Mind the gap between students and their mathematical textbooks

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Reading and comprehending mathematics textbooks means understanding the global meaning and for this to occur successful comprehension strategies are required. Drawing on the results of a pilot study with six grade 3 students, a relationship between the students' reading skills and their mathematical skills appeared. To examine this relationship further eighteen students from grades 1, 4 and 7, with different achievement levels were interviewed in this study. Both in the pilot study and in the current study the interview questions were inspired by the comprehension strategies of prediction, clarification, questioning and summarization from Palincsar and Brown's reciprocal teaching model. These strategies are connected to Halliday's *Systemic functional linguistics* to better understand how the textbook context affects students' use of comprehension strategies. The results show that all students had developed reading comprehension strategies that were more or less successful, starting already from grade 1. Furthermore, the results of this study highlights that all students, independent of their achievement level or grade, require explicit teaching concerning efficient comprehension strategies to grasp the mathematical content being presented in mathematics textbooks.

The reliance on textbooks in mathematics is considered a worldwide phenomenon (Despina & Harikleia, 2014; Fan et al., 2013; Haggarty & Pepin, 2002; Thomson & Fleming, 2004). This is also the case in Sweden where working individually in textbooks is common practice during mathematics lessons (Boesen et al., 2014; Johansson, 2006; Mullis et al., 2012). Thus, the textbook plays an important role in instructions because it influences how many students apply and learn mathematical concepts (Bryant et al., 2008). However, when students are expected to work individually with textbooks it is often up to each student to create their own understanding

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Segerby, C. (2020). Mind the gap between students and their mathematical textbooks. *Nordic Studies in Mathematics Education*, 25 (3-4), 115–138.

of mathematics from reading. This cannot be presumed to be a simple activity given that mathematical textbooks are generally written in a short, dense style with few contextual clues to decipher the mathematical content being expressed (Barton & Heidema, 2002). Further, much of the text in mathematics textbooks is generally multimodal and involves the use of multiple semiotic (meaning-making) resources to construct knowledge from words, illustrations and symbols. Consequently, making sense of this complex multimodal synthesis of meaning may present challenges for students' reading skills and differ from reading texts in other subjects (Noonan, 1990). Furthermore, a correlation between students' reading skills and mathematical skills has appeared from grade 4 (Möllehed, 2001; Vilenius-Touhimaa et al., 2008), and according to Korhonen et al.'s study (2012), this correlation seems to remain during students' whole school time. Despite this, students' development of reading skills in mathematics is not emphasized in the current syllabus for mathematics in Sweden (Skolverket, 2019).

To date, research has been limited in examining how students read their mathematical textbooks to comprehend the mathematical content being presented (Shepherd et al., 2012; Österholm, 2007). Previous research tends to report intervention programmes where students are introduced to different comprehension strategies in their mathematics lessons concerning problem solving tasks, but does not include data about the initial comprehension strategies of the students (Huber, 2010; Quirk, 2010), or instead only involves results of studies with secondary school and university students (Shepherd et al., 2012; Weinberg et al., 2012; Österholm, 2006). Less attention has been paid to examining younger students' use of comprehension strategies in mathematics even though the importance of identifying early signs of mathematics difficulties is stressed by several researchers (Dowker, 2005; Gersten et al., 2005). By early identification of signs, efforts can be implemented to support students' learning environments and thereby prevent future mathematics difficulties.

Drawing from a pilot study with grade 3 students (Segerby, 2014) this study, which is a developed version of a degree project in Special Needs in Mathematics (Segerby, 2018), is expanded to involve eighteen students: six in grade 1, six in grade 4 and six in grade 7, all with varying degrees of achievement levels in mathematics. In each of the grades, the students' legitimated mathematical teachers were asked to choose two high achievers, two middle achievers and two low achievers in mathematics, based on the students' academic performance in mathematics. This approach was also used in the pilot study (Segerby, 2014). Having some understanding of the students' academic performances provided the similarities and/ or differences between the students' use of comprehension strategies –

when reading and solving tasks about the place value in a mathematics textbooks – and how these might influence their possibility to grasp mathematical content is presented.

The following research question is addressed:

> What kind of comprehension strategies do students with different achievement levels in mathematics in grades 1, 4 and 7 use when they approach pages addressing place value in a commonly used mathematics textbook?

To inform this study, the theoretical perspective *Systemic functional linguistics* (SFL) is coordinated with the reading comprehension strategies of prediction, clarification, questioning and summarization adopted from Palincsar and Brown's (1984) *Reciprocal teaching model* (RT).

Reading the mathematical textbook

The text presented in mathematics textbooks is often compact and multimodal (Barton & Heidema, 2002; Love, 2008). The words in mathematics can be difficult to comprehend because certain mathematical words, such as "geometry", are only connected to the mathematical contexts (Lee, 2006). Furthermore, some mathematical words, such as "volume" and "table", have alternative meanings in conversational language compared to the mathematical language (Lee, 2006; Riccomini et al., 2015).

