Mathematics lecturers' views on the teaching of mathematical modelling

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The paper reports on the views and use of mathematical modelling (MM) in university mathematics courses in Norway from the perspective of lecturers. Our analysis includes a characterisation of MM views based on the modelling perspectives developed by Kaiser and Sriraman (2006). Through an online survey we aimed to identify the main perspectives held in higher education by mathematics lecturers and the underlying rationale for integrating (or not) MM in university courses. The results indicated that most respondents displayed a realistic perspective on MM when it came to their professional practice. There was a more varied response when it came to their views on MM in teaching. Regarding conditions influencing the use or non-use of MM in teaching, these mainly concerned the mathematical content and the institutional practices.

Mathematical Modelling (MM) is widely used in engineering, social and natural sciences. While research indicates that the teaching of MM is important and necessary (Blum & Niss, 1991), in practice MM often is not a part of the curricula of mathematics degree programmes. Even when MM is viewed as an important part of students' academic preparation, it is still unclear whether it ought to be taught on its own, as a separate course, or should be incorporated into existing university courses as a subset of skills to be learnt. It has been widely reported that students find MM difficult (Soon, Tirtasanjaya & McInnes, 2011); this adds to the reluctance of many lecturers to introduce MM as part of their teaching. All these factors prevent closing the gap between research and teaching of MM.

In the present study we aim to investigate mathematicians' views of modelling in their professional practice, as academics going about their

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research. We also wish to explore mathematicians' views on the use of modelling in university teaching, their aims in using MM, or their reasons for not using it. The research questions that we will address are:

1. What are lecturers' views on the aims of MM in professional practice and in teaching?
2. How do lecturers claim that they use MM in their teaching? If they do not use MM in their teaching, what are the reasons they give for not using it?
3. How would lecturers prefer to teach MM? What do they perceive as obstacles towards implementing their preferred way of teaching MM?

Answers to these research questions will allow us to gain a better understanding of the current teaching practices regarding MM at university and how these practices relate to what we already know from the research literature on MM. This will help to advance towards closing the gap between research and teaching of MM.

Review of the literature and theoretical perspectives

Many definitions of mathematical modelling are used in the mathematics education research literature (e.g. Garcia, Gascón, Ruiz & Bosch, 2006; Blum, Galbraith, Henn & Niss, 2007; Jablonka & Gellert, 2007; Frejd, 2011). In an attempt to bring clarity to the debate on modelling, Kaiser and Sriraman (2006) developed a categorisation of international perspectives on modelling in mathematics education. They distinguished five perspectives according to their central aims in connection with modelling. These perspectives are based on a review of the literature and although not exhaustive, make a well-defined list. Kaiser and Sriraman's categorisation is appropriate for the purposes of our research, and we have chosen to adopt it as a framework for this study. We will briefly describe each of the five categories presented in Kaiser and Sriraman (2006):

1. *Realistic (or applied) perspective*. The aims of modelling within this perspective are pragmatic or utilitarian, i.e. to solve authentic problems in the way that applied mathematicians would do in their professional practice. As Burkhardt (2006) stated, the reason why mathematics has such a large proportion of curriculum time in schools is due to its perceived utility in solving problems from outside mathematics. MM is therefore

a strategy, or a competence (Niss, 2003), to be acquired in order to solve such problems.

2. *Epistemological (or theoretical) perspective*. The aims of modelling within this perspective are theory-oriented, i.e. to develop theory without paying too much attention to the realistic aspects of a problem. In this perspective, all mathematical activity – including problems entirely within pure mathematics – can be identified as modelling. To encompass this view, Treffers (1987) distinguished horizontal mathematising (in the direction from reality to mathematics) from vertical mathematising (working inside mathematics). This perspective is mostly associated with the French traditions of Chevallard, Brousseau and others (e.g. Garcia & Ruiz, 2006; Dorier, 2006).

3. S*ocio-critical (or emancipatory) perspective*. The aims of modelling within this perspective are to develop critical understandings of the world and the role that mathematics – and in particular mathematical models and modelling – plays in making important societal decisions. Developing mathematical modelling competences through critical reflections of the modelling process and its application is important to this perspective. The socio-critical perspective is mostly associated with work in the area of ethno-mathematics and in critical mathematics education (e.g. D'Ambrosio, 1999; Barbosa, 2006).

4. *Contextual perspective*. The aims of modelling within this perspective are subject-related and relate to psychological development, i.e. modelling activity should elicit the invention, extension and refinement of mathematical (psychological) constructs. This perspective is mostly associated with American traditions, tracing its origins to the American Pragmatism of Pierce, Dewey and James, and modern descendants of Piaget and Vygotsky. More recently, Lesh and colleagues (Lesh & Doerr, 2003; Lesh & Sriraman, 2005) have expanded this perspective – referred to as the model eliciting perspective – with "the premise that modelling research should take into account findings from the realm of psychological concept development to develop activities which motivate and naturally allow students to develop the mathematics needed to make sense of such situations" (Kaiser & Sriraman, 2006, p.306).

