The language factor – what exactly is it? Bilingual speakers of Russian and Finnish solving mathematical tasks

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A lack of knowledge of the language of instruction is often believed to be the main reason for low achievement among students with an immigrant background. We regard language as a tripartite unit comprising aspects of concept formation, pragmatic language usage and the linguistic form. In this theoretical framework, we report two case studies of bilingual, Russian and Finnish speaking students' explanations of their procedures while solving mathematical tasks. The students' linguistic processing varied in terms of conceptualization, pragmatic meaning-making and grammatical form. In a bilingual context, the labelling of concepts and meaning-making through argumentation are simultaneously processed in two languages.

In this article, we report two case studies of talk-aloud problem-solving by three bilingual students (aged 14–16, in the 8th and 9th grades) whose mother tongue was Russian and the language of instruction was Finnish. In the first study, a pair of participants solved three PISA mathematical problems together, talking about them in Russian, and explaining their line of reasoning to the researcher in Finnish. In the second study, the participant solved three PISA mathematics problems alone and described his process and solutions to the researcher in Finnish. The aim of these studies was to obtain evidence of whether the verbal meaning-making process is in line with the results of problem solving. Furthermore, as problems with language have been highlighted by Schnepf (2007), Harju-Luukkainen et al. (2014), Kupari and Nissinen (2016), we wanted to investigate what kind of problems bilingual students may have.

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The discussion on whether language can be considered a problem, a right, or a resource (Ruiz, 1984; Planas & Setati-Phakeng, 2014) has raised questions regarding respect for diversity and concern for equity (Wagner & Herbel-Eisenmann, 2009), access, achievement, identity, and power (Moschkovich, 2012), including agency (Norén, 2011, 2015). Multilingualism is not only about cognitive processing, it is also about cultural traditions and the power-relations between these (Setati, 2005; Planas & Setati, 2009; Moschkovich, 2012; Jorgensen, Gates & Roper, 2014; Norén, 2015; Planas & Civil, 2015).

Language is an ambiguous concept, and in our article, we understand the polysemic notion of *language* as a tripartite whole. First, in everyday speech, the most common meaning for the word *language* is what de Saussure (1916), in his classical work, called *langue*, as a counterpart of *parole*: the linguistic system of, for instance, the Finnish/Russian/English language (Culler, 1976). This refers to the linguistic (i.e. phonological, morphological and syntactic) structure of the language. Second, the pragmatic emphasis deals with the situated, social aspects of language; in short, ”languaging” (Swain, 2006) or meaning-making (Blackledge & Creese, 2014). The pragmatic aspect of language resonates with the Saussurean idea of *parole* (Culler, 1976), and accordingly, we represent it as a distinct dimension of language. But in addition to natural language, a third dimension also exists, which is formal language, or the labelling function of the human language (Fiske, 1990). If languages are, broadly speaking, conceptualization systems, we should also refer to mathematics as a ”language” in the same way as we do to English or any other linguistic system. More than anything else, mathematical literacy is competence in the mathematical labelling system, but without the ability to communicate how the mathematical labelling system works, one's comprehension of mathematical literacy cannot be fully proven. Therefore, mathematical literacy is a bridging term for two aspects of language: the conceptual and the pragmatic.

The theoretical framework in this article is thus based on a tripartite definition of language. Language refers to

- conceptualizing: *language as a labelling system*, formal mathematical language
- pragmatics: *language as languaging*, math-talk, and
- linguistic (i.e. phonological, morphological and syntactic) systems: *language as ”langue”*.

The tripartite approach is parallel to Barwell's (2009) suggestion of three tension lines for multilingual mathematics classrooms: the first is between mathematics and language (cf. language as labelling system), the second tension line is between formal and informal language varieties (cf. language as languaging), and the third is between home languages and the language of instruction (cf. language as linguistic structure, "langue").

In an article published in a recent volume of content and language integrated learning (CLIL), Barwell (2016) analyses multilingual mathematics lessons in the framework of Bakhtinian dialogism, showing that the dialogical tension is present in all discussions during mathematics classes: the formal mathematical language (cf. language as a labelling system) exists only in relation to the informal component of language (cf. language as languaging), and therefore no separate "language focused" or "content focused" distinctions are needed. The Bakhtinian perspective is a sophisticated abstraction of how the different components exist in relation to each other, and seems to justify the bilingual practices of CLIL and teaching. Compared with the focus of this article, the Bakhtinian contribution appears to lack the specific conception of the third component: how to approach the various linguistic systems in the mathematics classroom, or the interplay between mathematical and linguistic components. This aim is taken up by Berger's (2016) research in the same volume, in which she uses the integrated language and mathematics model (ILMM) to reveal the cognitive process of individual learners (German) solving mathematical word problems in a foreign language (English), showing through talk-aloud protocols how the process iterates between linguistic and mathematical phases. Berger is also careful to remind us that the essential precondition to bilingual problem solving is the learner's access to first language terminology, since the focus group's unsuccessful attempts to construct meaning appeared to trigger translation strategies for learning mathematics through foreign language in lower secondary school. It is mainly through scaffolded translation strategies that the added cognitive value of bilingual learning is acquired.

