Theorizing the interactive nature of teaching mathematics: contributing to develop contributions as a metaphor for teaching

ANDREAS ECKERT

The teachers' role in teacher-student interaction in mathematics has received increased attention in recent years. One metaphor used to describe teaching in teacher-student interaction is to describe teaching as a learning process itself, in terms of learning to develop learning. The aim of the present study is to contribute to the conceptualization and understanding of this view of teaching mathematics. This is done by introducing and elaborating on a new conceptual framework, describing teaching as Contributing to Develop Contributions (CDC). The CDC framework is constructed by combining the theory of symbolic interactionism with a complementing metaphor for learning; learning as contribution. The CDC-framework is illustrated in the context of experimentation-based, interactive teaching of probability. The analysis shows how the CDC-framework helps in coming to understand how teachers develop their own contributions to manipulate the negotiation of meaning of mathematics in the classroom and thereby also develops the students' contributions. In the presented case we can see how CDC particularly helps in giving account of how a teacher develops her way of using symbols and indications and adjust her own interpretations during a whole class discussion where the teacher and students interpret the empirical results of a random generator. In addition, the analysis also illustrates how the framework draws our attention to how a teacher can contribute to the negation of meaning, and so, to students' opportunities to learn, by making her own interpretations and ways of ascribing meaning to objects transparent to the students in the interaction.

The word *to teach* generally mean *to impart knowledge to*, *instruct* or *give information about* (Oxford Dictionaries, 2016). Furthermore, the word has its roots in old English and Germanic language, *meaning to show*, *present*, *point out* or *represent*. Franke, Kazemi and Battey (2007) suggest a broader definition of teaching when it comes to mathematics education

Andreas Eckert
Linnaeus University

Eckert, A. (2017). Theorizing the interactive nature of teaching mathematics: contributing to develop contributions as a metaphor for teaching. *Nordic Studies in Mathematics Education*, 22 (3), 51–69.

research, namely as a collection of practices, e.g. lecturing, talking with parents or operationalizing the curriculum. This paper will deal with one practice, the classroom interactions of teachers and students related to mathematics. Lester (2005) problematizes this teacher-student interaction by raising what has become one of the big questions of mathematics education research: What is the teacher's role in instruction? Rephrased with the focus on teacher action, the question to ask can be: What is the teacher's contribution to interactions in the practice of students' learning? The epistemological assumption of the present study is that teaching – in terms of teachers' contribution to students' learning in teacher-student interaction – should be understood as a direct outcome of that interaction rather than an outcome of a stable knowledge construct of teacher knowledge (cf. Eckert & Nilsson, 2015).

Previous conceptualizations of teaching follow different paths. Some focus on distinguishing predetermined teacher attributes, such as teacher knowledge, beliefs and intentions, and how combinations of such attributes can be used to explain teachers' actions in the classroom (e.g Schoenfeld, 1999). Others focus on teachers' actions and strategies to create fruitful conditions for learning in the classroom (e.g Jaworski, 1994). Thus, there is a focus in the literature on teachers as organizers of a learning environment through strategies based on their knowledge of students and mathematics. Voigt (1994), and others with him, shifted the focus towards interactions in the classroom and showed the benefits of viewing meaning as something that arises in the interaction. If the meaning-making process is viewed as a reciprocal responsibility amongst teachers and students, teaching becomes something else than imparting knowledge of giving instruction. Jaworski (2006) takes the interactive perspective on teaching even further, saying that it is not only meaning-making that is distributed amongst teachers and students but learning as well. She suggests that teaching can be viewed as a process of learning, a process of *learning to develop learning*.

The aim of the present study is to further develop the notion of teaching as learning to develop learning by theorizing interactional processes in teacher-student interactions in classroom mathematics discourse. There is potential to learn more about teacher-student interaction by regarding teaching as part of a learning process in itself. To this end I draw on symbolic interactionism (Blumer, 1986) to conceptualize meaning-making in interactive settings and connect this conceptualization to the notion of contributions as a metaphor for learning (Stetsenko, 2008). The framework is then illustrated with transcripts of a lesson in the context of experimentation-based, interactive teaching of probability. The upcoming section is supposed to create an orientation of previous work on classroom interaction and the teacher's role and how the present theoretical construct of teaching as learning to develop learning fits.

Previous research on the role of the teacher

Walshaw and Anthony (2008) conclude their review of mathematics education research regarding the role of the teacher in classroom discourse[1] by saying that for instructional practices to be effective, students must actively participate. However, getting students to actively participate in classroom interaction and mathematical discourse relies on inclusive norms (e.g Wood, 2002; Yackel & Cobb, 1996) and implementing purposeful strategies by the teacher (e.g Ding, Li, Piccolo & Kulm, 2007; Walshaw & Anthony, 2008; Wood, 2002; Woodward & Irwin, 2005).