5. *Educational perspective*. The aims of modelling within this perspective are pedagogical, i.e. modelling should foster the understanding of mathematical concepts and structure the learning processes, developing mathematical modelling competencies. The modelling cycle described

e.g. in Blum and Leiß (2006), can be considered a useful pedagogical tool that represents the modelling process. This perspective can be differentiated into two sub-categories:

a. *Didactical modelling*, which aims to structure the learning processes by using modelling as a heuristic guide – a la Pólya (1957) in his famous book ”How to solve it” – that might make problems more accessible. Pedagogical instruction would aim to develop learners' understanding of all the parts of the modelling cycle.

b. C*onceptual modelling*, which aims to use modelling as a strategy or motivation to introduce mathematical concepts and develop their understanding. The starting point for teaching within this perspective is usually a ”realistic” problem (in the sense of Freudenthal's Dutch Realistic Mathematics) that is ”begging to be organised” using the mathematics to be learnt (Freudenthal, 1983). Once the problem has been ”mathematised”, a resulting mathematical model is an example of the mathematical concept to be learnt (e.g., a differential equation).

We complement the above categorisation with a new category that has not been explored in the MM research literature per se. It is usually referred to by professional mathematicians and mainly relates to mathematical work in highly abstract theoretical fields. We call this the *Enjoyment (or affective) perspective* in which the aim of modelling is the intrinsic satisfaction derived from engaging in modelling activity. Famous mathematicians such as Poincaré (1890) or Hardy (1941) talked about the beauty of mathematics and the enjoyment or pleasure they found in studying it, without reference to its utility in any sense.

We consider MM to be an essential part of mathematical work and, alongside many professionals involved in mathematics research and teaching, find this activity very pleasant, too. However, enjoyment of MM, in contrast to enjoyment of mathematics in general, has been referred to less frequently and often rather pragmatically, with a more pronounced emphasis on utilitarian aspects. For example Dym (2004, p. 4), the author of a popular textbook on MM, writes:

> Since the modeling of devices and phenomena is essential to both engineering and science, engineers and scientists have very practical reasons for doing mathematical modeling. In addition, engineers, scientists, and mathematicians want to experience the sheer joy of formulating and solving mathematical problems.

By incorporating this category into the existing Kaiser and Sriraman (2006) classification, we wanted to see if lecturers find this aspect of MM important. In this respect, we share the views of Pollak (2015, p. 275) in relation to the goals of MM:

> A number of people have written books entitled something like "The Joy of Mathematics". I should like to see a book entitled "The Joy of Mathematical Modeling", consisting of fifty to a hundred examples, taken mostly from everyday human experience. The joy I have had in my life of doing and teaching mathematical modeling should be transmitted: Will you join me?

Methodology of data collection and analysis

In this section we describe our methodology which uses the categorisation presented above.

The perspectives categorised in Kaiser and Sriraman (2006) provided the conceptual framework for the study and an analytical tool in data analysis. The five categories came about as a result of research into the teaching of modelling and relate specifically to the context of teaching. They were not developed with regard to lecturers' use of modelling in their own professional practice, and not all of them were relevant for this purpose. Hence, in order to gain insight into lecturers' use of modelling in their profession as researchers, we considered just three modelling perspectives. Two perspectives, the realistic and the epistemological, were taken from Kaiser and Sriraman's (2006) categorisation as the descriptions translated well into this new context. The third perspective used was the enjoyment category mentioned above. We developed and conceptualised this perspective based on the literature on mathematicians reflecting on and writing about their own practice. These three perspectives were considered to represent the aims of a professional mathematician using modelling for research purposes. We hypothesised that applied mathematicians would have a mainly realistic perspective and pure mathematicians a mainly epistemological perspective. We anticipated that the enjoyment perspective would characterise both groups.

In order to gain insight into lecturers' use of modelling *in their teaching* we considered all five of Kaiser and Sriraman's (2006) perspectives and, in addition, the enjoyment perspective.

Design of the questionnaire

To answer the research questions, we developed an online questionnaire consisting of questions and statements asking lecturers to express their view of modelling and of teaching modelling, and to provide information about their experience of using modelling in research and teaching. We will refer to both questions and statements as "items" that formed the content of the questionnaire.

The first part of the questionnaire was related to demographical data, such as work experience, gender, age, and research area. We also asked whether the respondent had any experience working with modelling in research or industry.

The second part of the questionnaire was concerned with views on modelling in relation to professional activities. The items in this section were of different forms. For instance, there were Likert scale items where participants were asked, on a scale of 1 to 5, to agree or disagree with a statement. Most statements were developed by consulting the literature referenced above, as well as textbooks on mathematical modelling such as Giordano, Fox and Horton (2013), Shiflet and Shiflet (2014), and Velten (2009). Examples of statements used in the Likert scale items are: "Models illustrate mathematical concepts", "Validation of a model against real data is vital for modelling", and "Group work is vital for modelling". One questionnaire item involved participants ranking statements about the aims of modelling. This item was designed based on the realistic and epistemological perspectives in Kaiser and Sriraman (2006) (three statements for each perspective) and augmented with a further three statements that focused on the enjoyment perspective. An additional option was "None of the above". The categorisation of the statements was not suggested to respondents. Respondents were asked to choose and rank three statements that best corresponded to their views. An excerpt from the online questionnaire is provided in figure 1.

