symbols and express geometrical relations using symbols (see example 6).

The tone in these discourses is impersonal and academic, and the pupil is positioned as someone who already knows algebra. However, a small number of tasks in these two discourses instead position the pupil as an apprentice.

- In the *arithmetical discourse*, algebra is constructed as a mostly human activity that consists of posing and solving number riddles. The pupil is asked to participate in this activity (see example 4).
- In the *(un)realistic discourse*, algebra is constructed as an artificial gaze upon a ”real world”. In this world, humans participate in many everyday activities, mostly buying and selling things or earning money. However, no one does algebra. Because the everyday activities described normally involve arithmetic, not algebra, the context is more or less artificial. The pupil is asked to express and interpret the activities of this ”real world” through symbols (see example 2).

The tone of these discourses is more personal and informal. The pupil is positioned either as a child (solving riddles) or as an artificial consumer (trying to make algebra out of everyday consumption). Further, the algebra is less challenging, in that it is presented as processes and not as objects or mathematical operations. However, a small number of tasks in these two discourses position the pupil as an explorer and participator.

- In the *scientific discourse*, algebra is constructed as depersonalized, in that human agency is obscured through passive verb forms. The pupil is required to perform calculations on physical phenomena.

Linguistic features such as dense nominal phrases (e.g. *the speed of sound at different temperatures*) give this last discourse its scientific tone (Palm Kaplan, 2018). Therefore, this discourse positions the pupil as someone who can handle more complex, scientific language.

In identifying the five school algebra discourses, some tasks in the studied algebra chapters were singled out because they were *hybrids* of discourses, *algebra outside the discourses* (see example 1) or simply *not algebraic* (Palm Kaplan, 2018). In the present study, the school algebra discourses are further examined to understand changes in their frequency and distribution in relation to the 2011 reform. This is presented in table 3. The interplay, changes and spread of the combinations of school algebra discourses and algebraic activities are visualized in table 4.

Results

The results are presented in three steps. The distributions and changes in algebraic activities and school algebra discourses are first given separately for each textbook series in tables 2 and 3, and then combined in table 4.

Algebraic activities, school algebra discourses and changes

Most of what is called school algebra in the textbooks involves algebraic activity connected to EEEI or M (see table 2). This is true regardless of which textbook or curriculum is considered. Though there are changes in the percentages for EEEI and M for all three series, the increase in EEEI in the F series is relatively modest. The changes in M are ambiguous, as the one-percentage-point decrease in M in the MD series involves four tasks fewer, and the two-percentage-point decrease in M in the F series involves 18 tasks more. The increase in FT in two series is notable, but this activity is still not common. In table 2, the most prominent changes (5 percentage points or more) are marked in bold.

Table 2. *Distribution of algebraic activities per textbook* *

Algebraic activity	Textbooks ordered by series, % in book					
	Y96 $n=338$	Y12 $n=455$	MD02 $n=160$	MD10 $n=154$	F07 $n=196$	F13 $n=254$
Generalized arithmetic, GA	1 %	2 %	4 %	1 %	2 %	2 %
Qualitative and proportional reasoning, QPR	4 %	1 %	1 %	3 %	4 %	4 %
Functional thinking, FT	**1%**	**10 %**	**0**	**5 %**	3 %	2 %
Equations, expressions, equalities and inequalities, EEEI	**42 %**	**47 %**	**48 %**	**57 %**	55 %	57 %
Manipulation, M	**49 %**	**33 %**	38 %	37 %	41 %	39 %
Not algebra	8 %	8 %	9 %	0	0	0

Note. * Several algebraic activities can be identified in one task. The percentages may not add up to 100.

In the school algebra discourses, most of the tasks in all series involve the symbolic discourse. In table 3, the most prominent changes (5 percentage points or more) in the school algebra discourses are marked in bold. On average, the changes in school algebra discourses are larger than the changes in algebraic activities. This means that the textbooks may be more influenced by reform messages carried by school algebra discourses than by algebraic activities. In general, there are no unambiguous changes in the algebra characteristics that apply to all textbook series.