When teacher and students interact they establish socio-mathematical norms, as a sort of rulebook, of how one may participate and contribute to mathematical discourse (Yackel & Cobb, 1996). These norms can include what would count as acceptable contributions as well as whether or not one is expected to make a contribution, e.g. in whole class discussions. For example, norms regarding what counts as a sophisticated and efficient solution can directly influence the students' development of reasoning and it is up to the teacher to establish such norms (McClain & Cobb, 2001). Rystedt, Kilhamn and Helenius (2016) show that socio-mathematical norms could also work as a resource for the teacher to sustain the dynamic flow of an interaction. A teaching strategy may, for example, entail relying on an established socio-mathematical norm of what constitutes an acceptable answer when introducing a task.

Walshaw and Anthony (2008) report on teachers' strategies for creating effective discursive interactions that develop students' thinking according to the idea that teachers must take an active role in the mathematical discourse by differentiating between students' contributions or by supporting their claims. It could mean encouraging students to expand on their ideas and make connections (Manouchehri & Enderson, 1999) or simply noticing reasoning so that one can act knowledgably at an appropriate time (Jaworski, 2004). An example of such a teaching strategy is revoicing. Forman, Mccormick and Donato (1998) show how teachers can share the responsibility to solve and justify solutions by revoicing students' utterances. In accordance to Forman et al. (1998) and Herbel-Eisenmann, Drake and Cirillo (2009), Eckert and Nilsson (2015) show that teachers' small variations of revoicing can be an effective action to support or differentiate students' claims and thereby take an active role in making discursive interactions effective.

Jaworski (2006), in this praxis to characterize teachers' strategies as a way of understanding teaching, points out that that theories of teaching are underdeveloped especially compared to theories of learning. There is no theory of teaching that compares with the big theories of learning, such as constructivism or sociocultural theory. To make sense of teachers' actions and their role one should first address the issue of teaching. She writes, "Teaching develops through a learning process in which teachers and others grow into the practices in which they engage" (p. 187). Furthermore, by viewing teachers as critical professionals, she describes teaching as a process of learning to develop learning and learning as teachers' ongoing critical alignment. The perspective has been used to research teaching communities through collaborative action research (e.g Edwards & Hensien, 1999; Raymond & Leinebah, 2000) and more recently to research professional development communities (e.g Potari, Sakonidis, Chatzigoula & Manaridis, 2010). The literature argues for a suitable frame to study the teacher's own development by regarding the teachers and the researchers as members of a community, either a community of practice (Wenger, 1998) or a community of inquiry (Jaworski, 2006). The teachers' engagements in different communities become the focal point of their analysis. As a result, they are able to gain insight into how collaboration, awareness and reflection of their own practice enable teachers to develop their teaching practice.

Preciado (2011) argues that the community-perspective of learning to develop learning has mainly focused on interactions amongst the community's participants, teachers and researchers, rather than how interactions influence classroom practice. An interesting next step could be to see what we gain by interpreting teachers' contributions to classroom interaction as continuous professional development. Meanwhile what is somewhat missing in research on the teacher's role is the perspective how teaching influences the mathematical practices of the discourse. What is the role of the teacher in regards to the development of content matters in interaction and the subsequent meaning-making processes? Studies regarding the teacher's role in the review by Walshaw and Anthony (2008) show how teachers guide and structure students' thinking and reasoning but do not take into account teachers' own development as they contribute to the meaning-making of mathematical objects. The Symbolic Interactionism perspective provides insights on how actions relate to meaning-making (Blumer, 1986), providing a theoretical foundation to take the concept of learning to develop learning into the context of student-teacher interactions. The next section serves to outline the three basic premises of Symbolic Interactionism and explain how mathematics education research has refined its theoretical constructs to describe and explain mathematics classroom interaction.

Symbolic interactionism

Symbolic interactionism has its roots in sociology and it is a part of influential frameworks in mathematics education research such as the emergent perspective (Cobb & Bauersfeld, 1995b). It has been used to focus the analysis on interaction in the classroom (Yackel, 2001). It shifts the focus from structural explanations of social processes to interactional explanations (Keys & Maratea, 2011). Instead of focusing on societal rules and norms, symbolic interactionism emphasizes the analysis of interactional aspects such as personal interpretations and how meaning emerges. Symbolic interactionism relies on three premises (Blumer, 1986):

- ”Human beings act toward things on the basis of the meanings that the things have for them.” (p. 2)
- ”The meaning of such things is derived from, or arises out of, the social interaction that one has with one’s fellows.” (p. 2)
- ”These meanings are handled in, and modified through, an interpretative process used by the person in dealing with the things he encounters.” (p. 2)