Tripartite meaning of language in mathematics classrooms

The discussion on language in mathematics classrooms is sometimes confused by an unclear referent of *language*. In the tripartite conception described above, the first two tension lines (Barwell, 2009) or meanings of language concern the conceptualizing and pragmatic dimensions. In these two senses, the multilingual classroom brings nothing new to teaching or learning mathematical literacy or mathematical meaning-making, because the first two tension lines are equally important for teaching foci in monolingual and multilingual classrooms – although a bilingual student must deal with the relationship between mathematics and language in two linguistic systems and the relationship between the formal and informal varieties of two languages. As for the third tension line (Barwell, 2009), *language as langue*, this deals specifically with multilingual learners' multilingual worlds.

Analysing the three dimensions of language separately seems a step backwards, if we compare it with recent concise grammar explanations such as functional grammar (Halliday, 2004), which sees language through ideational, interpersonal and textual metafunctions, and defines the ideational metafunction as a composition of the experiential and logic parts, thus resulting in a holistic description of meaning and form. In addition, the frameworks of cognitive (Langacker, 2008) and construction (Goldberg, 2006) linguistics emphasize the unity of the lexical and structural aspects. In contrast, research on learning and teaching language for specific purposes (LSP) puts strong weight on labelling systematicity and the particular lexical hierarchies in various epistemic domains (Arús, Bárcena & Read, 2014; Coxhead, 2012). However, LSP research does not represent a theoretical flashback in its analytic differentiation between lexicon and grammar; but as Johns (2012) shows, it is a more terminology-centred and classroom compatible approach to linguistic structure and textual genre (Martin, 2009).

From the linguistic viewpoint, Parkinson (2012) depicts special features of mathematical domain in her LSP-oriented article (although her focus is in the language of science and technology), stating that mathematical language differs from the standard language in vocabulary, genre and textual composition. These terminology- and register-bound aspects also intertwine with the conceptualizing and pragmatic notions of language, as mathematical discourse is also visual and thus multisemiotic (O'Halloran, 2000). Using an elegant and explicit mathematical language has a long tradition, and is also culturally determined. Standard language words and phrases, such as *simplify*, *prove*, and *below 14 years* have a specific, technical meaning in a mathematical context and in formal mathematical language. In addition, many verbal mathematical discussions include highly specialised vocabulary, the passive voice, subordination, complex question phrases, and abstract or impersonal presentations, which may in turn affect comprehension among certain groups of students (Abedi & Lord, 2001; Halai & Karuka, 2013; Jorgensen, Gates & Roper, 2014; Planas, 2014). The language for mathematics is more than new concepts – specialized vocabulary, new words and new meanings for familiar words or technical terms (Riordain & McCluskey, 2015) – it is also a socially extended language discourse that includes syntax, organization and discourse practices.

Focusing on discourse practices is a matter of the second tension line, formal versus informal usage of language and the pragmatic framework. Moschkovich (2007, 2012) as well as O'Halloran (2015) depicts mathematics classroom discourse as multimodal and situated, pointing out that the complexity of language in mathematics classrooms engages students in multiple modes (oral, written, receptive, expressive, etc.). In particular, observations of bilingual learners' communication have shown that it often includes gestures, dragging and diagrams as illustrative objects (Ng, 2016). In the mathematics classroom, we use multiple representations (including objects, pictures, words, symbols, tables, graphs, etc.), different types of written texts (textbooks, word problems, student explanations, teacher explanations, etc.), different types of talk (exploratory and expository), and different audiences (presentations to the teacher, to peers, by the teacher, by peers, etc.) (Moschkovich, 2007, 2012).

A typical content area in which the language-as-discourse factor is often challenging is statistics. As Bergvall, Wiksten Folkeryd and Liberg (2016) show, statistical tasks are often long and tightly packed with details and information that is partly illustrative by nature. A high proportion of long subject-specific words such as population, citizens and consumers provide a real-world background, but are retrieved from other school subjects and seldom used in students' everyday language. Names of persons, teams, or cities may be used to animate the context, but they also often complicate the text. Bergvall et al. (2016) add that linguistically, statistical tasks commonly use the passive voice, intensifying words such as *more*, *total*, *each*, and constructions such as *with certainty*, *closest estimate*, or *compared to* in order to increase precision. In a mathematical sense, statistical reasoning tasks require skills such as using direct proportionality and information from the table, the ability to calculate something that is not stated in the task, understanding implicit meanings to draw conclusions about the significance of the information, or identifying the information given by the average. In statistics, one has to be able to first understand and read charts, compare heights in a bar graph, forecast trends, and analyse data, before being able to answer questions concerning these (Bergvall et al., 2016).

