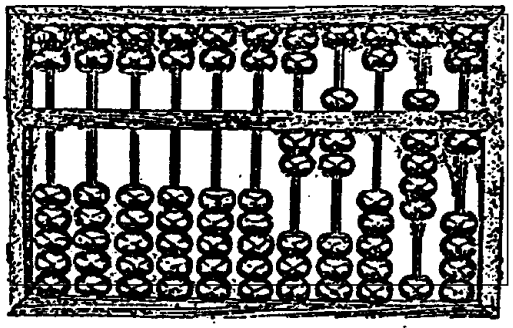
Numerals from Ancient China

*I kursen Skolmatematikens historia i Göteborg (Se Nämnaren nr 1, 1989.) medverkade professor **Frank J Swetz** som föreläsare. Frank J Swetz har studerat kinesernas matematik under många år. Hans föredrag, Numerals from Ancient China, handlar om sifferskrivning och räkning med hjälp av ”pinnar”. Detta sätt att räkna leder fram till kinesernas räkning med abacus.*

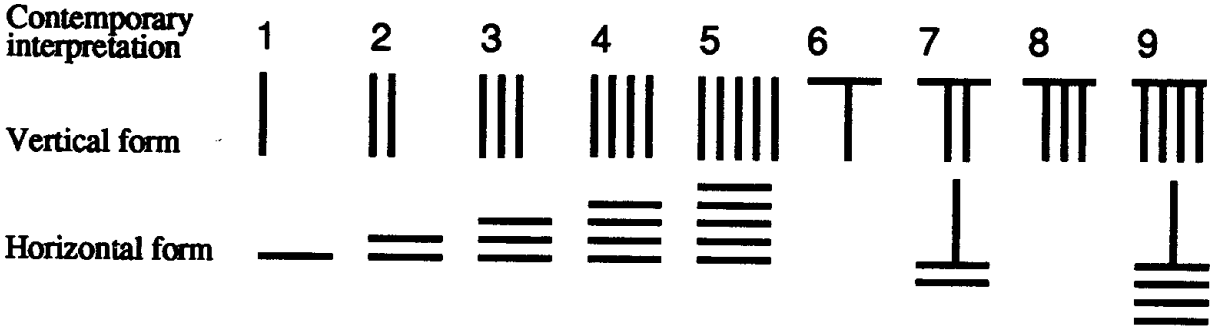


Abacus.

Ask a fellow teacher to tell you something about Chinese mathematics and he or she will probably say ”The Chinese do their mathematics on an abacus.” This is true. The bead-abacus is a popular computational device, evident today in Chinese classrooms, markets and shops. While in the minds of most people, the abacus is closely associated with the Chinese and their mathematics, relatively speaking, it is a rather modern Chinese device. The bead-abacus, suan pan or zhusuan as it is known in China, became popular in about the middle of the 15th century. But for a thousand years prior to this time, the Chinese demonstrated a computational proficiency and accuracy unique in the ancient world. They could extract square and cube roots of numbers to several decimal places and by 500 A.D. had obtained a value for π of 3.14159267, a value unknown elsewhere until after the 16th century. This amazing computational efficiency and accuracy was accomplished using a set of sticks whose configurations represented numbers. Written records of such stick configurations were composed of simple strokes and became, in themselves, a set of numerals.

Rod Numerals

Chinese rod numerals can be categorized into two groups, those of a vertical form and those of a horizontal form:



Rod numerals from one group would be used to designate; units, hundred, ten thousands ... and from the other group; tens, thousands, hundred thousands ... In representing a number, a Chinese mathematician would lay out his rods in a row progressing from right to left and alternating between horizontal and vertical forms. Such a procedure indicates a knowledge and use of positional notation based on powers of ten. Thus the number 438 would be represented by

and 1989 by

As conceived, the rod numerals possess certain properties:

1. Rod numerals 1-5 are obviously simply tally symbols by which a 1:1 correspondence is established between the objects being counted and the recording strokes. The symbols change to a coded symbol after a count of 5. Coded symbols are comprised of a stroke representing 5 and simple tally strokes. A close association with hand counting is evident.

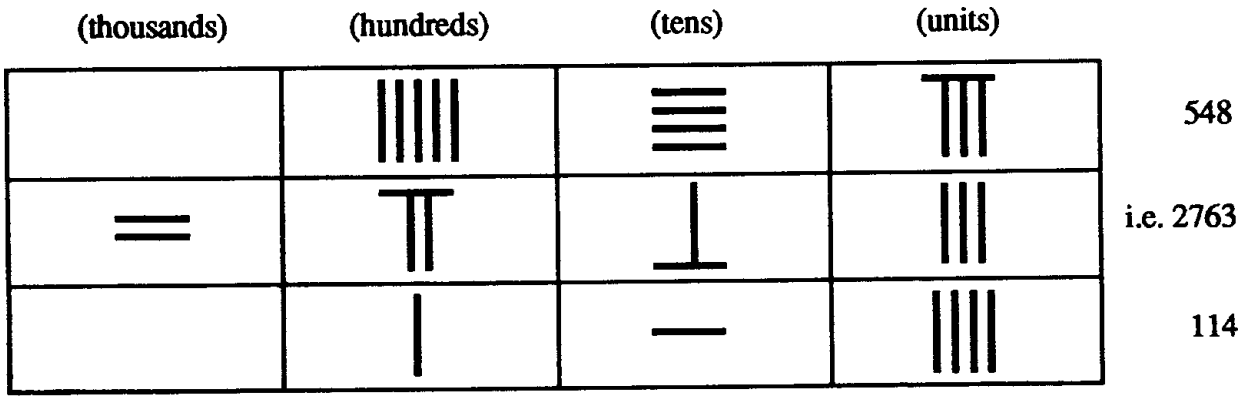
2. The symbols used to represent powers of ten are single strokes – either horizontal or verticle. Rod numeration is clearly decimal in nature and employs a positional ordering based on powers of ten.

