Proving the Pythagorean Theorem

—You have seen the rest but which is best?

I januari 1989 medverkade professor ***Frank J Swetz****, USA, i kursen Skolmatematikens historia i Göteborg*[1]*. Bland annat tog han på kursen upp att Pythagoras sats var känd och använd långt före Kristi födelse av folk i Egypten, Indien och Kina.*

Introduction

In a previous article by Paulus Gerdes (1988), he referred to the 370 different proofs of the Pythagorean Theorem published by Elisha Loomis (1972) and then went on to develop his own proof of the theorem. Gerdes further explained that his proof had an infinite number of variants, thus concluding that there exists an infinite number of proofs for the Pythagorean Theorem. While this fact is interesting, as a teacher of children, it helps me little. As a teacher, my needs are modest—I seek one or two simple and convincing proofs to use with my students. At the introductory level of instruction I seek a proof that is intuitively obvious; while, later in more advanced levels, I may seek more mathematically complicated and intellectually challenging proofs. So, for me, the question remains "What is a good introductory proof of the Pythagorean Theorem—one that is simple and appealing for children?"

[1] Se Nämnaren nr 1, årg 16, s 7.

Seeking an answer in history

In many respects, our forefathers first learned mathematical concepts in a manner similar to that used by young children. They learned primarily through inductive reasoning from concrete experiences and experimentation. Thus, it would seem that history might supply an answer to the quest for the required proof.

Upon seeking out and examining the history of the Pythagorean Theorem, a startling fact emerges—namely, that the discovery of the relationship that the sum of the squares of the sides of a right triangle is equal to the square of the hypotenuse did not originate with Pythagoras nor even with the Greeks! It was known and used by several ancient peoples: the Babylonians, the Egyptians; the Indians and the Chinese, well before the time of the Greeks.

In early agricultural and river-based societies, the determination of the seasons and seasonal changes was of prime importance for survival. Periods of flooding and those advantageous for the planting of crops had to be recognized and accommodated. Priest-mathematicians took observations of heavenly phenomena to note changes that marked the seasons. Their tools were simple; a staff placed vertically in the ground served as a gauge to measure the sun's shadow and shadow length became a convenient chronological time keeper. Resulting "shadow reckoning" concerned the relationship between the sides of a right triangle. Eventually, these priest-mathematicians and their societies became aware of the properties of a right triangle.

Of all the ancient agricultural and river-based societies perhaps none was more involved with astronomical observation and shadow reckoning than the Chinese, therefore it is not surprising to find the first documented proof of the Pythagorean Theorem in China. The proof is contained in a simple diagram comprised of four congruent right triangles and a small square. See Figure 1.

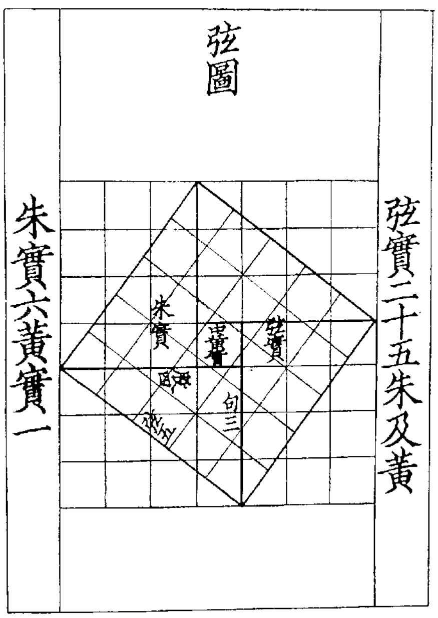
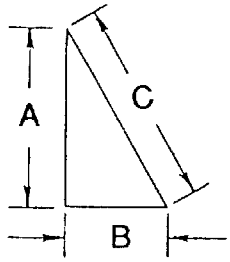
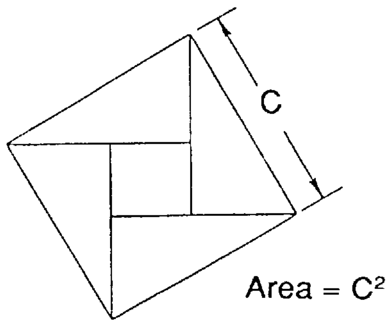
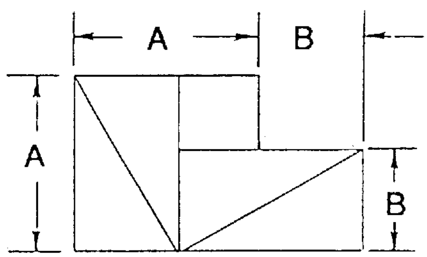