Statistical literacy, or reading and understanding statistical assignments, demands elaborated decoding skills of students. Reading statistics means decoding the information that is encoded in the technical terminology and re-verbalizing its essential parts in standard language. It is regarded as good practice in mathematics classrooms that teachers assist students in decoding word problems by verbalizing the assignments using informal language that is familiar to the students (Joutsenlahti, 2003). However this is not possible in test situations, when the student is left alone with the mass of text and has to be able to select the essential information from the descriptive narrative.

Compared with informal conversational language, formal academic language is more abstract, more contextualized, more specific, and more culturally determined. Moreover, unfamiliar representations and contexts may cause confusion and additional challenges. It is well recognized that visual representations can help, but these might not be sufficient – especially if the goals for the representations are not clear, or the purpose of the representation is incomprehensible (Truxaw & Rojas, 2014).

Finally, we come to the third tension line: the differences between the linguistic systems of the learners' own language and the language of instruction. This includes differences in languages' phonology, morphology and syntax, as well as the semantic and pragmatic conventions of language usage. Multilingual students switch from their first language to the language of instruction, while solving mathematical tasks, and using translation strategies in cases of difficulties (Berger, 2016). The switch may be due to mathematical solving methods, and the kind of processing they are about to require, or it may depend on the mathematical topic, since the student could prefer the language that has been used for mathematics instruction of that particular topic (Moschkovich, 2007).

The Finnish language has the reputation of being a "difficult" linguistic system, which of course is an experience-based statement and impossible to prove right or wrong. However, Finnish language is regularly found in the popular ranking lists of the most difficult languages in the world, listed by individuals as well as authorities such as the Foreign Service Institute (FSI) of the US government, which has a well-established School of Language Studies. Its experienced difficulty usually derives from two features: its synthetic morphology and its vocabulary bearing little resemblance to Indo-European word formation (Kaivapalu & Martin, 2007). The typological linguistic distance between a bilingual learner's first language and the language of instruction is obviously a significant factor in learning through a second language, although whether the perceived similarity between languages is even more significant than the objective or typological distance (Kaivapalu & Martin, 2017) is under discussion. Previous research has shown how the cognitive process takes place in solving mathematical word problems when the learner languages – the first language and the language of instruction –are typologically close to each other, such as German and English (Berger, 2016), but it is still unknown whether the cognitive process is different when the languages are typologically distant from each other, as in the case of Russian and Finnish.

Research questions

In our study, we sought answers to three questions on the tripartite meaning of language.

1 How does the participants' talk-aloud problem-solving relate to labelling systems?

2 How does the participants' talk-aloud problem-solving relate to situational and contextual meaning-making?

3 How does the participants' talk-aloud problem-solving relate to the morphological and syntactical complexity of language?

These questions rest on the hypothesis that relevant, applicable results of learning and using language in mathematics classrooms require dividing the meta-concept of *language* into a multi-dimensional entity. According to the three-dimensional frame for mathematics subject education presented in the previous chapters, language is simultaneously a labelling system, process of meaning-making, and a linguistic system.

Method

This article applies the "talk-aloud problem-solving" concept as the research method. In the first case study, two participants solve word problems that are given to them in the language of instruction (Finnish), and they solve the tasks by talking together about them in their first language (Russian), and later explain their process to the teacher in their school language. In the second case study, the participant whose first language was Russian received the word problems in Finnish, solved them alone, and afterwards verbally explained his process to the researcher in Finnish.

The talk-aloud protocol is a widely-used instrument for studying cognitive processes such as problem-solving (Krahmer & Ummelen, 2004; Berger, 2016). In this protocol, participants verbalize their thoughts while performing problem-solving tasks, and the researchers record all verbalizations. One of the advantages of using this method is being able to capture the problem-solvers' immediate thoughts and gain an insight into their metacognition. Therefore, in the field of mathematics education, the use of this method has proven fruitful and continuously produces new knowledge regarding the cognitive process of problem-solving (see Jacobse & Harskamp, 2012; Berger, 2016; Özcan, İmamoğlu & Bayraklı, 2017).

In this study, we were interested in using mathematical problems that could be viewed internationally, and selected a main survey item of PISA

2012, "Revolving door", and a field trial item, "Holiday apartment" (the tasks are presented in connection with the analysis) (OECD, 2013b). For the "Revolving door" task we had national comparative data of students with immigrant backgrounds (L2 students) (3.5 % of the population) and non-immigrant backgrounds (non L2 students). The Revolving door proved to be the most difficult task for Finnish L2 students (see table 1). The Holiday apartment was an example of a statistical task, and in PISA 2012, these statistical tasks seemed to be challenging for Finnish L2 students (Harju-Luukkainen et al., 2014).