The aim of practicing mathematical modelling (in your professional capacity) is

- to describe, explain and/or predict reality. ☐
- to advance mathematical theory. ☐
- to foster creativity. ☐

Figure 1. *Excerpt from questionnaire*

These statements represent the realist, epistemological and enjoyment categories, respectively. Other examples were "to solve real-life, authentic problems", "to increase our understanding of the real world" for the realistic perspective, "to gain mathematical insights", "to solve problems purely within (pure) mathematics" for the epistemological perspective, and "for interest and/or enjoyment", "to address/seek out challenges" for the enjoyment perspective. The formulations of the statements of the realist and the epistemological categories were lifted from the descriptions presented in Kaiser and Sriraman (2006). The formulation of the statements representing the enjoyment category was inspired by the literature on the enjoyment of MM mentioned above.

The third part of the questionnaire was concerned with views on the teaching of modelling. We collected data on teaching experience at the university level, and the level of teaching (i.e. undergraduate, postgraduate). We also asked whether the respondents had experience of using modelling in their teaching, the form in which modelling was taught, and to describe what would be their preferred way of teaching modelling. As in the second part, we included a Likert scale item with statements regarding the teaching of modelling as well as a ranking item, based on the categorisations of Kaiser and Sriraman. We augmented the five categories of Kaiser and Sriraman with the enjoyment category once again. To reduce the number of statements we used two statements representing each category, resulting in a total of 12 statements. Respondents were asked to choose and rank three that most corresponded to their view. An excerpt of the ranking item can be found in figure 2.

The aim of teaching modelling is for students

- to become more critically aware as citizens. ☐
- to gain conceptual understandings of mathematics. ☐
- to learn to apply models and modelling to different situations. ☐

Figure 2. *Excerpt from questionnaire*

These statements fit the remaining three categories of Kaiser and Sriraman (2006), namely the socio-critical, educational and contextual categories, respectively. Throughout the questionnaire, a number of statements included open comment boxes, with the aim of providing a degree of freedom and choice in the formulation of an answer. The questionnaire was piloted twice at the conferences of the Norwegian Centre for Research, Innovation and Coordination of Mathematics Teaching (MatRIC) with a mixed group of 10–14 mathematics and mathematics education researchers. On both occasions piloting led to changes in the design and content of the questionnaire.

Conducting the survey

The survey took place with both English and Norwegian mathematicians. Two similar sets of data based on the views of lecturers in English and Norwegian universities were obtained. However, at the time of writing the analysis of the English data was incomplete due to delayed returns of the questionnaire. Hence, we present our analysis for the Norwegian sample only.

The survey was conducted using SurveyXact, an online questionnaire software, and an invitation to participate was sent by individual emails to 498 mathematicians in Norway. The criteria for including an individual were that he/she had to be an academic member of staff, working in the mathematics department at a Norwegian university. The email list was compiled using lists of academic staff on university websites, augmented by the names from already existing, but incomplete mailing lists available to the authors. In a few cases where no staff lists could be found on the university websites, recipients at those universities were kindly asked to forward the email to their colleagues (in their mathematics departments). The questionnaire was sent out to all universities and to most university colleges that teach a significant amount of mathematics. This way we believe we have reached the majority of academic staff working in mathematics departments in Norway. We are aware that many people working in other departments (i.e. engineering, economics) also teach university level mathematics. Hence this survey is of university mathematicians rather than of teachers of university mathematics more generally. We have also included PhD students in our survey. There are two reasons for doing so. Firstly, in Norway, PhD students are often employed for (class) teaching mathematics and hence belong to our target group. Secondly, it was at times not possible to distinguish PhD students from academic staff listed on university websites.

We received 119 responses to the questionnaire which formed the sample used for analysis. We were pleased with this response rate which corresponded to 24 % of all questionnaires sent. Participants self-selected to take part, and there is a definite possibility that people with experience of and/or a clear view on modelling may have responded to a larger extent. However, the accompanying email clearly stated that we were equally interested in responses from lecturers who did not, and never have used modelling, thereby trying to reduce the risk of self-selection bias. We do not claim that the results are representative of the views and practices of all Norwegian university mathematicians.

38 of the 119 participants submitted partial responses. In some cases this was due to technical difficulties. In other cases the reason remained unclear and may have been due to a participant's decision to log out. Thus, on any particular item on the questionnaire the number of responses varied.