In addition to vocabulary, mathematics textbooks often contain numeric and non-numeric symbols (Barton & Heidema, 2002), where the reader needs to translate them into words and vice versa (Carter & Dean, 2006). Numeric symbols involve numbers such as 9 and 55, and non-numeric symbols involve symbols used for mathematical operations – addition, subtraction, multiplication and division – which efficiently tell the learner what to do (Hammill, 2010), such as "+" means "to add" two or more numbers, for example 13 + 7. Concerning numbers, students also need to understand that the value determined by the position or positions of the symbols (digits) within the numbers means the value increases by tens from right to left, and the value of a digit is equal to the digit times the value assigned to its position (Ross, 2002). Illustrations connected to words and symbols are also common in mathematical textbooks. Previous studies (Jellis, 2008; Segerby, 2014) show that students pay excessive attention to illustrations in mathematical textbooks. The purpose of the illustration is to make certain mathematical concepts more accessible and easier to learn (Jewitt, 2005). However, the purpose of the illustrations varies because sometimes the illustrations' function is to make the text more attractive and serve no instructional purpose. On other occasions, the illustrations are considered essential and important in order to make meaning of the mathematical content being expressed (Noonan, 1990). This issue makes it sometimes difficult for the reader to determine whether the illustrations are relevant or not for solving a mathematics task (Jellis, 2008). Watkins et al. (2004) suggest that, without guidance, students may not connect written sentences and visual representations to construct the author's intended meaning about the content, which means that guidance is required. Another important part of reading skills is to know how to read a text. This involves good reading comprehension skills in order to read to learn (Palincsar & Brown, 1984).

Comprehension strategies in mathematics

Comprehension strategies are consciously controlled mechanisms that readers use to interpret and organize text to improve their understanding of it and involves both decoding and linguistic comprehension skills (Vellutino, 2003). Decoding refers to the reader's phonological awareness, reading fluently and knowing how to decode the text (Vellutino, 2003), which in mathematics involves words, visual representations and symbols. This way of reading is uncommon in other contexts.

Linguistics comprehension involves understanding the separate meaning of words, illustrations and symbols, but also understanding how these separate purposes can be combined to comprehend the text (Love, 2008). Aiken's (1972) study showed that students with a higher reading level tended to solve textual problem-solving tasks better than the other students. According to Aiken, these students are not considered better at mathematics, but use successful strategies to handle their comprehension of the text. Similar correlation has been found connected to other kinds of texts, for example, fictional texts (Brown & Day, 1983; Palincsar & Brown, 1984; Pressley, 2002). Previous research about comprehension strategies in mathematics has mostly concentrated on investigating the result of the students' use of comprehension strategies after implementing comprehension strategies from the Reciprocal teaching model connected to specific problem solving tasks (Huber, 2010; Quirk, 2010; Yang & Lin, 2012). These sets of strategies include prediction, questioning, clarification and summarization. Prediction involves predicting about the content in a text and then activating prior knowledge. According to Brown and Day (1983), proficient readers combine information from several sources to predict the main ideas. Identifying the main ideas helps a student to activate prior knowledge so that new knowledge can be connected to this prior knowledge, which is essential for developing knowledge in an area (Carter & Dean, 2006). Clarification involves clarifying elements in the text (such as vocabulary), questioning involves asking questions about the content in a text, and summarizing involves summarizing the content (Palincsar & Brown, 1984).

In other fields, extensive research has involved both quantitative and qualitative data (e.g. Lederer, 2000; Pressley, 2000; Westlund, 2009) emphasizing these types of strategies to support students' comprehension of a text, but also in mathematics. For example, in Yang and Lin's (2012) study with twenty-two grade 9 classes, the reading-oriented tasks involved reciprocal teaching activities to facilitate students' comprehension of geometry proofs. A similar positive result was found among grade 4 students in Huber's (2010) and Quirk's studies (2010) among grade 5 students when reciprocal teaching activities were implemented to support students' reasoning competence to problem solving. However, limited research has been conducted to examine students' use of comprehension strategies before any formal instructions are provided (Shepherd et al., 2012; Österholm, 2007). The focus has then been on examining how university and secondary students' read their mathematical textbook. These results show that many of the students developed unsuccessful strategies, such as identifying incorrect main ideas. As a result, they activated incorrect prior knowledge (Shepherd et al., 2012; Weinberg et al., 2012) or concentrated on the operative meaning of the symbols and not on the semantic role of the text (Österholm, 2006). In the next section, the theoretical perspectives, which inform this study, are introduced.

Theoretical perspectives

In this study, a conceptual framework is used where *Systemic functional linguistics* (SFL) is coordinated with the *Reciprocal teaching* (RT) model. By using SFL, information about the students' linguistic choices in a specific situation and how the classroom context, textbook context and social context can have an impact on students' comprehension strategies can be provided. As SFL is used to analyse texts, it could not be used by itself to describe specific comprehension strategies, and Palincsar and Brown's (1984) framework – reciprocal teaching – is used to describe the strategies and these strategies are linked to SFL (see table 1).