3. The positional ordering in rod numeration follows modern conventions. Chinese mathematicians used decimal based, place-value systems of recording numbers from approximately the 8th century BC onward.

Computing with Rods

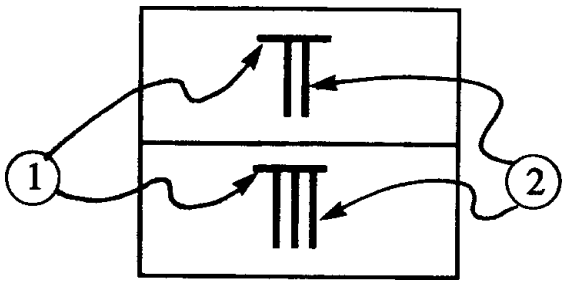
In order to assist with computional chores and preserve positional integrity, the rods were used in conjunction with a computing board whose surface contained a grid comprised of columns and rows. Rods representing a number would be laid out across a grid row and individual rod configurations would lie in particular columns. Thus a simple addition problem might be laid out on a board as follows.

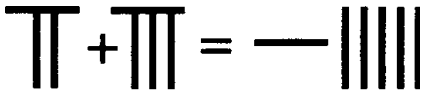
The visual pattern of the rod numerals



facilitate the performance of simple arithmetic operations. For example, consider how the addition of 7+8 was accomplished. A trained computer would first note the five's strokes, i.e. horizontal strokes, ① combine them and carry 1 horizontal stroke into the next column, the remaining vertical stroke would then be added together ②.

In performing this procedure, a rod was





(7 + 8 = 15)

physically "carried to the next column and the ancient Chinese used this term to describe the process of moving ahead one tens place on the left in their numeration scheme. In the subtraction process, they used the term "borrow" to indicate the moving of a ten to the right to permit a subtraction. Thus, in principle, their algorithms for performning addition and subtraction were very similar to ours.

The use of a computing board grid permitted the designation of empty positions within a number, for example, 107 would be represented with rods on a board as:
and be written accordingly as

Mathematical scribes would be able to

discern the number of empty position places in a numeral and interpret the number correctly. A written symbol to zero, 0, was adopted for use by the Chinese in the 12th century. Thus from that time onward, 107 would be recordes as
On a computing board, rod entries placed

to the right of the unit's column represented decimal fractions. The Chinese are historically distinguished as being the first people to mathematically emply a system of decimal fractions.

Another amazing fact concerning rod numerals is that from the 2nd century B.C. onward they came in two colors, black and red, to represent positive and negative numbers respectively! An examination of old Chinese arithmetic manuals reveals rules for working with negative numbers that are exactly the same as those we use today.

By the time of the Ming dynasty (1368-1644 A.D.), commercial considerations forced a need for a computing device that was faster and more popular than computing rods. To meet this need the bead-abacus was developed but its working principles were directly derived from the computing rods and board.

On the Chinese abacus, columns of seven beads are divided into two groups comprised of 2 five-counters and 5 unit counters. Thus, the Chinese abacus is the terminal product of a two thousand year old computing tradition that began with the fingers of the human hand, moved to a set of computing rods and finally ended with a fixed

frame containing columns of vertically movable beads.

Useful Learning Experiences

In lessons on number systems, children are often exposed to Egyptian or Babylonian numeration systems. Chinese rod numerals are easier to understand and simpler to work with than other ancient numeration systems. Number recognition exercises employing match sticks and an overhead projector (i.e. lay number configurations composed of match sticks on the face of the projector forming images for class identification), sharpen mathematical intuition. Children also enjoy manipulating sticks to form their own numerals and challenging their friends to correctly identify the number represented.

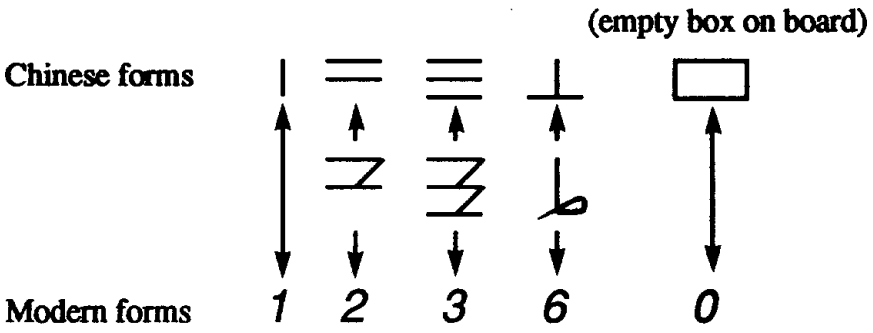
A class examination and discussion of rod numerals reveals several historically relevant facts, namely:

1. The Chinese were the first people to use a decimal place value numeral system.
2. They were also the first to use decimal fractions.
3. Their arithmetic was the first in the world to accommodate negative numbers.

Able pupils can build on their experience with rod numerals to further explore the arithmetic of computing rods. Using rods, how was subtraction performed? Multiplication? Root extraction?

Indeed, there can be many questions asked and fruitful learning experiences constructed around the Chinese rod numerals. In fact, some historians hypothesis that our present day numeral system evolved from the Chinese rod numerals. In support of their conjecture, they offer the following speculative evidence

What do you think?



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