In analysing the data, we used both quantitative and qualitative methods. The quantitative analysis is mainly descriptive, with additional statistical analysis of some of the category data. This consisted of comparative analysis aimed at eliciting differences in response patterns among different groups of the respondents. Qualitative analysis was appropriate for the written responses that participants provided in comment boxes. These were analysed using qualitative methods of analysis such as interpreting, coding, and categorising (Cohen, Manion & Morrison, 2008).

Results

Starting with an overview of the demographical data, most respondents (98 %)[1] said that they were working in higher education, at a university or university college, with 44 % having done so for more than 16 years. 19 % of the participants indicated 3 years or less, which most likely represented PhD students. There was an almost even split of 13 %, 14 %, 10 % for the other three groups, 4–7, 8–11, 12–15 years. The responses were almost identical when we asked for how long participants had been teaching, instead of working in higher education. Gender balance male to female was 4:1. Regarding age, 35 % indicated 51 and over, 26 % and 22 % indicated 31–40 and 41–50 respectively, and 17 % indicated 21–30. Concerning PhD specialism, the 119 respondents were fairly equally divided between pure mathematics (31 %), applied mathematics (31 %) and other subjects, including physics, statistics and mathematics education (27 % in total); 11 % of respondents did not have a PhD. A large majority of the participants (87 %) indicated that they were active in research, mostly in mathematics or statistics, but there were also a number of respondents doing research in physics, engineering, mathematics education and other fields.

Lecturers' views on the aims of MM

To answer the first research question, we analysed responses to two ranking items on the questionnaire. The first of these related to participants' views on the aims of mathematical modelling in their professional work. Out of the 91 respondents who answered this item an overwhelming majority opted for statements we had categorised as corresponding to the realistic perspective on modelling (table 1). 45 % of respondents selected all three statements from the realistic perspective and a further 35 % selected two statements from the realistic perspective. None of the respondents chose all three statements that corresponded to the epistemological or to the enjoyment perspective. Only 9 % and 3 % chose two statements (plus one other) corresponding to the epistemological and the enjoyment perspective, respectively. A further 8 % chose a mixture of statements and hence no categorisation could be made.

Table 1. *Choice of category*

Statements chosen	Participants	
3 realist statements	41	(45 %)
2 realist statements	32	(35 %)
3 epistemological statements	0	(0 %)
2 epistemological statements	8	(9 %)
3 enjoyment statements	0	(0 %)
2 enjoyment statements	3	(3 %)
Mixed (one chosen from each category)	7	(8 %)
Total	91	(100 %)

Regarding the ranking of the statements, we argued that the respondents' highest ranked statement was the one that better reflected their views. Hence we considered which statement participants selected as their first choice (table 2). As a result, the realistic perspective became even more pronounced, with 85 % of respondents selecting, as their first choice, one of the three statements corresponding to the realistic perspective.

Table 2. *Participants' first choice*

Statement chosen	Participants	
Realist	77	(85 %)
Epistemological	9	(10 %)
Enjoyment	5	(5 %)
None of the above	0	(0 %)
Total	91	(100 %)

Next, we compared the distribution of answers of two groups, namely those selecting all three realist statements and those not selecting three realist statements, with the latter implying a different or less strongly held realist perspective. Comparing the distribution of answers from these two groups, no significant differences were observed. Respondents in the realist group tended to be male and to have used modelling in their own research to a larger extent than the other group. But, as mentioned, neither of these differences was statistically significant. One might have expected an overrepresentation of respondents in the realist group who have a PhD in applied mathematics but the data and our analysis did not show such a connection. On the contrary, the distribution of PhD subject areas in the two groups was very similar. Hence we believe it is fair to conclude that, in relation to professional activity, the majority of respondents took a realist view on mathematical modelling, regardless of their field of research.

We now consider the second ranking item on the questionnaire related to participants' views on the aims of teaching mathematical modelling. There were two statements corresponding to each perspective. Nearly half (47 %) of the 81 respondents answering this item chose two statements from the same category. We interpreted this choice as indication of the commitment to a particular view on the aims of teaching modelling. We found that the realistic perspective was represented most often, with 35 % of all responses including two realist statements (table 3). We also found that each of the six perspectives was represented by at least one participant. On the other hand, 53 % of respondents chose statements from different categories so that we could not relate the response to any single perspective. This response pattern could be interpreted as indicating a wider variation in views on the teaching of modelling.

Table 3. *Participants' choice of category*

Statements chosen	Participants	
2 realist	28	(35 %)
2 epistemological	3	(4 %)
2 enjoyment	3	(4 %)
2 socio-critical	2	(2 %)
2 educational	1	(1 %)
2 contextual	1	(1 %)
No two from the same category	43	(53 %)
Total	81	(100 %)

Again, focusing on respondents' first choice, we found that 50 % of respondents selected a realist statement as their first choice and 20 % of respondents opted for a contextual statement as their first choice (table 4). It should be pointed out here that the difference in frequency between the two contextual statements was very large, with 15 respondents selecting one of the two statements and one respondent the other. Furthermore, the more popular statement of these two ("The aim of teaching modelling is for students to learn to apply models and modelling to different situations") could be seen as consistent with a realistic perspective if "different situations" were interpreted as pertaining to "real-life situations".