In the first two premises above, participants of an interaction react towards objects depending on how they interpret the meaning of these objects. Meaning is dependent on how others have acted upon the object in prior interactions. As in the case with the addition symbol, participants of the mathematical community proceed to do the mathematical operation of adding for example integers when encountering the addition sign. They do so since they have interpreted others’ actions in relation to this sign in the past and thus ascribed meaning to the addition sign. Objects are anything that could be referred to in interaction, i.e., the objects can be physical, social or abstract. The meaning of an object is not viewed as inherited, or even fixed, but continuously negotiated through a string of ongoing interactions in line with Blumer’s third premise. Symbols are objects that evoke meaningful interpretations. In the perspective of symbolic interactionism, symbols can consist of concrete tools or abstract (theoretical) ideas and are highly situational. A triangle can be a powerful symbol while interacting in the mathematics classroom but perhaps not in other situations, depending on the circumstances of the interaction. The most powerful symbol of all is language with its power to evoke meaningful interpretations through concepts in almost any interaction (Mead, Morris, Huebner & Joas, 2015).

Symbolic interactionism and learning

Drawing on von Glasersfeld's definition of learning as self-organization when operationalizing learning in symbolic interactionism, Cobb and Bauersfeld (1995a) form the notion of the emergent perspective. Learning is viewed as a constructive activity through the processes of assimilation and accommodation as the individual interacts with others. In the language of symbolic interaction, personal meanings of symbols are formed through the process of interpreting others' actions in interaction (Yackel, 2001). The concept of personal meanings is used to represent knowledge, and the fact that it is formed through a personal process indicates that the type of knowledge referred to is knowledge-objects that can be acquired. Effective interaction requires participants to utilize common symbols, symbols which are interpreted to have the same meaning communicated through gestures and indications for all participants (Blumer & Morrione, 2003). In the case of mathematics education, it could mean that a teacher talks about right-sided triangles and at the same time sketch one triangle roughly in the air by finger gestures. Interaction remains effective as long as the students interpret this to be about a generic triangle with one angle of 90°.

A complementary interpretation of the concept of common symbols, and thereby effective interaction, is the shared definition of the situation. Knowledge is not viewed as shared per se, but as "taken-to-be-shared" where teachers and students can interact as if they share the definition of the situation (Voigt, 1996). As meaning is negotiated in the classroom, effective interaction can occur if participants act as if the meanings evoked by symbols in the interaction are shared amongst the other participants. Consequently "Learning is characterized by the subjective reconstruction of societal means and models through negotiation of meaning in social interaction" (Bauersfeld, 1988, p. 39). Participation in the negotiation of meaning is viewed as crucial for the learning process. Krummheuer (2007) highlights this further by arguing that learning mathematics is dependent on students' participation in communicative processes such as collective argumentation. The researcher aims at separating analytically between the dynamics of a learning process and that of an interaction process to clarify connections between social interaction and mathematical learning.

Skott (2013) argues for the opposite view, that social processes and learning processes should not be separated. He stresses the second premise of symbolic interactionism in trying to disentangle the shifts of participation in different social practices. Inspiration from symbolic interactionism together with a participation perspective on learning mathematics forms the Patterns of participation perspective. Instead of separating between learning processes and the participation in the classroom community, he views learning mathematics as a process of becoming able to participate in mathematical practices (Skott, 2013). It is a process of self-development to advance from a peripheral participation to fuller types of participation. An example would be students developing their mathematical language and alignment with the norms of the practice. It is a transformation of human doing rather than a transformation of people (Sfard, 2015), keeping up with the focus on actions rather than states of mind as is the case with symbolic interactionism.

Mead et al. (2015) talk about participation from a purely symbolic interactionist view with emphasis placed on participation in interaction and participation in the other. For effective interaction, participants need to envision the perspectives of the others, participating in the other by envisioning possible intentions and interpretations. When taking the role of the other, we can establish patterns (Blumer, 1986) and thus form strategies for potentially effective interaction. By envisioning others' possible interpretations and intentions, we can act in ways that make sense in the eyes of the other participants. If one is to, for example, interact with a student in a mathematics classroom and discuss the area of different geometrical shapes, one can reason according to different geometries. It becomes vital for effective interaction to participate in the other to anticipate how one's use of symbols might evoke different interpretations amongst the participants of the interaction. If it is an elementary school classroom, perhaps the most common interpretation of a geometry is the Euclidean and it would make most sense to reason when using common symbols such as the triangle, rectangle, circle and so on. Interaction would probably become less efficient if one participant reasons with symbols from for example spherical geometry without considering that those symbols might evoke other interpretations than the participant intended.

Voigt (1995) argues that patterns of interaction are methods of structuring the interaction into themes. They are routines that minimize the risk of collapse of effective interaction. Wood (1994) for example identifies what she calls the focusing pattern, a pattern of asking questions that intends for the student to focus on critical aspects of a problem to help them solve it. Skott (2013) on the other hand views patterns of interaction as patterns of participation, a methodological tool to understand interactions in the classroom. Patterns of participation are a way to "understand how a teacher's interpretations of and contributions to immediate social interaction relate dynamically to her prior engagement in a range of other social practices" (Skott, 2013, p. 549). Whether the focus lies on patterns of actions or patterns of prior engagements, both perspectives highlight teacher's contribution to the interaction. Contributions, as a form of active participation, become the centre of attention when trying to understand interaction in the mathematics classroom.