This approach using SFL and comprehension strategies from the RT model has previously been used in the pilot study (Segerby, 2014). In next section, SFL will be discussed further.

Systemic functional linguistics (SFL)

Systemic functional linguistic is a socio-semiotic theory where language is viewed as a system of meaning which can be analysed by the clause level in order to understand how contexts are reflected in participants'

Table 1. *Coordination of the RT model and SFL to a conceptual framework*

Comprehension strategies in the RT model	SFL's metafunctions
Prediction, Questioning, Clarification & Summarization	*Textual metafunction*: Semiotic resources, such as words, symbols and illustrations, and how they together contribute to cohesiveness.
Prediction, Clarification, Questioning & Summarization	*Ideational metafunction*: Naming relevant for the context, such as ten digits and hundred digits.
Questioning	*Interpersonal metafunction*: The use of imperatives, such as ”write” and interrogative, such as ”How much” when constructing a question.

linguistic choices. According to Halliday (Halliday & Hasan, 1985), any situation is an instance of culture, and a culture lies behind all the types of situation that can occur, so every text is developed through the context of situation, which, in itself, is surrounded, by the *context of culture* (Halliday & Hasan, 1985). The context of culture refers to what occurs outside language such as the events and conditions of the world and the social processes involved, and involves certain expectations and assumptions (Halliday, 1978). For example, context of culture in Sweden can refer to how mathematics education is designed with the use of the mathematical textbook and how that affects a student's role. The *context of situation*, where a student makes sense of their textbook, is the ”environment in which meanings are being exchanged” (Halliday & Hasan, 1985, p. 12). The context of situation is comprised of three elements: field, tenor and mode. They are materialized through lexico-grammatical choices, which in turn are realized by three metafunctions: ideational, interpersonal and textual. The concept of metafunction is one of a small set of principles necessary to describe how language works by explaining how meaning is expressed through language (Halliday & Hasan, 1985).

The first function – the *ideational metafunction* – operates as the field (Morgan, 2006). The *field* ”refers to what is happening, to the nature of the social action that is taking place: what is it that the participants are engaged in, in which the language figures as some essential component” (Halliday & Hasan, 1985, p. 12). The naming of objects is connected to the field and can include such mathematical terms as ”position system”, ”number” and ”place value”. The second function – the *interpersonal metafunction* – is provided through the tenor (Morgan, 2006). The *tenor* indicates who is taking part, their role and status, and what kind of relationship exists between them, both in temporary and permanent relationships (Halliday & Hasan, 1985). The relationship between the participants can be shown by examining the ”voice” through identifying the use of interrogatives, imperatives, modality and personal pronouns (Herbel-Eisenmann, 2007). Interrogatives involves question words such as ”What” and ”How much”. Imperative verbs command listeners or readers, for example, to do something such as ”count”, whereas modality indicates the level of certainty associated with particular actions, such as ”can”. Personal pronouns, such as ”you” and ”I”, identify those considered to be the main and secondary actors in the text.

The third function – the *textual metafunction* – is provided through the mode (Morgan, 2006). *Mode* refers to what role language plays in developing the functions of the text within the context (Halliday & Hasan, 1985). For instance, word, illustrations and symbols are important semiotic resources with important functions in mathematics text. However, since SFL focusing on linguistic choices and not on analysing illustrations, Kress and van Leeuwen´s (2001) framework, which draws on understandings of SFL, needs to be adopted.

The textual metafunction also involves cohesive relations between the semiotic resources in a text. To visualize the students' comprehension strategies the RT model is used.

Reading comprehension strategies in the RT model

Four comprehension strategies – prediction, clarification, questioning and summarization – are involved in the Reciprocal teaching model (Palincsar & Brown, 1984). These four strategies aim to help students understand the factors that interact with and influence their comprehension of a text (Palincsar, 2003).

Prediction involves students predicting future content, which concerns assumptions about the main ideas through making sense of different components, such as title, headings and images (Palincsar, 2003).

Clarification requires students to engage in the clarification of words and/ or phrases, symbols and visual representations, which are essential for grasping an understanding of the content being expressed.

Questioning involves students asking questions about the content to check their own understanding (Palincsar, 2003). This helps to increase students' awareness of the important ideas in the text (Palincsar & Brown, 1984). It also helps the reader to identify which information in the text is important enough to construct a question (Palincsar, 2003). The questions can have different characters, which means they can be closed and then involve only one answer (Sullivan & Liburn, 2002). The opposite to close questions are open questions and these generate several different alternative answers. Open questions provide students with the possibility of active participation by encouraging them to visualize different thoughts and solutions. *Summarization* is concerned with distinguishing important information (main ideas) from the less-important information in a particular part of the text (Palincsar, 2003).