Figure 1

When the diagram is visually or physically dissected and the pieces rearranged, the Pythagorean relationship becomes obvious:







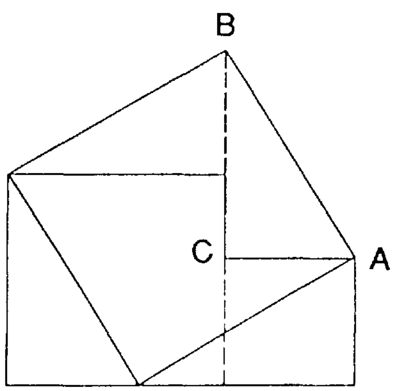
$A^2 + B^2 = C^2$

$\text{Area} = A^2 + B^2$

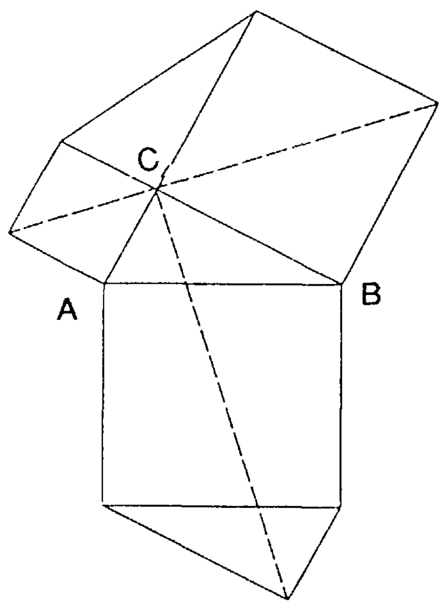
In the Chinese situation, the vertical gauge was called "*Gu*", meaning a scale, and the shadow cast by the sun was called "*Gou*". Thus, the mathematical concept known in the West as the Pythagorean Theorem, is known in China as the *Gougu* theorem. Its proof is found in *Zhoubi suanjing* (*The Arithmetical Classic of the Gnomon and the Circular Paths of Heaven*), the oldest known Chinese mathematics text whose origins can be traced back at least to the 6th century B.C. The proof is an example of a dissection proof which combines algebraic manipulation with geometric intuition. It is elegant in its simplicity and is also aesthetically appealing.

So from ancient China I have found the proof I can use with my children. They easily understand the proof and enjoy working with it as a learning activity whereby they cut out and rearrange the pieces themselves arriving at the appropriate conclusion. Furthermore, this proof lends itself nicely as a demonstration on an overhead projector.

History reveals many interesting dissection proofs. Two more of these are left as learning-challenges for the reader. Can you rearrange the pieces of the following to prove the Pythagorean proposition?



Devised: by Tabit ibn Qorra (826—901)



Leonardo da Vinci (1452—1519)

References

Paulus Gerdes, "How many proofs of the Pythagorean Theorem do there exist?" *Nämnaren,* Årgång 15 (1988) Nr 1, pp. 38—41.

Elisha Scott Loomis, *The Pythagorean Proposition* The National Council of Teachers of Mathematics, 1972.

Frank J. Swetz, *Was Pythagoras Chinese?* The National Council of Teachers of Mathematics, 1977.