Eckert and Nilsson (2015) use patterns of interaction in a similar fashion as Voigt (1995) with attention turned towards content-matter and the role of the teacher in negotiation of meaning. They use revoicing as an example of how a teacher's action might be understood in the framing of negotiation of meaning. Eckert and Nilsson (2015) differ from e.g. Voigt (1995) in their use of patterns of interaction in that they ascribe more authority to teachers' professionalism as actors in an ever changing classroom. Instead of describing revoicing as a theme, attention is turned to details in the negotiation of meaning allowing for different types of revoicing depending on their contribution to the negotiation. The identified differences were connected to how much the teacher's own intentions and interpretations were made available to the students. According to Mead et al.'s (2015) notion of taking the role of the other, effective interaction depends on the participants' ability to envision the others' intentions and interpretations. The window to a participant's intentions and interpretations lies in the actions, and whether or not that action is transparent in respect of intentions and interpretations.

To summarize, the teacher is understood, from the symbolic interaction perspective, as an active participant of the learning practice in mathematics education research. Whether it is as a representative of the mathematical community or an upholder of the classroom micro-culture, the teacher influences the classroom interaction on multiple levels. But most importantly, for this paper, the teacher actively contributes to the students' learning practice by its acts towards symbols that have the power to evoke meaningful mathematical interpretations. Teachers learn to develop learning through the ever on-going negotiation of meaning by participating in symbolic interaction. This process is supported by teachers and students engaging in role-taking with each other by being transparent with interpretations and intentions in their actions. However, symbolic interactionism does not specify what learning is. The framing of teachers as active contributors to the students' learning practice needs further development for it to make a significant impact on teaching as learning to develop learning. The next section presents a metaphor of learning as contribution that transcends the colloquial meaning of the term contributions.

Teachers as contributors in symbolic interaction

The symbolic interactionism perspective views social interaction as a continuous flow of actions and interpretations of these actions (Blumer, 1986) and meaning becomes negotiated in the process. Stetsenko (2008) argues that learning can be understood as contribution to the continuous flow of actions as part of a collaborative purposeful transformation. That is, by contributing to the negotiation of meaning you transform the collective understanding as well as your own of the negotiated objects. Individuals play the active role since their actions transform their world just as the world transforms them (Stetsenko, 2008). So, by contributing to the negotiation of meaning, you actively transform the negotiation and you also transform your own understanding of prior events. Thus, learning could be viewed an act of contribution. Learning to develop learning becomes an act of contribution to the continuous flow of actions in the classroom, or contributing to develop contributions (CDC).

With the focus on action in terms of symbolic interaction, human activity becomes contributions (Stetsenko, 2010). It can be narrowed down in the case of mathematics educations to contributions with a mathematical content, since the overarching aim of the analysis is to understand the teacher's role and actions when negotiating the meaning of mathematical objects in the classroom. A contribution is when participants add in any fashion to the negotiation of meaning by means of indications, gestures and symbols. It could be by uttering one's reasoning and interpretation of an object, just as well as contributing a symbol that might evoke meaningful interpretations and therefore help in interpreting one's actions. As Mead et al. (2015) suggest, spoken language is the most important symbol but in mathematics education there are also numerous objects that in the right context can be symbols available to create effective interaction.

The mathematical content is what makes the symbolic interactions in the mathematics classroom unique. It becomes the direction of the flow of actions as well as being a constituent tool in the social practice (Stetsenko, 2010). The negotiated meanings of the general mathematical community point out in which direction it is preferable that the classroom negotiation goes. It is also the necessary tool to initiate symbolic interaction in classroom as there is no symbolic interaction without objects and the interpretation of actions towards those objects (Blumer, 1986). Mathematics becomes the home of symbols and room for role-taking in the continuous flow of actions making up the negotiation of meaning.

So to summarize teachers in CDC make their intentions and interpretations of symbols from the mathematical community available to students through transparent actions, and by doing so contribute to the negotiation of meaning of mathematical objects with their students. The teacher directs the class' constant flow of actions and interpretations and aligns the flow of the negotiation of meaning according to the mathematical norm in the process of contributing to it. Teachers may act in ways to ensure more favourable conditions for effective interaction by making available their intentions and interpretations in consistent actions and by their consistent use of symbols. Transparency in actions enables students to take the role of the other and interpret the teacher's contributions in accordance to the teacher's own intention. As teachers contribute to the negotiation, they influence it and at the same time transform their own understanding for prior events, framing teaching as learning to develop learning in terms of contributing to develop contributions. The metaphor is layered; the first part focuses on the teacher's processes, signalling that teaching entails contributing to the negotiation of mathematics in the classroom at the same time as the teacher transforms its own understanding of prior events. The second part signals that the teacher influences the way students contribute to the negotiation, and thereby develops the way students interpret and contribute to the continuous flow of actions. The next section exemplifies key concepts of the current state of contributing to develop contributions in a classroom example.