Table 1. *Revolving door – descriptions and solving percentages*

Revolving door	Description[1]	Solving percentages[2]	
		Not L2	L2
Q1	Calculating the central angle of a sector of a circle	69.2 %	47.5 %
Q2	Interpreting a geometrical model and calculating the length of an arc	7.3 %	2.7 %
Q3	Identifying information and constructing a model to solve the problem	55.0 %	38.5 %

Notes. 1 (OECD, 2013b), 2 (PISA, 2012)

In PISA 2012, mathematics tasks were categorized according to the underlying mathematical processes, content areas and contexts. The processes used in PISA 2012 were *formulating* (i.e. being able to recognize and identify opportunities to use mathematics and then provide mathematical structure to a problem presented in some contextualized form), *employing* (i.e. being able to apply mathematical concepts, facts, procedures, and reasoning), and *interpreting* (i.e. being able to determine whether the results are reasonable and make sense in the context of the problem). The four mathematical content areas used were *change and relationships* (connections, functions, equations), *space and shape* (measuring, geometry, spatial sense), *quantity* (working with numbers, basic calculations and operations), and *uncertainty* (probabilities and statistics). In PISA 2012, the tasks were embedded into four different contexts. The *personal context* focused on the activities of one's self, one's family or one's peer group. The *occupational context* category emphasized the world of work. The *societal context* focused on one's community, and the last context, the *scientific context*, related to the application of mathematics to the natural world, and issues and topics related to science and technology (OECD, 2013a).

The space and shape task, Revolving door, was embedded in a scientific context and required employing and formulating processes. The statistics item, Holiday apartment, was embedded in a societal context and mostly required interpretation skills.

Participants

In the first case study, two students, Inna (pseudonym, aged 16, in the 9th grade) and Polina (pseudonym, aged 15, in the 8th grade) solved PISA tasks. Inna had started school in Finland at the age of 14, and had studied mathematics for six years in her mother tongue before moving to Finland. She claimed that mathematics was difficult and boring, and her last grade had been either 6 or 7 (on a scale of 4–10, 10 being the highest grade), but that it used to be easier for her when she studied in her mother tongue. Polina had started school in Finland at the age of 10, before which she had studied mathematics for four years in her mother tongue. She described mathematics as easy and fun, and her last grade had been 9. The task was given to the students in Finnish, which was the language of instruction at their school.

In the second case study, Pavel (pseudonym, aged 15, in the 8th grade) had moved to Finland from Russia at the age of 11. He was a strong English speaker, and after three years in Finland he reported needing English to solve mathematical problems, using trilingual processing. The recording in this study was made after four years in Finland, and he no longer expressed the need for English in the mathematical problem-solving process. He claimed to solve the problems ”in Finnish and a little bit in Russian”, so in his own words, he had changed from a trilingual to a bilingual learner of mathematics.

Language as labelling

We divided the results of our study into three subcategories representing the three dimensions of language: language as labelling, language as languaging and language as morpho-syntactic structure.

In the first two examples, we show how, in learning mathematics, the students' talk-aloud protocol contains the language as labelling approach. The first excerpt is from when Inna and Polina discuss the PISA task called Holiday apartment (OECD, 2013b). This task had a short introduction and then some statements. The introduction was as follows:

> Christina finds this holiday apartment for sale on the internet. She is thinking about buying the holiday apartment so that she can rent it out to holiday guests.
>
> | | |
> |---|---|
> | Number of rooms: | 1 x living and dining room |
> | | 1 x bedroom |
> | | 1 x bathroom |

Size:	60 square metres (m^2)
Parking spot:	yes
Travel time to town centre:	10 minutes
Distance to the beach:	350 metres (m) in a direct line
Average usage by holiday guests in the last 10 years:	315 days per year

The questions had alternative yes/no answers. On the basis of 315 days per year being the average usage of the apartment by holiday guests over the last 10 years, students had to decide whether the three facts given could be deduced from this information.

In the following section, we show the students' bilingual process when they were solving these tasks. First, Inna and Polina discussed the Holiday apartment task by themselves in Russian, which is here translated into English. After they had agreed on the solution, they explained and reasoned their process to their teacher in Finnish.

Example 1. *täsmälleen* "exactly"

> *Voidaan sanoa varmuudella, että lomavieraat käyttivät lomahuoneistoa täsmälleen 315 päivänä ainakin yhtenä vuonna viimeisten 10 vuoden aikana.*
>
> It can be said with certainty that the holiday apartment was used on exactly 315 days by holiday guests in at least one of the last 10 years. (OECD, 2013b)

1 P: Здесь можно ответить «нет», потому что в задании сказано, что пусть и 315 дней 10 лет подряд, но это в среднем. А тут именно сказано – täsmälleen, – что они прям …
2 I: Как бы, это уже точно… Да, да.
3 P: Да. То есть – «нет».
4 I: То есть – «нет». Хорошо.