To visualize students' comprehension strategies – prediction, clarification, questioning and summarization – the reading comprehension strategies are connected to SFL. All the four strategies involve understanding different components and relate to SFL ideational metafunction, i.e. what is happening in the text where naming is central. This also includes representation that appears in the text and if they contribute to cohesiveness, which relate to SFL's textual metafunction. Questioning also concerns SFL's interpersonal metafunction and relates to the type of questions students propose by choosing interrogative, imperative or personal pronouns as the focus of analysis. To visualize, the strategies are coordinated with SFL and in table 1, the coordination of the conceptual framework is provided to be able to answer the research question: "What kind of comprehension strategies do students with different achievement levels in mathematics in grades 1, 4 and 7 use when they approach pages addressing place value in a commonly used mathematics textbook?"

Method

This interview study was conducted with eighteen students in grades 1, 4 and 7 at the end of their academic year. In each grade, the class' legitimated mathematical teacher was asked to choose six students, two who were considered as high achievers, two as middle achievers and two as low achievers based on the students' academic performance in mathematics. Having some understanding of the students' achievement levels provided the possibility to link this to the students' use of comprehension

Table 2. *Students in the study*

Level	grade 1	grade 4	grade 7
High	1A1	4A1	7A1
High	1A2	4A2	7A2
Middle	1B1	4B1	7B1
Middle	1B2	4B2	7B2
Low	1C1	4C1	7C1
Low	1C2	4C2	7C2
Total	6	6	6

strategies. Same approach was used in the pilot study (Segerby, 2014). Table 2 visualizes how the students are referred to.

The students in grade 1 read three pages in Prima Matematik and performed six tasks on these pages regarding place value (Brorsson, 2014, p. 8–10, tasks: 1, 4, 10, 12, 18 and 21). The tasks the students were asked to perform in grade 1 did not have any number so they are named after their positions on the pages. Grade 4 read page 12 in the mathematics textbook Matte Direkt Borgen and performed five tasks concerning place value (Falck et al., 2011, p.12, tasks: 27a, 28a, 29a, 30a and 31). In grade 7, the students read three pages in Matematikboken X regarding place value and performed four tasks (Undvall et al., 2011, p. 9–11, tasks: 1002, 1004, 1007a and 1007b).

Table 3. *Questions in the interviews*

Questions	RT	SFL
1. What is the first thing you look at on the page/pages?	Predict	Textual
2. What did you do to find out what the text is about?	Predict	Ideational & Textual
3. What do you consider the text is about?	Predict	Ideational & Textual
4. What is a digit and what is a number?	Clarify	Ideational & Textual
5. Make a task that someone who has worked with these pages will be able to solve.	Question	Ideational, Interpersonal & Textual
6. After reading and working with a number of tasks on the pages, if you explain to a friend what they are about what would you say?	Summarize	Ideational & Textual

Interview questions

Six questions were posed in the study and in table 3 the questions are connected to the theoretical framework.

Similar questions were asked in the pilot study (Segerby, 2014), where students answered orally. Yet, in this study, all of the students were asked to mark on the pages where they first looked (question 1). Furthermore, students in grades 4 and 7 were also asked to write down notes to questions 5 and 6. Another difference was that all of the students in this study were asked to perform some of the tasks on the current pages in the mathematics textbook, but before answering questions 5 and 6. Furthermore, question 5 concerning strategy questioning was not included in the pilot study.

Data collection and analysis tool

All the eighteen interviews were conducted individually and recorded with a video camera and a dictaphone, and then transcribed. A qualitative content analysis was conducted based on the theoretical perspectives, the RT model coordinated with SFL visualized in tables 1 and 3. Thereby, the content analysis is concept driven since it is based on theories (Schreier, 2012). In table 3, the coordination to each of the questions is shown. Further, all of the students' notes connected to the textbook pages were collected.

To examine students' use of comprehension strategies, the current text needs to be take into consideration because the features of a text can have significant impact on comprehension, and different texts place different demands on students' reading comprehension (Sweet & Snow, 2002). Thereby, a content analysis of the textbook pages in this study was conducted using SFL's three metafunctions: ideational, interpersonal and textual. Depending on the study's theoretical perspectives, methodological considerations and the type of generated data, the focus on the metafunctions can be different in different studies (Herbel-Eisenmann & Otten, 2011). The ideational metafunction involved identifying the main ideas on the textbook pages. Interpersonal metafunction involved examining the construction of the textbook tasks where interrogative and imperative were in focus. Finally, textual metafunction involved the text as was presented, i.e. which semiotic resources appear and how they contribute to cohesiveness in the text.

Results and analysis

Initially, the results and analysis of the textbook pages are provided, which is followed by the interview study.

The mathematical textbook pages

All the mathematical textbooks being explored in this study, were structured in similar ways and the content involved place value, number and digits, which refer to SFL's ideational metafunction. Initially, a heading presents the mathematical content, which is followed by an exposition multimodal text (information box) with examples that aim to illustrate a generalized method followed by tasks.