Table 3. Distribution of school algebra discourses per textbook

School algebra discourse	Y96 $n = 338$	Y12 $n = 455$	MD02 $n = 160$	MD10 $n = 154$	F07 $n = 196$	F13 $n = 254$
Symbolic discourse	**54 %**	**36 %**	53 %	56 %	45 %	44 %
Geometrical discourse	12 %	10 %	18 %	16 %	**20 %**	**28 %**
Arithmetical discourse	9 %	8 %	9 %	7 %	16 %	12 %
(Un)realistic discourse	**12 %**	**25 %**	**7 %**	**15 %**	15 %	13 %
Scientific discourse	5 %	1 %	0	0	0	0
Hybrids	0	3 %	2 %	1 %	2 %	2 %
Algebra outside the discourses	**0**	**7 %**	0	0	1 %	0%
Not algebraic	9 %	10 %	13 %	1 %	1 %	0

The algebraic activities are at least partly constructed in different discourses (see table 4). For example, all tasks categorized as GA are in the arithmetical discourse, and FT is limited to tasks in the arithmetical and (un)realistic discourses. M dominates the symbolic discourse, and this is the most common combination of algebra characteristics in the tasks (see

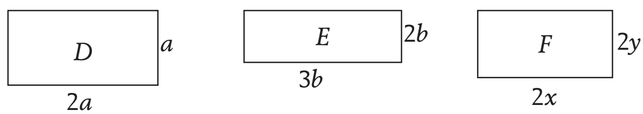
Table 4. The distribution of algebra characteristics in the textbooks *

School algebra discourse	Curriculum	Algebraic activity					Discourses in % of Total**
		GA	QPR	FT	EEEI	M	
Symbolic	1994	0	1 (1.5)	0	9 (4.0)	**42 (6.4)**	**51 (3.9)**
	2011	0	1 (1.1)	0	10 (5.5)	**32 (3.1)**	**42 (8.5)**
Geometrical	1994	0	1 (0.3)	0	15 (3.2)	0	16 (3.6)
	2011	0	1 (0.7)	0	16 (6.7)	0	16 (7.4)
Arithmetical	1994	1 (0.8)	1 (0.6)	0 (0)	9 (2.0)	1 (2.4)	11 (3.7)
	2011	1 (0.3)	0 (0)	1 (1.0)	6 (2.0)	1 (1.3)	9 (2.2)
(Un)realistic	1994	0	0 (0.3)	1 (1.3)	**10 (2.1)**	1 (0.8)	**12 (3.4)**
	2011	0	0 (0)	4 (1.0)	**15 (4.2)**	2 (1.2)	**20 (4.9)**
Scientific	1994	0	0	0	2 (2.2)	0	2 (2.2)
	2011	0	0	0	1 (0.5)	0	1 (0.5)
Hybrids	1994	0	0	0 (0)	1 (0.8)	0 (0)	1 (0.8)
	2011	0 (0)	0	1 (0.7)	1 (0.2)	0 (0)	2 (0.7)
outside the discourses	1994	0 (0)	0 (0.4)	0 (0)	0 (0)	0 (0.2)	0 (0.2)
	2011	0 (0.1)	0 (0)	1 (0.5)	3 (2.5)	0 (0.2)	4 (3.3)
Algebraic activity in % of Total**	1994	2 (1.6)	3 (1.3)	**1 (1.3)**	47 (5.3)	**44 (4.4)**	Total $n = 694$
	2011	2 (0.7)	2 (1.4)	**7 (3.0)**	51 (4.7)	**35 (2.6)**	Total $n = 863$

Notes. * Standard deviation in parentheses. ** The row percentages do not add up to the total, as a task can entail more than one algebraic activity.

example 3). In contrast, EEEI is distributed across all of the discourses. The most prominent changes (5 percentage points or more) are marked in bold in table 4.

Some of the standard deviations in table 4 change over time. This indicates that the textbook series have interpreted the syllabus differently. For instance, one series increases its combination of EEEI and the geometrical discourse (see example 7), and another shows a small increase in its combination of EEEI and the symbolic discourse (see example 8). The former includes representing symbolically in a geometric situation, while the latter includes reasoning and more conceptual understanding.

> Example 7. Write simplified expressions for the rectangles'
> a) area
> b) circumference
>
> 
>
>
> (F13, p. 201)

> Example 8. The equation 5x + 2y = 29 contains two different unknowns.
> a) Can you find any solution to the equation?
> b) How many solutions do you think there are? (Y12, p.238)

Two series increase their combinations of EEEI and the (un)realistic discourse (see example 2) and have small but notable increases in FT (see example 5). These increases emphasize representing and problem-solving, and representing and generalizing, respectively. Despite the many changes in discourse within EEEI, the overall change in EEEI is quite small. One change makes the textbook series more similar to each other: the reduction, in one series, in the combination of M and the symbolic discourse (see example 3).

The textbooks in relation to the syllabi

There are no clear trends in the changes for all of the textbook series, regardless of whether algebraic activities, school algebra discourses, or combinations of these characteristics are considered. This implies that textbook authors have interpreted the reform differently.