1 P: Here we can answer "no" because the problem says that, despite it being 315 days 10 years in a row, this is an average. But here it says – *täsmälleen* – that they exactly ...
2 I: Well, that's for sure ... Yes, yes.
3 P: Yes. So – "no".
4 I: So – "no". OK.

In line 1, Polina highlights that the key word to the solution is *täsmälleen* "exactly". The answer is right, and in their reasoning, the participants use their lexical knowledge, or the labelling function of the language. The explicit meaning of one word, a clear label, led them to the right answer.

This word resonated with their knowledge of the mean. Mean, in contrast to mode, is the number you arrive at when you add together all the numbers in a set and then divide the sum by the total count of numbers. Accordingly, in this case, one of the values may be 315, but not necessarily.

The second example also depicts the importance of register-specific vocabulary. The Pisa Holiday apartment task was given to Inna and Polina in Finnish, but they discussed it in Russian.

Example 2. *joka vuosi* ”every year”

> *Teoriassa on mahdollista, että lomavieraat käyttivät kyseistä huoneistoa useampana kuin 315 päivänä joka vuosi viimeisten 10 vuoden aikana.*
>
> Theoretically it is possible that in the last 10 years the apartment was used on more than 315 days every year by holiday guests
>
> (OECD, 2013b)

1 P: Ну, мне кажется, в принципе … То есть последние десять лет.
2 I: Ну, я поняла, да. Я просто соображаю сейчас, как …
3 P: То есть они использовали комнаты чаще, чем 315 дней. Нет, потому что в среднем они использовали только 315…
4 I: А нет, подожди, это в среднем. То есть они могли в какие-то дни, допустим …
5 P: В какие-то дни.
6 I: Так что я бы ответила – «да».
7 P: у, да. Тут да.

1 P: Well, I think that ... that is for the last ten years.
2 I: Yes, I got that, yes. I’m just thinking now how ...
3 P: So they have used the rooms more often than 315 days. No, because on average they have used only 315 ...
4 I: Oh! No, wait! It’s an average. So on certain days they might have, let’s suppose ...
5 P: On some days.
6 I: So I would answer ”yes”.
7 P: Well, yes. Here, it is ”yes”.

In this task, the students did not notice that the statement read *every year* (joka vuosi), and their answer was wrong. Polina, in line 3, nearly had the right idea. However, in line 4, Inna seemed stuck in the same argument as in the first statement. The values may be different from the average. She ignored the words ”every year”, and did not notice that the average cannot be smaller than all the values used to calculate it. Again, the labelling function of the language was determinative. However, the languaging function was also part of the argumentation process. The students seemed to only partially read the task and therefore ended up with the wrong answer. As a term, *joka vuosi*, is frequent both as an everyday construction and as a precise term. Its frequency makes it easy to remember, but there is a hypothetical chance that it could be confused with *joku vuosi* ”some year”, which would make their answer right. Thus, this error could be a linguistic mistake. This is in line with what Berger (2016) has noted in her research on CLIL mathematics classes in lower secondary school, namely that learning mathematics in a foreign language lengthens the linguistic phase in the mathematical word problem solving process, and if the propositional meaning is not properly construed, the creation of a mathematical model can be destroyed.

The participants explained the same task in Finnish to the teacher after working it out together in Russian. Inna took the lead in talking and continued to repeat that ”315 is only an average”. Interestingly, she then drifted from the original text and explained that ”it is also no because the holiday times are different in different countries” and ”a guest from Russia, they have different vacation times to, for example, Thai people”, and therefore ”on those days they did not use the rooms”. The answer reflects a difficulty in making choices between relevant and irrelevant verbal information. However, Inna was capable of languaging different imaginative options and holding on to the genre of verbal assignments in Finnish – and in this sense, she was developing her ability to talk ”mathematics”.

Language as meaning-making

The second dimension of the language we focus on is the pragmatic meaning-making function of language. In the third example, Inna and Polina continue with their discussion about the Holiday apartment task (which began in Examples 1 and 2).

Example 3. *lainkaan* ”at all”

> *Teoriassa on mahdollista, että jonakin vuonna viimeisten 10 vuoden aikana lomavieraat eivät käyttäneet kyseistä huoneistoa lainkaan.*
>
> Theoretically it is possible that in one of the last 10 years the apartment was not used at all by holiday guests. (OECD, 2013b)

1 I: Ну, да. Но это в среднем если считать, то да. Ну, как бы да.
2 P: Ну, в среднем, там же … В среднем 315, то есть … Я думаю, что и последний

тоже можно сказать «да», потому что некоторые комнаты они могли использовать, если …

3 I: А-а, ну, да. Они могли меньше их использовать или наоборот больше.

4 P: Ну, тут [по-фински читает задание] … lainkaan. Lainkaan – это, типа, никогда, или как-то так. Нет?

5 I: А я не помню, что значит.
[I. asks the teacher about the meaning of *lainkaan* and the teacher answers that it means ”not at all”.]