Table 4. *Participants' first choice*

Statement chosen	Participants	
Realist	41	(50%)
Contextual	16	(20%)
Educational	8	(10%)
Epistemological	7	(9%)
Socio-critical	5	(6%)
Enjoyment	4	(5%)
Total	81	(100%)

Looking more closely at the seeming dominance of the realistic perspective, we investigated the responses of the 41 participants who had selected all three realist statements in the item regarding the aims of mathematical modelling. Of these, 36 had responded to the second ranking item, about the aims of *teaching* mathematical modelling. Of these 36, 20 had selected both realist statements in the second item, and could thus be said to hold views that were firmly realist. Thus, there is a small group of 16 respondents who display strong realist views on the aims of modelling, but more varied views on the aims of teaching modelling. On the other hand, 10 of those in the second group (not having selected all three realist statements in the first item) selected both realist statements in the second item. Analysing the answers of these 10 respondents in more detail, we find that they have for the most part selected realist statements to the first item. Hence, these 10 respondents can also be thought of as holding views that were mostly realist. In conclusion, the realistic perspective on the aims of both modelling and the teaching of modelling was a prominent one among respondents to the questionnaire.

Lecturers' claims on the use of MM in teaching

The analysis presented in this section is aimed at answering the second research question: "How do lecturers claim that they use MM in their teaching?" To answer this question, we asked lecturers about their teaching practice and their current and past use of modelling in teaching mathematics. This included information about extent of teaching experience, the level of teaching and the kinds of students taught, and whether teaching of modelling was organised as a part of a course or as a separate course. Our aim was to elicit responses that could help us to characterise the use of MM in teaching. In figure 3 we list the items that we analysed in order to answer this research question.

1. Do you use, or have you used models and/or modelling in your teaching?

2. If you use(d) models and/or modelling in your teaching, which statement best represents how you use(d) it?

– Mathematical modelling is taught as a separate course.	
– Small mathematical pure units are followed by applications where modelling is used.	
– Mathematical models are used as illustration of theory.	
– Mathematical modelling is used as motivation for introducing new theory.	
– None of the above. (Can you provide more details below?)	

3. If you do not, or have not use(d) models and/or modelling in your teaching, please explain briefly why not.

Figure 3. *Questionnaire items related to second research question*

Item 1 (in figure 3) separated our sample group into those who had experience of teaching MM and those who had not. Both subsets were important in advancing our understanding of use and non-use of MM in teaching, with each subset of participants then answering either item 2 or item 3. Item 2 consisted of statements that related to the use of MM in structuring the course (as a separate course or as small units) and in structuring the teaching and learning of modelling (to illustrate theory, for motivating new theory). We also included an option for participants to provide their own description. Item 3 was an "open response" type as we anticipated a varied response to why participants did not use MM in their teaching.

We found that 90 % of all respondents had current or past experience of teaching at tertiary level, mostly to undergraduates in mathematics and statistics but a significant number also indicated teaching master level and PhD students. Nearly half of all participants (48 %) had been teaching in higher education for more than 16 years which matches the responses given to an earlier item, on the length of time working in higher education. Of the 82 participants who answered the first item, 74 % (61 participants) indicated that they used models or modelling in their teaching.

The second item gave an indication of *how* participants used modelling in their teaching. This was a ranking item where we considered that the statement ranked the highest most clearly reflected the respondents' use of modelling. Of those who indicated that they did use modelling in their teaching 30 % said they used models or modelling "to illustrate theory". We interpreted this as providing examples of mathematical concepts already taught. A further 16 % answered that they used models or modelling "to motivate new theory", which we interpreted as an indication that modelling preceded the more traditional teaching of mathematical concepts. Within these two options modelling was a part of the mathematical instruction. In contrast, 16 % of the respondents said they taught modelling as a "separate course", which implied that the instruction was not focused on teaching mathematics but rather on the development of modelling skills. A further 9 % taught modelling in "small units followed by applications where modelling was used" which is similar in structure to the previous response. Hence 46 % gave a mathematical reason, 26 % referred to institutional level affordances for using models or modelling and 26 % did not use MM at all. These results are tabulated in table 5.

Table 5. *Use of MM in teaching*

How MM was used in teaching	Participants	
To illustrate theory	25	(30 %)
To motivate new theory	13	(16 %)
Separate course	13	(16 %)
Small pure units	7	(9 %)
Own description	2	(2 %)
None of the above	1	(1 %)
Did not use MM	21	(26 %)
Total	82	(100 %)

The third item was aimed at eliciting responses that could give insights into the reasons for not using modelling in teaching and corresponded to the second part of research question 2. Responses to this item were in the form of written comments. These were qualitative in nature and hence analysed using qualitative methods of interpreting and categorising. Some of the responses did not address the question and were removed resulting in 20 coded comments.