In the tasks in grade 1, students were asked to ”put a circle”, ”fill in” and ”write different numbers”. For example, in one of the tasks (Brorsson, 2014, p.10, task 21) the students are asked to ”Write tens” (Skriv tiotal) in an illustration where 70 is written in the middle square and the students are asked to write which tens that comes before and after 70, see Figure 1.

	70	

Figure 1. *A redrawn version of task 21 (Brorsson, 2014, p. 10)*

In grade 4, students were asked to solve the tasks starting with the interrogative ”how much” and the imperative ”write”, and one of the questions was a ”yes or no question”. For example, in task 28a (Falck et al., 2011) the question is ”How much is the digit 4 worth in the number 1674” (Hur mycket är siffran 4 värd i talet 1674).The tasks in grade 7 involved ”constructing” and ”writing” different numbers. For example, task 1002 (Undvall et al., 2011) asks the students to ”Construct with the digits 2,3,1,9 and 5 a number that is as close to 20 000 as possible (Bilda med siffrorna 2,3,1,9 och 5 ett tal som är så nära 20 000 som möjligt)

The use of interrogatives and imperatives refers to SFL's interpersonal metafunction (Herbel-Eisenmann, 2007). All the questions provided are closed and involve only one (short) answer with numbers, which is a common approach in mathematics (Sullivan & Lilburn, 2002). In all of the texts, several semiotic resources appear to express the meaning, such as illustrations, words and symbols, and the use of the semiotics resources and construction of the page refers to SFL's textual metafunction. The illustrations on the pages have different aims. In grade 1 one of the illustrations is decorative and does not involve the mathematical content being presented. The other illustrations on the pages are essential for making meaning about the content (Noonan, 1990), both in the information box and in the tasks. In grade 4, one of the illustrations is also essential but the other illustration is important but not essential since all the information is presented in the text in the task. In grade 7, the illustrations are essential in the information boxes, but the illustration in one task is decorative.

The interview study

The result of the questions is divided into the reading comprehension strategies prediction, clarification, questioning and summarization, and analysed by SFL's ideational (naming), interpersonal (interrogative and imperatives) and textual metafunctions (semiotic resources); see table 3.

Prediction

Question 1 ”What is the first thing you look at on page 8?” was used to find out how individuals predicted the content of the page. The students were only able to choose one answer and the result is presented in table 4.

Table 4. *The results of Question 1 ”What is the first thing you look at on page 8?”*

Alternative		Students	Total
Heading	The heading	3 (4A2, 7A1, 7B1)	3
Information-box	The illustration in the information box	6 (1A2, 1B1,1C1, 4A1, 4B1, 4B2)	6
Tasks	Illustrations in the task	7 (1A1, 1B2, 1C2, 4C2, 7A2, 7C1, 7C2)	
	Texts in the task	2 (4C1 and 7B2)	9

What caught more than half of students' attention was the information presented in the tasks, where seven of the students looked at the pictures and two looked at the text. In addition, six of the students looked at the illustrations in the information box and three students looked at the heading. In table 5, the results of Question 2 ”How can you find out what the pages are about?” are presented.

To the next question ”What are the pages about?”, four students (1A2, 1B2, 4C2 and 7B2) were unable to relate to the specific mathematical content presented. For example:

> Räkning. (Counting.) (1A2)

> Plus och minus. (Plus and minus.) (7B2)

The remaining students relate to digits and/ or numbers that are essential mathematical concepts in order to grasp the content, such as:

> Tal och siffror. (Numbers and digits.) (4B2)

However, only five of these students (1A1, 4A1, 4A2, 4B1 and 7C1) mention something about the place values (position system) that are essential on these pages, and one student expressed:

> Om siffror och deras värde. (About numbers and their value.) (4A1)

14 of the students expressed that the pages are about digits and numbers, but only five of the students related the content to place value, which is an essential term to grasp the mathematical content on the pages. Current important concepts that students relate to involves SFL's *ideational metafunction*.

The resource, being most frequently used both initially when predicting and when students identify the mathematical content was the pictures. Furthermore, it appears that the high-performing students use more appropriate sources, such as the heading and information box where they read the text and the images, compared with other students when predicting the content. It is also worth noting that high-performing students' average value of the chosen resources is 3.2, while

Table 5. The results of Question 2 "How can you find out what the pages are about?"