6 I: То есть никогда.

7 P: Угу. Хотя нет, если 315 дней – это, в принципе, на 50 дней… 50 дней они могли не использовать. А тут вообще никогда, в принципе, из десяти лет.

8 P: Из 365. И то есть думаю, что если из 315 и тут одна olohuone, ruokailu ... то есть одна гостиная, одна столовая, одна спальня и одна ванная, то все комнаты, конечно же, использовались. Потому что, в принципе, тут только две комнаты.

9 I: Тогда значит нет.

10 P: В принципе, если подумать, то если приезжало всего по одному гостю …

11 I: Тогда все окей, да.

12 P: Нет. Даже если один гость, он все равно мог быть в гостиной. Мы же не знаем плана, во-первых, комнаты. Это было бы хорошо знать, потому что, если, допустим, гостиная здесь, здесь вот столовка рядом – значит, нету прохода, тогда да. Ну, я бы ответила «нет» все-таки.

1 I: Well, yes. If we calculate it in average terms, then yes. So yes.

2 P: Well, on average, there is ... On average it is 315, that is ... I think that for the last one we can also say ”yes” because some of the rooms they might have used, if ...

3 I: Oh, yes! They might have used them more or, on the contrary, less.

4 P: Well, here it is [reading the problem in Finnish] ... lainkaan. Lainkaan means never or something like that, doesn’t it?

5 I: I don’t remember what it means.
[I. asks the teacher about the meaning of *lainkaan* and the teacher answers that it means ”not at all”.]

6 I: That is ”never”.

7 P: Yes. However, 315 days is 50 days less ... for 50 days they might not have used them. But here, it is never in ten years.

8 P: Out of 365. So, I think that if out of 315 ... and there is one olohuone, ruokailu... that’s one living room, one dining area, one bedroom and one bathroom; then of course all the rooms have been used. Because in fact there are only two rooms.

9 I: So, that means ”no”.

10 P: Well, if we think ... if only one guest has come at a time ...

11 I: Then it is OK, yes.

12 P: No. Even if there has only been one guest, they might have been in the living room. We don’t know the layout of the room, to start with. That would have been good to know because if, say, the living room is here, the dining area is here and there is no connection – then yes. However, I would still answer ”no”.

The right answer was ”yes”. It is possible to still have an average of 315 days of occupancy even if one year is zero.

For the third statement, Inna first suggests the right answer, and Polina agrees with her. The following discussion, however, interestingly has some word-semantic relevance, but its focus misleads the solving process. The students start by discussing what ”using an apartment” really means. Polina introduces a new factor (in line 2) to the discussion by saying that ”some of the rooms they might have used” – as if they first had to solve the question about the utilization rate of the different rooms in the apartment. Inna first rejects (line 3) the idea that ”they might have used them more or less”, but Polina continues languaging by asking what exactly *lainkaan* means. Inna assumes it means ”never” and addresses the question to the teacher. The teacher specifies the meaning by saying it means ”not at all”. The students' uncertainty about the adverb *lainkaan* is natural, since the word is literary and has a more frequent parallel in everyday Finnish (*ollenkaan*). Both adverbs only occur in combination with the negation verb *ei* (*ei lainkaan*, *ei ollenkaan* ”not at all”), but here the formal mathematical language of the national PISA translation favours the less frequent and more literary term *lainkaan* instead of the everyday word *ollenkaan*. In line 8, Polina names the different rooms and concludes that ”there are only two rooms”. Inna asks what she thinks is the right answer (lines 9 and 11). Polina says it is ”no”, and the reason is that even a single guest ”might have been in the living room”. The reasoning is cryptic, but the students explain it further in Finnish.

The students explained their response to the third statement in Finnish to the teacher (the original transcript is translated here into English).

1 I: In the last question, we answered no, because the apartment has only two rooms.
2 P: There are only two rooms, like the bathroom, like the living room, dining room, bedroom and the bathroom. If one person was there in the whole house, I don't know about that person, but I would sort of be in every room because I am interested in how ...
3 I: She sleeps in the bedroom, for example. She might be in the living room the evening, for example, watching tv. In the dining room she can, for example, eat.
4 P: For example. [laughing]
5 I: In the bathroom, she washes. So, all the rooms are in use.

In the Finnish explanation, the students focus on describing the apartment and its different rooms. They form a reasonable answer to a fictional question about whether the guests used all the rooms of the apartment. However, the PISA task did not ask this question. We do not know whether the participants could have solved the problem through lengthening the linguistic phase, using a more precise translation strategy and returning to the text (as suggested by Berger, 2016). We can also speculate about why the discussion drifted away from the actual assignment, leading to them thinking about language as a morpho-syntactic structure.