In conducting this analysis we distinguished 4 categories of reasons why MM was not used: the nature of mathematics, institutional issues, (lack of) teaching skills and student profile. We provide an explanation of the categories developed from the analysis and some of the participants' responses taken from the questionnaire (see table 6).

Half of all comments (50 %) related to reasons that linked modelling closely to the nature of mathematics and to the content that had to be taught so that modelling was not relevant, or it was not appropriate to include it. We particularly noted that some comments related to modelling not being relevant when teaching pure mathematics. Approximately one third (35 %) of all comments related to institutional constraints. Here 3 out of the 7 participants stated that they did not have the opportunity

Table 6. *Reasons for non-use of MM in teaching*

Category	Examples	No. of comments
(Nature of) mathematics itself. Comments referred to the nature of mathematics or the mathematical curriculum/content of modules, in particular modelling is not relevant for pure mathematics.	”not relevant for my courses”, ”not in the curriculum”, ”I had no need for that”, my subject is ”purely theoretical”.	10 (50 %)
Institutional issues. Comments related to teaching practices (institutional level constraints) where individuals had no opportunity to include MM or were under time constraints that prevented them from incorporating MM.	”I did not have the opportunity”, ”no time to teach modelling”, ”modelling taught in a separate course”.	7 (35 %)
Teaching skills. Comments referred to a lack of skills or experience for teaching modelling.	”Not familiar with …”, ”I have little experience with modelling”.	2 (10 %)
Student profile. Comment was about education (as context).	”students lack quite a few mathematical skills…”.	1 (5 %)
Total number of responses		20

to include or teach modelling. Others stated that they did not have the time to do so or that modelling was taught as a separate course at their university.

Two participants (representing 10 % of the respondents) claimed that they were not familiar with or did not have the necessary skills to teach modelling. 1 participant (5 %) cited students’ lack of mathematical skills for not including modelling in teaching.

The results from the latter analysis will be combined with what we learnt about participants’ views on how modelling could, or should be included in teaching. There is some overlap between responses on ”best use” and on not using MM which is discussed in the section *Lecturers’ preferred way of teaching MM*.

Additional analyses of group dynamics

We conducted additional analysis to elicit any differences between groups of respondents. However, when comparing, for instance, respondents with applied mathematics PhDs with the rest of the respondents, ”strong realists” with the rest of the respondents, or users of mathematical modelling in research with the rest of the respondents, we found very few significant differences. For instance, there were no significant differences between the strong realists and the remaining respondents on any of the other category items.

Of the few significant differences that we did find, unsurprisingly, respondents with PhDs in applied mathematics had used MM in their research to a significantly greater extent than the rest of the respondents ($p < 0.005$). Furthermore, respondents not having used MM in their teaching had significantly less teaching experience than those who had used it ($p < 0.05$). Finally, and highly relevant to the current study, when comparing the use of MM in teaching among those respondents who had experience of using MM in research and those who had not, we found that respondents having used MM in their research were significantly more likely (with a fairly large effect size – an odds ratio of 6.6) to also use it in their teaching ($p < 0.005$). This is perhaps not so surprising, but it does suggest that lack of research experience with MM might well be an obstacle to using it in teaching.

Lecturers' preferred way of teaching MM

In this section we present the analysis related to our third research question. We asked all participants, including those without teaching experience (of mathematics and/or MM) at tertiary level, what they considered to be the best way to teach MM. We also asked under what circumstances they would be most likely to teach MM this way (figure 4).

The first item was aimed at finding out lecturers' views on the "best way" of introducing and using modelling in teaching. The item was formulated in almost identical way as the item on lecturers' use of modelling in teaching in the previous section. Hence "best use" could be compared with "actual use" in analyses. In contrast to the item in the previous section – where a response in the form of a single tick was required – here

1. In your opinion, what would be the best way to use mathematical modelling in teaching? Choose three of the following statements that best fit your view and rank them in order of importance (where 1 is most important, 2 is second most important and 3 is third most important).

– Mathematical modelling is taught as a separate course. ☐
– Small mathematical pure units are followed by applications where modelling is used. ☐
– Mathematical models are used as illustration of theory. ☐
– Mathematical modelling is used as motivation for introducing new theory. ☐
– ____________________ ☐
[You may add your own statement which should be one of those ranked]

2. When (under what circumstances) would you be most likely to adopt this way (ranked 1 above) for your own teaching?

Figure 4. *Questionnaire items related to third research question*

participants had to choose three statements and rank them. We categorised the statements exactly the same way that we did in the previous section on the use of MM. Hence, two statements were categorised as relating views (on the "best use" of modelling in teaching) to the structuring of the course (as a separate course or as small units) or to the structuring of the teaching and learning of modelling (to illustrate theory, for motivating new theory). We also included an additional option for lecturers to enter their own description. The second item was aimed at investigating circumstances surrounding MM in teaching that could be regarded as providing a barrier to introducing MM in teaching as well as conditions that could be regarded as facilitating its introduction and use. This second item was formulated as part of an open comment box to allow participants to formulate and write their own answer. We viewed the latter item as contributing to our analysis of item 3 of the previous section, which was aimed at eliciting responses from those who did not use MM in teaching.