1. We believe that some minor changes may relate to the clarified competence perspective. The decrease in M involves transformations of equations and expressions, or procedures and skills (see example 3). The increase in EEEI partly involves reasoning and communicating in the

combination with algebra outside the discourses (see example 1), and more conceptual thinking in the combination with the symbolic discourse (see example 8). However, the majority of EEEI tasks still focus on representing and modelling (see examples 2 and 7), and the share of M is still large.

2. The role of problem-solving in everyday situations was enhanced in the 2011 syllabus, the goal being to boost pupils' mathematical confidence. This may explain the increase in EEEI in the (un)realistic discourse (see example 2).

3. Concerning the progression in the 2011 syllabus, the increase in FT is limited (see tables 1 and 2), and the FT tasks in the algebra chapters concern patterns of matches, beans or numbers. This is more consonant with the syllabus for year 1–6 than year 7–9. In Swedish textbooks for primary school, topics related to FT vary across the school years (Bråting et al., 2019). This may be true for lower secondary school as well. We acknowledge that topics related to FT are found in chapters on *Relations and changes* and in year 7 and 9. However, FT does not seem to be enhanced in year 8 algebra chapters.

4. In the 2011 syllabus, algebra is specified as many different aspects of the ability to generalize. This seems to have had little impact. The share of exercises with GA is very low in all investigated textbooks, both before and after the reform (see table 2). These results are in line with those of Kongelf (2015) and Bråting et al., (2019).

Beyond this, the low figures for QPR and the scientific discourse are not surprising, given that neither the 1994 nor the 2011 syllabus stresses these issues explicitly. However, GA and QPR activities are in accordance with the competence perspective, and the scientific discourse may afford situations for problem-solving. These potentials are not explored by the textbooks.

Conclusions

As mentioned above, researchers in education have underscored the importance of textbooks in reform processes, but little is known about textbooks' relation to the syllabus in contemporary reforms of Swedish school mathematics. Johansson (2003) argued that power-coercive strategies have been used to implement curricular reform in Sweden. The current steering system for implementing a school algebra reform lacks textbook review and development of new curriculum materials initiated by national school authorities; it instead involves a free textbook market. The present study reveals that these strategies are not so coercive after all.

Our results indicate that during this reform, textbooks for lower secondary school were conservative. Manipulations and equation-solving continues to be a stable Swedish textbook tradition. In relation to the competence perspective and algebra, the textbooks are conservative in the sense that their producers have not included tasks that require new types of algebraic activities. We wish to point out that the 2011 syllabus is not an obstacle to introducing such activities. The results confirm the claim made by Boesen et al. (2014), which is that implementing content seems easier than implementing competences.

Another aspect of conservatism is that the greatest change in the textbooks involves a well-established theme in Swedish school mathematics, i.e., grounding maths in everyday situations. Already in the 1919 syllabus, the purpose of school mathematics was formulated in relation to everyday contexts (Prytz, 2007). Johansson (2003), who studied textbooks from 1979 onwards, found that problem-solving exercises mainly concerned private economic matters.

However, the textbooks also were transformative here: what the (un)realistic discourse constructs as algebra is not necessarily in line with what is emphasized in the syllabus. We see this as an example of when textbook producers drive changes not initiated by the curriculum reform. Recall that the reform message involved preparing pupils to use mathematics in everyday decisions and increasing their confidence in using mathematics. The (un)realistic discourse involves looking at the "real world" *through* algebra, rather than solving problems in the real world *with* algebra (Palm Kaplan, 2018).

Our study shows how the textbook seems to be conservative and transformative in relation to the 2011 syllabus. Because we know that textbooks are important in teaching (Hill & Charalambous, 2012; Stein et al., 2007), it would be interesting to investigate in what ways these conservative and transformative traits of the textbooks play out in the classroom.

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Notes

1 Unless stated otherwise, the term syllabus refers to the set of documents regulating teaching of a school subject. Here we also include the commentary material. Curriculum refers to a larger set of documents that regulate all school subjects and other school practices.

2 The 1994 syllabus specifies goals for year 5 and 9. The 2011 syllabus describes core content for year 1–3, 4–6 and 7–9.

3 For practical reasons, the discourses are described in the Methods section. For the exhaustive method of analysis, see Palm Kaplan (2018).

4 This is 18 tasks fewer than the *not algebraic* in the five school algebra discourses identified in Palm Kaplan (2018). The difference is due to the fact that early algebra tasks are included in this study.

5 All examples are labelled as in table 1.

6 Not combinations of EEEI and M, however, as these activities are constructed as mutually exclusive.

Appendix

Sources

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