Teaching as contributing to develop contributions

The following transcript is from a lesson about probability with 12–13 year old students. It is the last lesson from a series of five, where the intention for the students was to learn about random processes and the law of large numbers in an experimentally based probability context. The first lesson revolved an opaque bottle filled with an unknown amount of coloured balls that constituted the class' unknown sample space. Every time they turned the bottle over, one ball became visible in a small space at the cork and could be recorded. The other three lessons were about developing mathematical tools and symbols to investigate the sample space of the opaque bottle. During these three lessons, the class interacted with transparent bottles, providing the students with the opportunity to see what the bottle contained and how the random process worked. Two blue, two red and two white balls were added to each bottle. The students worked in small groups. Each group had its own bottle and were asked to create two samples of 25 observations. Based on this organisation, each group had their unique experience of the random processes, which were discussed at the end of each lesson. All the groups' samples were also added together at the end of the lesson to create a large sample to compare with the sample space in the transparent bottle. By experiencing these different sized samples, 25, 50 and 350 observations, it was intended for the students to recognize that it was better to base their arguments on large samples rather than small ones. In the following two transcripts, two students, Derek and Eva, negotiate the meaning of the results of samples made with the opaque bottles with the teacher, Tilly. Neither Tilly, Derek or Eva know the sample space but they do have the observations from each group and a joint sample with 350 observations to justify their reasoning. Derek believes that the sample space of the opaque bottle is the same (uniform) as the clear one in the earlier lessons, whereas Tilly and Eva argue otherwise.

Derek: I don't think so, I think it is 2:2:2.

Tilly: You believe they are the same. Explain your thinking.

Derek: If you think, over there, red 40, blue 152, white 158, they are pretty much the same.

Tilly: Yes, those two are pretty much the same.

Derek: Hmm, so they are surely the same amount.

Tilly: Mmm.

Derek: And if you think about those, that one too, over there red was much much, erm, more than, what's its name, the others.

Tilly: Mmm

Derek: Or ... much much ... but ...

Tilly: May I stop you there? Because now you are saying much much more. But if we look at it, here it is, if we are looking at amounts now. Many of you are doing other stuff right now. If we look at amounts, the difference is ... the difference is ...

Derek: 110, 102, 112!

Tilly: 112 if we look at it. If we were to do a chart of it the difference would be significant, we would get something approximately like this, something like that. If we look at the last one we did, we thought there were quite a lot of difference but then it differed about 30.

Means and goals of symbols

The object in focus in the interaction is the result, or more precisely the differences between results, of class joint investigation with the opaque bottle. Tilly and Derek seem to assign similar meaning to the results regarding the blue and white ball. Tilly acknowledges that 152 and 158 are pretty much the same and Derek interprets the result as evidence that there are as many blue as white balls in the bottle. As they move on to the red ball, their interpretation differs significantly. Derek's contributed opening statement, that the ratio of the sample space is 2:2:2 as was the case with the clear bottle, indicates that his interpretation of prior events with the bottle experiment seems to influence his way of interpreting the present results. He interprets differences in the big sample of the opaque bottle similar to the differences he experienced with the small and big samples from the clear bottle. This becomes clearer in the following transcript where he contributes with the justifications of his claims based on results from the previous trials with the clear bottle when he experienced differences in the absolute frequencies even though the sample space was uniform. Tilly on the other hand contributes by clarifying that they are discussing amounts, absolute frequencies. She then contributes the chart, here thought of as a symbol because of its potential to evoke interpretations based on statistical reasoning, to effectively negotiate another interpretation of the differences more in line with hers. A chart in this context is probably meaningful to the students as a conveyor of facts through values and statistics. By drawing a chart on the board and gesturing towards it while saying *like this* and *like that* she tries to evoke an interpretation that a difference of 112 observations, as in the case of the opaque bottle is significant and that a difference of 30 (in a sample of 350 observations), as was the case of the clear bottle, was due to random variation and therefore insignificant. It becomes apparent at the end of the transcript that by contributing in this fashion to the negotiation of meaning of the differences between the three colours, Tilly begins to interpret prior events differently. In the previous lesson, a difference of 30 observations was interpreted as a big difference that had to be explained. Now she interprets 30 as a small and insignificant difference due to random variation. Tilly's contribution of the chart also affected Derek's reasoning, who does not adhere to Tilly's interpretation of the result but develops his own reasoning by justifying it with relative frequencies (see the transcript below). The negotiation of the results continues and it becomes more apparent how Tilly contributes symbols and her interpretations of them in her contributions and how it influences the negotiation of meaning. Especially notice how Tilly struggles to make her interpretations of the symbols in play available and how Eva picks up Tilly's arguments and develops them further through her contributions.