Of the 90 participants who answered the first item on the best use of MM in teaching, almost 60 % selected (as their first choice) one of the two statements focusing on mathematical content/syllabus. Here responses were divided between "to motivate new theory" (32 %) and "to illustrate theory" (slightly less at 27 %). However, this is a reversal of preferences compared with the responses on actual use of MM where clearly more participants said that they used MM "to illustrate theory". In addition, 21 % chose "small units followed by applications", and 11 % chose "as a separate course". Thus 59 % expressed a view that focused on the use of MM in mathematics and the mathematics curriculum while 32 % focused on the structure of a course as the best way of embedding MM in teaching, leaving aside the mathematics.

We completed the analysis of the first item by considering the responses of the 9 % of participants who offered their own description. These were written responses in comment boxes that we analysed qualitatively. Responses varied, with about half in favour of using modelling when it was "helpful", "fitted with the purposes of teaching" or was "natural" in the given context.

A summary of the results of "best" use of MM is presented in table 7 alongside the analysis from the previous section on participants' actual use of MM.

The responses provided to the second item gave insights into lecturers' perceptions of barriers to, or conditions that would favour introducing MM in teaching. A total of 86 comments were received but 28 were omitted as not relevant in addressing the question. Through a qualitative analysis of the remaining 58 comments we distinguished 4 categories of

Table 7. *Actual and best use of MM in teaching*

Use of MM in teaching	Actual use	Best use (rank 1)
To illustrate theory	25 (30 %)	24 (27 %)
To motivate new theory	13 (16 %)	29 (32 %)
Separate course	13 (16 %)	10 (11 %)
Small pure units	7 (9 %)	19 (21 %)
Own description	2 (2 %)	8 (9 %)
None of the above	1 (1 %)	- (0 %)
Did not use MM	21 (26 %)	- (0 %)
Total	82 (100 %)	90 (100 %)

conditions that participants referred to as leading them to adopt MM in teaching: the nature of mathematics to be taught, due to a decision taken at institutional level, "Always" implying that MM is integral to teaching mathematics, and student profile. We present our analysis with an explanation of the categories developed from the analysis and cite some of the participants' responses taken from the questionnaire (see table 8).

Slightly more than half (52 %) of responses related the use of MM to the nature of the mathematics that had to be taught. Respondents stated that they would use MM if the circumstances were right with respect to the curriculum/mathematical content of the course. In particular, participants stated that they would use modelling to illustrate theory, when teaching dynamical systems, to model ecology and evolution, or to show an application. These are all specific descriptions related to curricular issues which indicate a dependence on the curriculum (and on the mathematics to be taught) for using modelling in teaching. 28 % of respondents would introduce MM in teaching when circumstances allowed. In particular, participants would use modelling in teaching when it fitted naturally, when time allowed, and when they had more freedom or control over the course or the curriculum. A dependence on these kinds of circumstances seemed to suggest a constraint or influence of institutional practices (maybe locally only as "classroom practices"). 14 % of respondents answered strongly with "Always" or "in most courses". This suggests to us that the individual saw modelling as integral to mathematics teaching and would use modelling as much as he/she could and in most courses. 7 % of respondents referred to circumstances that related to student profiles and were dependent on the cohort of students taking a course, in particular students' interest and students' ability or lack of ability in mathematics. These were all specific descriptors relating to students, a dependence on students' profile for using modelling in teaching. The analysis is presented in table 8.

Table 8. *Conditions for including and excluding MM in teaching*

Category	Examples	No. of comments
Mathematics specific (nature of mathematics): Dependent on the mathematical content of a course	modelling ”to illustrate theory”, ”when teaching dynamical systems”, ”to model ecology and evolution or to show an application”.	30 (52 %)
Dependent on institutional level decisions: Influenced by institutional practice	”Whenever possible”, ”when it fitted naturally”, ”when time allowed”, ”when [having] more freedom or control over course or curriculum”.	16 (28 %)
Integral to teaching: Most common response was ”Always” implying that modelling is integral to teaching	”Always”, ”... in most courses”.	8 (14 %)
Student profile: Dependent on students' interest or ability	”enough students ... are interested”, dependent on ”students' ability”.	4 (7 %)
Total number of responses		58 (100 %)

Discussion

In this paper, we have characterised how MM has been used by the participants in our survey, both in research and in teaching. The respondents have indicated how they would prefer to use MM, and which context and conditions they regarded as favouring the introduction of MM in teaching. Answers to these questions provided significant insight and new knowledge on the use and aims of MM in higher education.

There is a vast research literature on MM in education and, as discussed above, within this literature there seems to be disagreement about what it means to do MM (or its definition) and the aims of MM for education. In our project, we attempted to shed light on how practitioners (i.e. lecturers of mathematics) view MM, its role in teaching mathematics and on factors that hinder or facilitate the implementation of MM in teaching practices.