	Heading	Information box			Tasks	Total	Average
The students: A= High B= Middle C= Low	Read the heading	Read the text in the information box	Look at the illustrations in the information box	Read the text in the tasks	Look at the illustrations in the information box		
1. 1A1		X	X			2	
2. 1A2		X	X	X	X	4	
3. 4A1	X	X	X			3	19/6= 3.2
4. 4A2		X	X	X	X	4	
5. 7A1		X	X		X	3	
6. 7A2	X	X	X			3	
7. 1B1		X	X			2	
8. 1B2						0	
9. 4B1				X	X	2	13/6= 2.2
10. 4B2		X	X	X	X	4	
11. 7B1	X	X	X			3	
12. 7B2	X			X		2	
13. 1C1			X		X	2	
14. 1C2					X	1	
15. 4C1			X	X	X	3	
16. 4C2					X	1	9/6=1.5
17. 7C1	X					1	
18. 7C2	X					1	
Total:	6	9	11	6	9	41	

the average of low-performing students is 1.5. From an SFL perspective, these aspects involves the choice of semiotic resources and refers to SFL's *textual metafunction*.

Clarification

The students should clarify (explain) digits and numbers, which are essential concepts to understand the content. Four of the students (1B1, 1C1, 1C2 and 4C1) replied that they did not know how to explain the concepts. On the other hand, six of the students (1A1, 1A2, 4A2, 4C2, 7A2 and 7C2) were able to explain digits and numbers in a well-functioning manner. Examples from these students' responses include:

> 10 tal är över nio och ental är 9 och neråt 8–0. (Tens are more than nine and unit digits are 9 and down to 8–0.) (1A1)

> Siffra är upp till 9. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Tal är siffror som 12. (Digit is up to 9. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Numbers are digits like 12.) (7A2)

> Siffra är ett tal. Ett tal kan användas för att ta reda på saker som t ex ett pris. (Digit is a number. A number can be used to find out things such as a price.) (4A1)

On the other hand, six students (1B2, 4A1, 4B2, 7A1, 7B1 and 7C1) had difficulty distinguishing between digits and numbers. For example, one student responded:

> Siffra det är ett tal, 3, 5, 1, 78, 50 massa olika tal. (Digit it is a number: 3, 5, 1, 78, 50. A lot of different numbers.) (7B1)

Two students related numbers to task, and expressed:

> 10 + 7. (7B2)

> Tar 1 + 2 och då blir det 3 och då blir det ett tal. (Take 1 + 2 and it becomes 3, and then it is a number.) (4B1)

Only one third of the students, independent of age and achievement level, could clearly explain digits and numbers, which are essential concepts to understanding the content on the pages and refers to SFL's *ideational metafunction*. Worth noting is that none of the students mentioned place value, i.e. the position of a digit in a number refers to its value (Ross, 2002), which was essential information expressed in all of the textbooks.

All the students who were able to provide an explanation to numbers and digits used several forms of semiotic resources, except for three students who only used words or symbols. The use of semiotic resources relates to SFL's *textual metafunction*.

Questioning

All of the students were able to ask a question, except for one student (1C1). In the students' questions three different categories appeared. Five of the students' (1B2, 1C2, 4B1, 7C1 and 7C2) provided questions that did not involve the main ideas presented on the pages, which is the first category. For example:

> Ella har 9 kr och hon vill köpa ett tuggummi som kostar 3 kr. Hur många kan hon köpa då? (Ella has 9 kr and she wants to buy one chewing gum, which costs 3 kr. How many can she buy?)(4B1)

> Bilda tal med siffrorna 5,7,1,6 och 3. (Construct numbers with the digits 5, 7, 1, 6 and 3.) (7C2)

The second category that appeared among seven of the students' (1B1, 4A2, 4B2, 4C1, 4C2, 7A1 and 7B1) notes relates to questions that concerned the mathematical main ideas but were more or less imitated from the tasks on the provided pages in the mathematical textbook. For example:

> Vad är före 70 och efter 70? (What is before 70 and after 70?) (1B2)

> 5836 hur mycket är siffran 3 värd i talet? (5836 – How much is the digit 3 worth in the number?) (4C2)

The last category of questions concerned the main mathematical ideas on the pages and were formulated differently than the questions posed on the current pages in the mathematical textbooks. Five students, 1A1, 1A2, 4A1, 7A2 and 7B2, asked these kinds of questions. For example:

> Klura ut om man lägger tiotal 10 stycken och 5 ental. (Find out if you add 10 tens and 5 unit digits.) (1A2)

> Skriv talen med siffror trettioåttatusen femhundrasex. (Write the number with digits thirty-eight thousand five hundred and six.) (7 B2)

To be able to construct a question, the students needed to identify important aspects in the text relating to the content (Palincsar, 2003), which involves SFL's *ideational metafuntion*. One third of the students could not relate their question to the current mathematical content place value and some of them wrote questions that involved addition. However, questions involving the current mathematical content appeared among several of the students, but mostly by imitating the tasks provided in the textbook. Only a few students, mostly high achievers, were able to construct a question to the current mathematical content place value that were not imitated from the mathematical textbook.

Interpersonal metafunction concerns the use of interrogatives and imperatives in the students' own constructed questions, which visualize the relation between different actors involved (Halliday & Hasan, 1985).The interrogatives that appeared involved is (är), what is (vad är), has (har) and how much (hur mycket), and the imperatives that appeared were figure out (klura ut), take (ta), write (skriv) and construct (bilda). These interrogatives and imperatives exclusively involve students following directions and completing tasks. All of the tasks are closed and involve one correct answer.