Tilly: And if we look at the percentages, they were pretty even compared to here.

Derek: But, if we regard what we got in our group, when it was 80-20.

Tilly: Mmm.

Derek: That time there were equal amounts, and it has to be like that this time as well, hmm, it could just as well been, hmm, 50 blue, 20 red and 30 white.

Tilly: Mmm, and then chance … chance affect … it is almost even more obvious and affects even more in a small sample, that became really palpable in your case. We were really lucky you got those results since it becomes so obvious how chance can influence. Since it was only you who got that result even though all bottles were the same. Eva?

Eva: Everybody got the same the same bottle and it would have been different if everybody would have gotten, like, a lot of different, like 20 blue and 40 red and 40 blue.

Derek: But I still think …

Eva: But now when, like, now everybody has the same bottle and we don't know what is inside them, hmm, almost everyone got approximately the same amount of red. It can't be chance because if it were chance it would have been more differences.

Transparency in action

Tilly contributes with a symbol, percentages, to shift the negotiation of meaning of the result in line with her interpretation of the results. The percentages, or relative frequencies, have the advantage of being more comparable and could evoke other interpretations than the absolute frequencies. It can be difficult to size differences in absolute numbers, especially as the sample grows large. Derek continues to compare the class' results with his prior experience with the small sample, but now in relative frequencies. His result with the clear bottle revealed to be a rather extreme case with observations distributed 80-20-0 (red, white, blue). He uses this small sample to argue that the large sample can be an extreme outcome as well and that the sample space in the opaque bottle is probably uniformly distributed, as was the case in the clear bottle. Derek's contribution enables Tilly to interpret the present and past meanings of the result differently and she struggles to make her interpretations available to the students. She contributes chance as another symbol and how it applies to Derek's reasoning in an attempt to make her interpretations and intentions more transparent to the students. The concept of chance, in contrast to sample size that she also indicates shortly afterwards, is here interpreted as a symbol as it acts as a catalyst to transform the negotiation. Both Tilly and Eva seem to start interpreting past and present events differently after Tilly's transparent contributions, and both became able to contribute differently in the following discussion. It becomes transparent to the participants that Tilly intends to use chance alongside the frequency data to steer the negotiation towards what seems to be an interpretation of the results that indicate a non-uniform sample space. She now interprets previous events with Derek's extreme results as a matter of chance and highlights how such interpretations of extreme results can contribute to their negotiation of meaning of chance. Tilly continues to contribute by pointing out that only one group had such an extreme result, indicating her interpretation that sample size is an important factor when assessing the influence of chance on the results.

Contributing to develop contributions

It becomes apparent at the end how Tilly's and Derek's contributions have influenced the negotiation of meaning of the results. Eva now argues, in light of Tilly's and Derek's previous contributions, how Derek's case should be viewed as extreme, and a matter of chance. She adds that the small samples with the opaque bottle should have been more diverse if a matter of chance could dismiss the difference in the large sample. It serves to exemplify that even though their discussion, and negotiation about the differences of each colour's absolute frequency does not reach consensus, Tilly's contributions have developed the way Eva and Derek are now able to contribute. Why Eva chose to reason in line with Tilly could have multiple explanations, perhaps Tilly's contributions were more transparent than Derek's, or perhaps Tilly's role as a teacher is a key factor in the interaction. Anyhow, it becomes apparent that Tilly's contributions have more impact on this particular interaction. More classroom data is needed to comment with any certainty about the role of the teacher. But it exemplifies the idea that teaching could be viewed as contributing to develop contributions with symbolic interactionism to conceptualize meaning-making in interactive settings and that it might offer alternative interpretations of classroom interaction. It becomes apparent in the two transcripts that Tilly engages in learning as she teaches. Through Tilly's contributions, one can see how she develops her interpretation of symbols as well as of prior events, which benefits the students' contributions as well. She transforms her interpretation of the concept of chance as well as transforms her view of the present and the past results.

Discussion

Skott (2013) discusses the teacher's role in emerging classroom practices and concludes that teacher's actions should be viewed as meaningful participation in past and present practices. It is through participation teachers transform practices in a process of re-engaging in other past and present practices. This paper starts with a similar proposition: Teaching means to actively participate in a learning practice (i.e. learning to develop learning). Compared to Skott (2013), contribution is the primary metaphor for learning rather than participation. The difference is intended to shift focus from re-engagement in other and past practices to collaborative transformation, which enables an analysis of teacher's contributions to the negotiation of meaning. In the case of Tilly, we try to understand her actions and role in the classroom in the light of collaborative transformation called negotiation of meaning. Instead of focusing on how prior practices might influence her actions, the focus lies on how her shifting contributions influence the present negotiation of meaning.