In our analysis of the data from the Norwegian lecturers, we found that their views on the aims of MM in relation to professional practice were largely consistent with a realistic perspective, i.e. MM is a tool to solve ”real world” problems. This result was surprising to us, as we expected that applied mathematicians would have a realistic perspective and pure mathematicians an epistemological perspective, given the different nature of their research. We were also surprised that the enjoyment perspective of MM was emphasised by very few lecturers.

Based on the questionnaire data we could not detect any significant differences concerning patterns of use of MM between respondents with a strong realistic perspective and the remainder of the respondents. However, the questionnaire data contained very little information on *how* MM was used in teaching. This is an area we intend to study further, using interviews and lecture observations to investigate the possible impact of a modelling perspective on teaching practice. To this end we intend to conduct interviews with mathematicians adhering to a realistic perspective as well as with those adhering to one of the alternative perspectives.

While respondents' views on the aims of using MM in their professional practice were overwhelmingly realistic, the dominance of the realistic perspective on the aims of MM in teaching was less pronounced. This can partly be explained by a larger number of statements in the questionnaire item on the aims of MM in teaching. Still, there were many respondents who selected statements from three different perspectives. We see this as indicative of a wider variation in the views on the aims of MM in teaching among the university mathematics teachers who took part in our study. To some extent this contrasts with the education research literature on MM at school level, where researchers have found that practitioners typically adhered to one particular perspective. In this context it is also worth noting that the educational and contextual perspectives, prevalent in MM research at school level, were rare amongst the participants in this study. It is conceivable that this reflects a difference between mathematics education at school level and at university level, both in terms of content and pedagogy.

One of the few significant differences found between groups of respondents was the fact that respondents who used MM in research were significantly more likely to use it also in their teaching. We have observed how the reasons given by participants for not using MM in teaching to a large extent were concerned with the nature of mathematics. Focusing on respondents who had neither used MM in research nor in teaching, we found the reasons they stated for not using MM in teaching mostly related to the nature of mathematics. A possible interpretation is that these arguments might be explained at least in part by a lack of experience of what MM can be in practice.

One of the motivations for conducting this study was to identify factors that could support or hinder the introduction and use of MM in university teaching practices. Considering the analyses from the previous sections we now discuss and summarise how four factors, namely the nature of mathematics, institutional issues, teaching skills and student profiles can support or hinder the use of MM in teaching.

1) *The nature of mathematics*: Participants in the study expressed the view that the inclusion of MM had to be considered alongside the mathematical content to be taught. Although our sample consisted of pure and applied mathematicians in equal numbers, the dominant perspective on modelling was realistic and related to using MM in the more applied

mathematical fields. Some participants expressed the view that modelling was an integral part of teaching (all) mathematics while others commented that MM was not relevant in pure mathematics. If more practitioners could become aware of the multiple aims of MM this might influence its wider use at university.

2) *Institutional issues*: These were mentioned far less than mathematical content or curricula. However, it seems that some participants would teach MM if it were an element or part of a course (e.g. if it fitted naturally). Reasons given for not using MM were lack of time, lack of opportunity and instances when MM was taught as a separate course (i.e. they would not teach it additionally). Considering these issues could help influence MM practices if institutional policies reflected the important role of MM in mathematics curricula. Given the few comments received about these issues we are unsure to what extent institutional changes could positively affect the use of MM in teaching.

3) *Teaching skills*: A lack of experience or skills in using and teaching MM was expressed by some respondents and related to not supporting MM in teaching. However, the dominant discourse of our sample group of mathematicians is a realistic one with many comments on modelling skills acquired and used in industry. Hence we hypothesise that many mathematicians have the skills but maybe not the confidence to teach modelling in their courses.

4) *Student profiles*: Some participants made references to students and students' ability or lack of ability. Participants indicated that they would use, or felt encouraged to use MM if students were interested and mathematically able, and not use it if they considered their students to be mathematically weak.

This summarises possible factors that have been expressed either as a view (actual use or "best" use) of modelling in teaching or as a reason given for not using it. While most respondents focused on the mathematical content and curricula as determining the use or non-use of MM in teaching, it is less clear to what extent institutional support in terms of curriculum and course structuring could positively affect the use of MM in teaching.

Some implications from this study – coming from a relatively small sample who stated a clear opinion on the matter – for university teaching are: (a) MM should be explicitly documented in the curriculum, and (b) mathematics education researchers and lecturers should collaborate closer so that the latter become aware of other aims of using MM in teaching (as detailed in Kaiser and Sriraman, 2006, for example) that could be advantageous for learners and create diversity in the learning of mathematics.

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Notes

1 Two respondents (2 %) answered that they were not working at a Higher Education College or University. A closer look at the responses given by these two individuals has led us to conclude that they were academics who either moved away from Higher Education or recently retired.

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