Furthermore, all of the students' tasks were multimodal, involving both symbols and words and referred to SFL's *textual metafunction*, except for five of the students where only words appeared (no symbols). For example, "Bilda ett högt tal med siffror" (Make a high number with digits) (7C1).

Summarization

More than half of the students (1A2, 1B1, 1B2, 4A1, 4A2, 4B2, 4C2, 7A1, 7A2, 7B2 and 7C2) were able to summarize the content in a clear way, which is visualized by two examples:

> Siffror, tiotal och ental. (Digits, tens and unit digits.) (1B1)
>
> Jag har lärt mig vad udda och jämna tal är och vad siffran i ett tal betyder. (I have learnt what odd and even numbers are and what the digit in a number means.) (7A2)

However, 1A1, 1C1, 1C2, 4B1, 4C1, 7B1 and 7C1 did not provide a clear definition of digit values and provide examples to clarify their explanations. For example:

> Siffror. (Digits.) (4C1)
>
> Hur man räknar ut läsuppgifter i matte. Man lär sig att klura ut lite svårare uppgifter. (How to count reading tasks in mathematics. You learn how to figure out more difficult tasks.) (4B1)
>
> Lägga tal. (Lay numbers.) (1C2)

For the students to be able to summarize means that they needed to identify the main ideas in the text (Palincsar & Brown, 1984). In this study, more than half of the students were able to summarize the current mathematical content, place value which refers to SFL's *ideational metafunction*. Nevertheless, for the other students – even after reading the information provided in the information box and performing some of the tasks – this failed to help them embrace the intended mathematical content of place value and relate the content to digits and solving text tasks. This could mean that their individual understanding of place value is not developed.

Few of the students' texts were multimodal, i.e. contain different forms of representation to clarify the content and relate to SFL's *textual metafunction*. Only words were used and no specific examples with numbers were presented in the students' texts, except for two of the students' notes that both involved digits and words. Thereby clarification of the intended mathematical content, place value, was unclear. Furthermore, no student performed any visual representation of place value.

Discussion

The reliance on textbooks in mathematics is a worldwide phenomenon, but specific significant in Sweden where these textbooks are often the main resource for informing students about mathematics (Johansson, 2006). Furthermore, in Sweden a common practice during mathematics lesson is students working individually in their mathematical textbooks (Boesen et al., 2014; Johansson, 2006), so it is often up to individual students to create their own mathematics understanding. Consequently, knowing more about which strategies students use to make sense of the content in textbooks is important.

In this study, the reading comprehension strategies of eighteen grade 1, 4 and 7 students with different achievement levels were investigated. In alignment with the pilot study (Segerby, 2014), high achieving students independent of grade had, to a larger extent, developed successful strategies. Just as low achieving students independent of grade had developed less successful strategies than their peers, which started in grade 1. However, independent the students' achievement level all of them need to develop their reading comprehension strategies further.

In this study, Halliday's Systemic functional linguistic (SFL) is coordinated with Palincsar and Brown's reciprocal activities to get an understanding of how the context of situation and the context of culture influence the students' comprehension strategies.

Context of situation

The context of situation in this study involved grade 1, 4 and 7 students making sense of pages concerning place value in the area number sense, in commonly used mathematics textbooks. The use of particular reciprocal comprehension strategies indicate the meaning students give to the reading process, such as how they interpret symbols and words, as well as the meanings they gain from performing some of the tasks. This indicates that students' perceptions of the context of situations from the provided information, as well as from prior knowledge, helps them interpret and conclude what they should be learning from the process.

The results show that all of the students developed more or less successful comprehension strategies to grasp the mathematical content, although probably without any previous formal instruction. This can have long-lasting effects on students' mathematical skills, which Korhonen et al.'s (2012) study showed. The same strategies that students in grade 1 in this study used can also be found in studies with high school and university students (Shepherd et al., 2012; Weinberg et al., 2012; Österholm, 2006). Thereby these strategies seem to continue to be used during their entire educational career and thus influence the students' opportunity to grasp the mathematical content.

In this study, when the students predicted the content, the most common semiotic resource used was pictures. Thus, it seems that pictures have an important role when students approach texts in mathematics. According to Jewitt (2005), the purpose of the illustrations is to make certain mathematical concepts easier to learn. However, some of the images in the text on the textbook pages are decorative, such as an image of a party hat (see Brorsson, 2014, p. 8) that has no function to clarify the mathematical content presented (Noonan, 1990). Other pictures are essential and some of them are important but not essential (Noonan, 1990), which can be very confusing for the students.

The high-achieving students used several sources when predicting the content, which is similar to the results in Browns and Days's (1983) study and the pilotstudy (Segerby, 2014), where high-performance readers combine information from multiple sources to make sense of the intended content.