Contribution to negotiation of meaning serves as the alternative interpretation to what Walshaw and Anthony (2008) call activities to scaffold students' thinking, fine-tuning their thinking through language and shaping their mathematical argumentation. Instead of characterizing teaching as something that develops students' higher mental functions, teachers' actions become the influencing factor in shaping the mathematical content. Tilly contributed with her interpretations as well as with symbols to influence the negotiation of meaning of the results from their experiment. Her acts are understood as acts to contribute to the mathematics and a process of learning as she develops her own understanding of present and prior events. The activity systems in the review of Walshaw and Anthony (2008) of teacher's role in classroom discourse gives valuable insights on how teachers create favourable conditions for mathematical discourse. This framing of the role of the teacher aims to provide insights on the dual nature of teaching and interaction and an alternative understanding of actions in the classroom.

A teacher as an educator as well as learner is the outcome of an interactive perspective on the classroom practice. Jaworski (2006) argues for the learning to develop learning as a perspective that captures the development of teachers in their practice. The aim of this paper was to add to that notion to frame teaching as an ever-developing practice and teachers as participants of the students' learning practice by drawing on aspects of Symbolic interaction. Jaworski and researchers citing her work provide insight on the development of the teacher within a community of professionals, often researchers and teachers working together. Their original stance on learning to develop learning assumes a degree of awareness of the teacher's development, a sort of explicit goal to develop the practice, making it suitable to research professional development. The course of argument in this paper proposes that the teaching practice is one of constant development even in interaction between teachers and students and that teachers' development is an implicit process in teaching. As teachers actively contribute with their own perspective in a mathematical discourse, their interpretation of past and present events develops as they aim to develop the students' contribution. Subsequently, teaching is viewed as contributing to develop contributions. As exemplified in the transcripts above, these contributions provide a perspective to explain a teacher's shifting contributions to the discourse by a set of theoretical constructs including symbols and learning as contribution. The challenge now is to develop this metaphor further, past the state of a metaphor for teaching, to a more complete theory; a theory with the capacity to describe and explain a wide range of teacher actions in interaction with students in interactive experimentation in the classroom.

References

Bauersfeld, H. (1988). Interaction, construction and knowledge: alternative perspectives for mathematics education. In T. Cooney & D. Grouws (Eds.), *Perspectives on research on effective mathematics teaching*. Hillsdale: Erlbaum and Reston.

Blumer, H. (1986). *Symbolic interactionism : perspective and method*. Berkeley: University of California Press.

Blumer, H. & Morrione, T. J. (2003). *George Herbert Mead and human conduct*. Walnut Creek: AltaMira Press.

Cobb, P. & Bauersfeld, H. (1995a). The coordination of psychological and sociological perspectives in mathematics education. In P. Cobb & H. Bauersfeld (Eds.), *The emergence of mathematical meaning: interaction in classroom cultures*. Hillsdale: L. Erlbaum Associates.

Cobb, P. & Bauersfeld, H. (1995b). *The emergence of mathematical meaning: interaction in classroom cultures*. Hillsdale: L. Erlbaum Associates.

Ding, M., Li, X., Piccolo, D. & Kulm, G. (2007). Teacher interventions in cooperative-learning mathematics classes. *The Journal of Educational Research*, 100 (3), 162–175.

Eckert, A. & Nilsson, P. (2015). Introducing a symbolic interactionist approach on teaching mathematics: the case of revoicing as an interactional strategy in the teaching of probability. *Journal of Mathematics Teacher Education*, 1–18. doi: 10.1007/s10857-015-9313-z

Edwards, T. G. & Hensien, S. M. (1999). Changing instructional practice through action research. *Journal of Mathematics Teacher Education*, 2, 187–206.

Forman, E. A., Mccormick, D. E. & Donato, R. (1998). Learning what counts as a mathematical explanation. *Linguistics and Education*, 9 (4), 313–339.

Franke, M. L., Kazemi, E. & Battey, D. (2007). Mathematics teaching and classroom practice. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 225–256). Charlotte: Information Age Publishing.

Herbel-Eisenmann, B., Drake, C. & Cirillo, M. (2009). "Muddying the clear waters": teachers' take-up of the linguistic idea of revoicing. *Teaching and Teacher Education*, 25 (2), 268–277.

Jaworski, B. (1994). *Investigating mathematics teaching: a constructivist enquiry*. London: Falmer.

Jaworski, B. (2004, July). *Grappling with complexity: co-learning in inquiry communities in mathematics teaching development*. Paper presented at PME 28, Bergen, Norway.

Jaworski, B. (2006). Theory and practice in mathematics teaching development: critical inquiry as a mode of learning in teaching. *Journal of Mathematics Teacher Education*, 9 (2), 187–211.

Keys, D. & Maratea, R. J. (2011). Life experience and the value-free foundations of blumer's collective behavior theory. *Journal of the History of the Behavioral Sciences*, 47 (2), 173–186.