Language as a morpho-syntactic structure

The linguistic, or more explicitly, the morpho-syntactic dimension of language arises if we analyse the linguistic features of Finnish in the previous example 3 above. The morphology of Finnish may be one explanatory factor for the wrong answer. The Finnish assignment is shown in morphemic gloss[1] below.

> Theoretically it is possible that in one of the last 10 years the apartment was not used at all by holiday guests.

Teoria-ssa	*on mahdollista että*	*jonakin vuonna*
theory-IN	is possible-PART that	some-ON year-ON

viimeisten	*10 vuoden*	*aikana*	*lomavieraat*
last-PL.GEN	10 year-GEN	time.ON	the-holiday-guests.PL

eivät käyttäneet	*kyseistä huoneistoa lainkaan.*
not-PL used-PL.PCPL	mentioned.PART apartment.PART at-all

The difficulty was obviously in the last phrase of the sentence: *eivät käyttäneet kyseistä huoneistoa lainkaan* ”did not use the apartment at all”. The students seem to question the meaning of the verb *käyttää* ”to use”. Second, they are unaware of the exact meaning of *lainkaan* ”at all”. Third, the difficulty may also have arisen from the Finnish word for ”apartment”, *huoneisto*, which is derived from *huone* ”room”. So, it may be that the students mixed the two words and misread the statement (”*the room* was not used by the holiday guests”). This would explain why they focused so intensively on the different rooms. And fourth, in addition to *lainkaan*, the statement contained another literary word, namely *kyseinen* ”mentioned” (partitive case *kyseistä*). This adjective has a similar function to that of the definite article in English: ”*the* apartment” – *kyseistä huoneistoa*. In spoken, colloquial Finnish the pronoun *se* is widely used in the function of a definite pronoun: *sitä huoneistoa*. Although the students did not mention this adjective in their discussion, it is likely that it confused the clarity of the sentence.

Although the Finnish language is known for its complex morphology, these data and the students' talk-aloud reasoning show that the difficulties experienced were more lexical than morphological or syntactical. The literary words *kyseinen* and *lainkaan* do not exist in spoken Finnish. The meaning of *käyttää* "use" is flexible in practical language, and the word *huoneisto* "apartment" is very close to its stem word *huone* "room".

One challenge in the task was choosing the relevant pieces of information and ignoring the irrelevant ones. It seems that the table of information regarding the apartment led the participants astray. The statements were only about the concept average, and intended to test their understanding of this concept. Namely, (1) average does not mean that any of the values have to be the same as the average, (2) all the values cannot be bigger or smaller than the average, but some of them can, and (3) average does not reveal anything about distribution and range. Perhaps the participants thought that they should use this table for something.

The fourth example focuses narrowly on numeral inflection and the complexity of the language-specific word morphology of the Finnish language. The third participant of this study, Pavel (9th grade) solved a PISA task called Revolving door.

The task wording plays a minor role here, but it is given for contextualization. The introduction to the Revolving door task read (OECD, 2013b) (see figure 1):

> A revolving door includes three wings which rotate within a circular-shaped space. The inside diameter of this space is 2 metres (200 centimetres). The three door wings divide the space into three equal sectors.

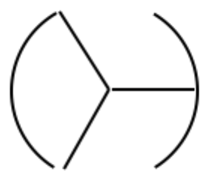

Figure 1. *Revolving door sectors*

In the picture, the door wings in three different positions are viewed from the top. The entrance and the exit, as well as the diameter of the circle and the wings, are all marked in the picture. In question 2 (OECD, 2013b), another picture (figure 2) also helps the student understand the following information.

> The two door openings (the dotted arcs in the diagram) are the same size. If these openings are too wide the revolving wings cannot provide a sealed

space and air could then flow freely between the entrance and the exit, causing unwanted heat loss or gain. This is shown in the diagram opposite.
What is the maximum arc length in centimetres (cm) that each door opening can have, so that air never flows freely between the entrance and the exit?

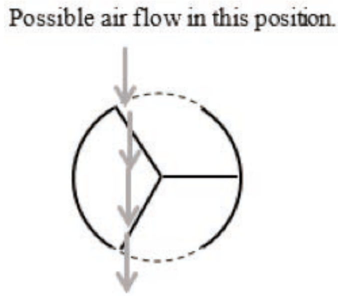

Figure 2. *Revolving door opening. Additional information for question 2*

In Example 4, the participant explains his calculation very briefly (the translation below gives the Finnish numerals in digits).

Example 4. Revolving door – this is difficult

1 Sit mä pyöristin siitä kolmestasadastaneljästätoist
2 Heh, tää on vaikeet
3 Kolmellesadaksiviidelletoistaksi

1 Then I rounded it up from 314
2 Hah, this is difficult
3 On-into-315

His solutions were right, and he could express the right results even with a limited morphological knowledge of Finnish.