Several of the other students in this study showed limited prediction skills and did not use appropriate sources when identifying the main ideas concerning "place value". For example, few of the students used the title as a source, which is a suitable source for identifying the content after which the title is usually a brief summary of the intended content (Carter & Dean, 2006). Instead, several students chose to focus on the information presented in the tasks. Some of the students only concentrated on the information in the tasks and not on the explanatory parts of the text, which was a common strategy used by high school and university students, which also usually lead to wrong conclusions about the mathematical content presented (Shepherd et al., 2012; Weinberg et al., 2012). This is problematic because activating inappropriate prior knowledge can mean that new knowledge about place value is not connected to the students' prior knowledge. Thereby, their knowledge about number and digits might not be developed (Carter & Dean, 2006).

The texts in mathematics are often very compressed with few clues to understand the mathematical content (Barton & Heidema, 2002), which means every word is important to understand (Lee, 2006; Möllehed, 2001) and involves the Reciprocal Teaching strategy clarification. In this study, the concept of "number" was confusing for some students, since numbers have two meanings in mathematics (numbers such as 12, and task) and in another context numbers (tal) mean speech. The issue of mathematical words having another meaning in another context is also raised by several researchers (Lee, 2006; Riccomini et al., 2015).

Providing questions after reading and performing some tasks on the pages was difficult for all of the students, except for the high achieving ones. Several of the other students' questions did not involve the current mathematical content or were imitated from the questions posed on the pages. All of the provided questions by the students were closed, i.e. involved one correct answer (Sullivan & Lilburn, 2002). This is not surprising because if the students are not used to providing tasks by themselves then the tasks provided in the textbook would be the only ones they are familiar with.

Also, when summarizing the content, the writing skills of most of the students in grade 4 and 7 were limited, and few of the students used several semiotic resources. One reason for this might be that it was a challenge for many of the students to communicate by writing an explanatory text in mathematics. As Watkin et al. (2004) suggest, students might not make the connection between written sentences and visual representations to express meaning. When discussing students' use of comprehension strategies there is also a need to consider the context of culture, which operates around the context of situation (Halliday & Hasan, 1985).

Context of culture

The context of culture highlights how a reliance on mathematical textbooks in many classrooms has been identified in Sweden (Johansson, 2006; Mullis et al., 2012).

In Sweden, the structure of the textbook pages in mathematics is consistent across a wide range of Swedish textbooks, and thus is a part of the context of culture. The tasks provided in the mathematical textbooks then become a strong part of the context of culture, which influence how students engage in mathematics. However, none of the tasks provided on the textbook pages asked students to explain or describe their thinking. Instead, the questions are closed and the answers involve a number or a word.

Furthermore, by working individually in the textbook, which is a common approach in mathematics in Sweden (Boesen et al., 2014; Johansson, 2006), it is up to each student to create their own mathematical understanding from reading the textbook. This is problematic since according to this study, students develop comprehension strategies from grade 1 that are more or less successful in order to grasp the mathematical content in a mathematics textbook. According to Askew (2015), by identifying individual problems can lead to possibilities to improve learning for all. Furthermore, Dowker (2005) and Gersten et al. (2005) stress the importance of early identification of mathematics difficulties to prevent future difficulties in mathematics. Thus, it seems important to teach about reading comprehension strategies to help students to comprehend mathematical content presented in a text. However, complementing tasks might be needed because it is not common that students are asked to predict, clarify, ask questions or summarize the mathematical content being expressed.

Conclusion

This study indicates that it is not a question of whether or not we should teach reading comprehension strategies in mathematics. Nevertheless, the number of students investigated is small. It is also unclear whether it is students' knowledge of mathematics supports or do not support their reading or whether their reading comprehension strategies result in higher achievement in mathematics. Regardless of which way this relationship operates, the study shows that low achieving students, as well as all students in general, need to develop their comprehension strategies to grasp the mathematical content being presented in mathematics textbooks. Specific guidance is then needed about how to read and how to write multimodal texts (Huber, 2010; Quirk, 2010; Watkin et al., 2012) because students do not learn strategies that they are not exposed to (Hiebert & Grouws, 2007).

However, in other classrooms and countries the context of culture and context of situation are likely to be different. The result of this study highlights a need to not isolate mathematical reading comprehension strategies from consideration of the contexts of culture and the contexts of situation that operate in those countries where the mathematical textbook has a significant role in mathematics education.

Further research is needed to provide more insight into whether a relationship exists between reading comprehension strategies and achievement level in mathematics. Eye tracking could be one suitable approach to use. Research is also needed to examine how to prevent future difficulties students might meet when reading mathematics textbooks.

Acknowledgement

I would like to thank Birgitta Lansheim for constructive comments during writing the degree project in Special needs in mathematics. Furthermore, I would like to thank the three helpful teachers and the eighteen students that participated in this study.

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