Krummheuer, G. (2007). Argumentation and participation in the primary mathematics classroom: two episodes and related theoretical abductions. *Journal of Mathematical Behavior*, 26 (1), 60–82.

Lester, F. (2005). On the theoretical, conceptual, and philosophical foundations for research in mathematics education. *ZDM*, 37 (6), 457–467.

Manouchehri, A. & Enderson, M. C. (1999). Promoting mathematical discourse: learning from classroom examples. *Mathematics Teaching in the Middle School*, 4 (4), 216–222.

McClain, K. & Cobb, P. (2001). An analysis of development of sociomathematical norms in one first-grade classroom. *Journal for Research in Mathematics Education*, 32 (3), 236–266.

Mead, G. H., Morris, C. W., Huebner, D. R. & Joas, H. (2015). *Mind, self, and society: the definitive edition*. The University of Chicago Press.

Oxford Dictionaries (2016). *Definition of teach in English*. Retrieved from http://www.oxforddictionaries.com/definition/english/teach

Potari, D., Sakonidis, H., Chatzigoula, R. & Manaridis, A. (2010). Teachers' and researchers' collaboration in analysing mathematics teaching: a context for professional reflection and development. *Journal of Mathematics Teacher Education*, 13 (6), 473–485.

Preciado, A. P. (2011). *Conversations held and roles played during mathematics teachers' collaborative design: two dimensions of interaction* (Ph.D. thesis). Simon Fraser University.

Raymond, A. M. & Leinebah, M. (2000). Collaborative action research on the learning and teaching of algebra: a story of one mathematics teacher's development. *Educational Studies in Mathematics*, 41, 283–307.

Rystedt, E., Kilhamn, C. & Helenius, O. (2016). What's there in an *n*? Investigating contextual resources in small group discussions concerning an algebraic expression. *Nordic Studies in Mathematics Education*, 21 (1), 5–26.

Schoenfeld, A. H. (1999). Models of the teaching process. *Journal of Mathematical Behavior*, 18, 243–261.

Sfard, A. (2008). *Thinking as communicating: human development, the growth of discourses, and mathematizing*. New York: Cambridge University Press.
Sfard, A. (2015). *Creating numbers: participationist discourse on mathematics learning*. Paper presented at Rôles et places de la didactique et des didacticiens des mathématiques dans la société et le système éducatif, Grenoble.
Skott, J. (2013). Understanding the role of the teacher in emerging classroom practices: searching for patterns of participation. *ZDM*, 45 (4), 547–559.
Stetsenko, A. (2008). From relational ontology to transformative activist stance on development and learning: expanding Vygotsky's (CHAT) project. *Cultural Studies of Science Education*, 3 (2), 471–491.
Stetsenko, A. (2010). Teaching-learning and development as activist projects of historical Becoming: expanding Vygotsky's approach to pedagogy. *Pedagogies*, 5 (1), 6–16.
Voigt, J. (1994). Negotiation of mathematical meaning and learning mathematics. *Educational Studies in Mathematics*, 26 (2-3), 275–298.
Voigt, J. (1995). Thematic patterns of interaction. In P. Cobb & H. Bauersfeld (Eds.), *The emergence of mathematical meaning*. Hillsdale: Lawrence Erlbaum Associates.
Voigt, J. (1996). Negotiation of mathematical meaning in classroom processes: social interaction and learning mathematics. In L. P. Steffe, P. Nesher, P. Cobb, G. A. Goldin & B. Greer (Eds.), *Theories of mathematical learning* (pp. 21–50). Mahwah: Lawrence Erlbaum Associates.
Walshaw, M. & Anthony, G. (2008). The teacher's role in classroom discourse: a review of recent research into mathematics classrooms. *Review of Educational Research*, 78 (3), 516–551.
Wenger, E. (1998). *Communities of practice: learning, meaning, and identity*. Cambridge University Press.
Wood, T. (1994). Patterns of interaction and the culture of mathematics classrooms. In S. Lerman (Ed.), *Cultural perspectives on the mathematics classroom* (pp. 149–168). Dordrecht: Kluwer.
Wood, T. (2002, July). *What does it mean to teach mathematics differently?* Paper presented at MERGA25, Sidney.
Woodward, J. & Irwin, K. (2005, July). *Language appropriate for the New Zealand numeracy project*. Paper presented at MERGA28, Sydney.
Yackel, E. (2001, July). *Explanation, justification and argumentation in mathematics classrooms*. Paper presented at PME 25, Utrecht, The Netherlands.
Yackel, E. & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education*, 27 (4), 458–477.

Note

1 ”Discourse” refers here to the specialized and situated communication of mathematics that includes some actors and excludes others (Sfard, 2008)

Andreas Eckert

Andreas Eckert is a doctoral student in mathematics education at Linnaeus University, Växjö, Sweden. His research interests include teacher-student interaction in the mathematics classroom and teachers' in-practice professional development.

andreas.eckert@lnu.se