In his explanation, Pavel made a comment on the difficulty of verbalizing the numeral expression ”from 314 into 315”. The numeral on the first line, *kolmestasadastaneljästätoist* is grammatically correct, but on the third line, the numeral is inflected incorrectly and is thus almost incomprehensible. However, together with the written output, it is sufficient to prove to the teacher that the process is understood. The difficulty with numbers in Finnish is a common morphological difficulty for learners, as numbers are old Finnish words and many of them are irregular in their inflection. There is no major problem with small numbers, but verbalizing numbers over ten is complex because all parts of the combined words need case endings. In Finnish, the number 315 is *kolme/sataa/viisi/toista* (slashes added for clarity), and ”into 315” spells *kolmeksi/sadaksi/*

viideksi/toista. All parts, ”three”, ”hundred” and ”five” happen to have irregular inflection, which here means changes in stem vowels and consonant gradation. It is unlikely that a speaker would be able to remember the rules and produce a correct form at a normal speech rate with the help of formation rules. Instead, the production of correctly inflected Finnish numerals results from frequent repetitions that automatize the irregular forms. Multimodality appears a helpful strategy (O'Halloran, 2015), as the meaning is made in two parallel modes, the written numerical and spoken linguistic modes. The written mode stands for the mathematical result, and the spoken mode reveals the cognitive process. This type of dual modality functions well in a learning situation at school, but it is also easy to find practical examples in which this type of approximate knowledge of the morphology of numerals would cause problems: let us imagine, for example, an emergency call in which the speaker should be able to verbally guide the ambulance to the right address, or less dramatically, ask about bus routes in a city. This raises a question about the goal of school mathematics: to what extent should receptive skills be emphasized, and to what extent is it a question of productive and communicative skills? If communicating about mathematics is the goal, the demand of morphological precision grows.

Discussion

This study does not divide the language factor into three parts, but illustrates it as a tripartite unity, a three-dimensional notion with conceptualizing, pragmatic and linguistic aspects that overlap and depend on each other. In the context of mathematics classrooms, the conceptualizing phase of language means that language functions as a labelling system for mathematical concepts. This causes lexical challenges in even monolingual contexts, and in a bilingual context, vocabulary-based challenges are doubled. The pragmatic aspect of language is seen in mathematics classrooms as varying discourses, shifts from formal and text-based wordings to more informal and spoken languaging, which also makes use of visual support and contextual temporary meanings. In addition, language is also a linguistic system with morpho-syntactic structure.

This study scrutinized these three dimensions of language through small empirical data on three bilingual, Finnish-Russian participants in a mathematics classroom context. The major obstacles for understanding were terminological and vocabulary-based challenges that belong to the conceptualizing aspect of language. Further, languaging and verbalizing the mental processes appeared possible despite limited morphological skills. Meaning-making or languaging does not presuppose faultless linguistic form. As regards the linguistic and pragmatic aspects, the study shows that languaging became difficult when linguistic explicitness was demanded – this is seen in the fourth example of a participant who had no difficulty in solving the PISA task correctly, but who struggled with linguistic problems while verbalizing the results.

This research supports previous results (Jacobse & Harskamp, 2012; Moskovich, 2012; Jorgensen, Gates & Roper, 2014; Truxav & Rojas, 2014; Berger, 2016; Ng, 2016; Özcan, İmamoğlu & Bayraklı, 2017) regarding the importance of mathematical talk and verbal meaning-making processes in the mathematics classroom, by highlighting their importance and thus encouraging the strengthening of the processes. This study also supports the idea that the process of learning mathematics through a foreign language (CLIL, see Berger, 2016) is in essence similar to learning mathematics through a second language, if the learners have scaffolded access to their first language, as in the case of Russian learners of mathematics in the Finnish-speaking school context. As CLIL studies (Berger, 2016), this research also clearly shows that the linguistic phase preceding the mathematical mode construction is extended when the language of instruction is something other than the student's first language, and the participants showed a tendency of transferring too quickly to the mathematical processing of the word problems. Moreover, this study gives reason to believe that in the case of two morphologically distant languages, morphological difficulty may be compensated with the multimodality of the mathematical register (see also O'Halloran, 2015). The morphological difficulty in the language of instruction did not hinder the cognitive process, which is obviously performed in the first language (see Planas & Setati, 2009), but it led to an unintelligible result in oral languaging.

More research is needed on mathematical word problem processing in two morphologically distant languages, including less-known first languages and cases in which the learners do not have due access or teacher-led scaffolding to mathematical labelling, or the mathematical register in their own first languages. On a theoretical level, this study suggests that the three-dimensional conception of language could be utilized and refined in further studies of language learning in mathematics classrooms.

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Notes

1 The gloss explanations: IN inessive case, "inside"; PART partitive case, ON essive case for temporal "on, during"; GEN genitive case; PL plural; PCPL participle.

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