# How many proofs of the Pythagorean Theorem do there exist?

*Paulus Gerdes, rektor vid lärarhögskolan i Moçambique, tar upp en gammal fråga — Hur många bevis finns det för Pythagoras' sats?*

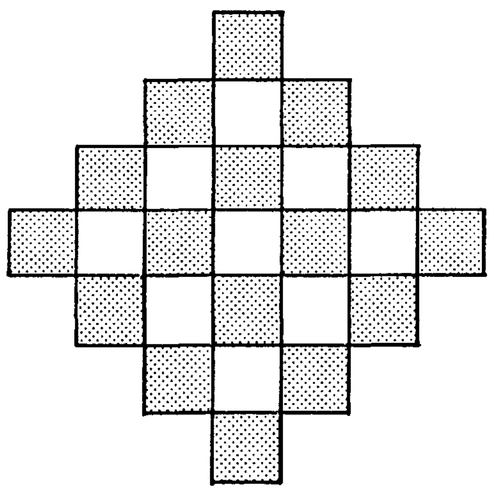
## Introduction

In his well-known study "The Pythagorean Proposition"—edited by the National Council of Teachers of Mathematics—professor Elisha Scott Loomis gives ". . . in all 370 different proofs, each proof calling for its specific figure. And the end is not yet" (Loomis, 1972, 269). And the end is not yet . . .? Loomis challenges his readers with the following remark: "Read and take your choice; or better, find a new, a different proof, for there are many more proofs possible, whose figure will be different from any one found herein" (Loomis, 1972, 13).

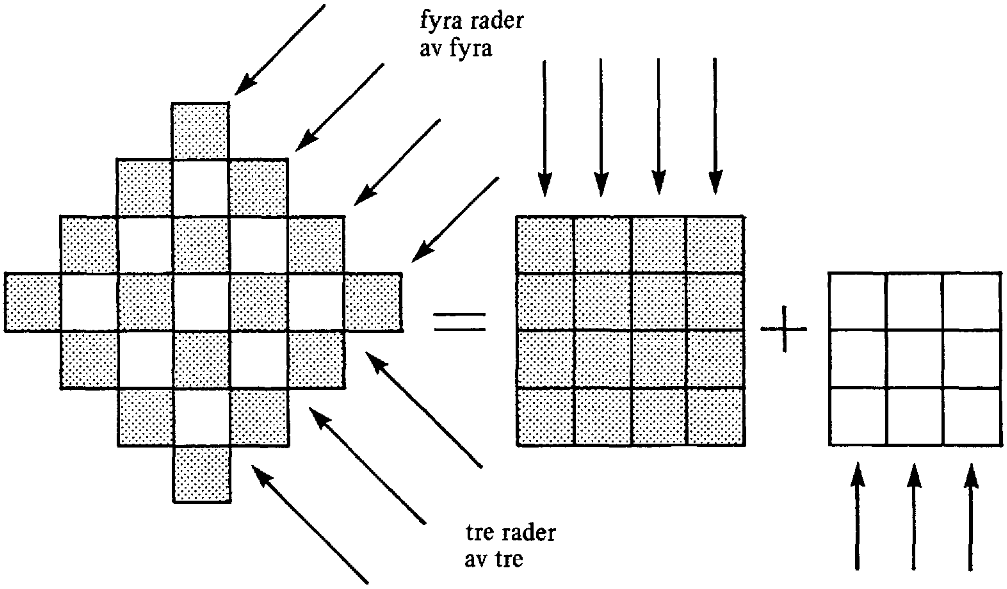
But how many demonstrations will be possible?

## Plaiting patterns and the discovery of the Pythagorean Theorem

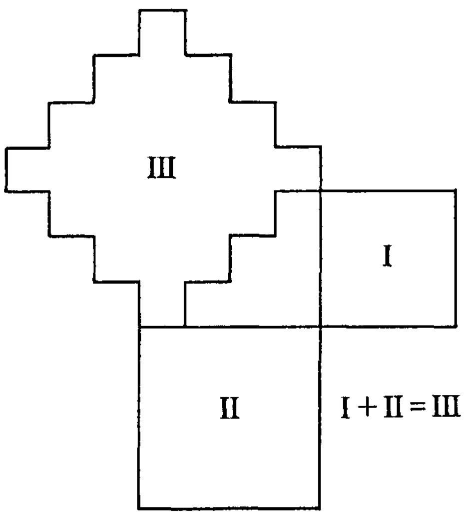
While I was doing research on mathematical aspects of basketweaving, I found that a very old and common plaiting pattern



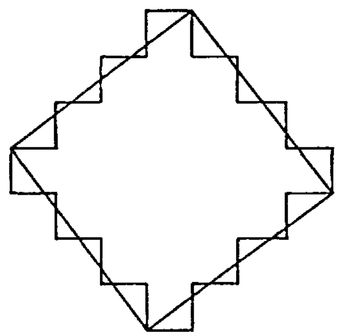
could have stimulated to the discovery of the Pythagorean Theorem (and of the socalled Pythagorean triples, cf. Gerdes, 1985). Looking at the number of unit squares on each row of the 'toothed' square,



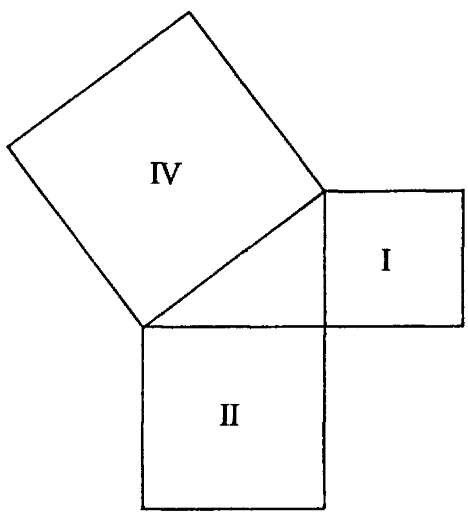

it is easy to see that a toothed square is equal (in area) to the sum of two 'real' squares. If we draw the three squares together in one figure,




we arrive at a *particular case* of the Pythagorean Theorem, as the toothed square can be easily transformed into a real square of the same area,



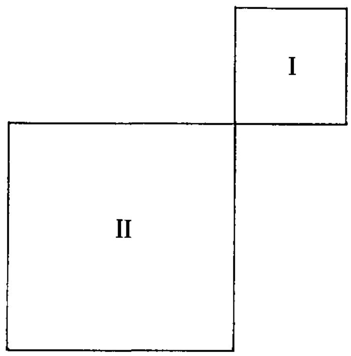
and therefore I + II = IV.



Does this discovery process suggest any new demonstrations of the Pythagorean proposition?

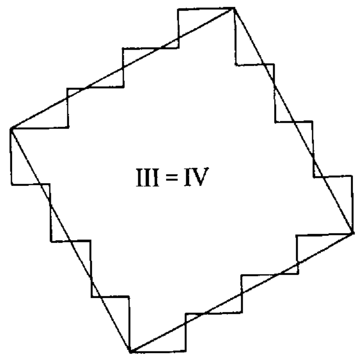
What happens, when the unit squares of the two squares I and II have different sizes?

## Variations



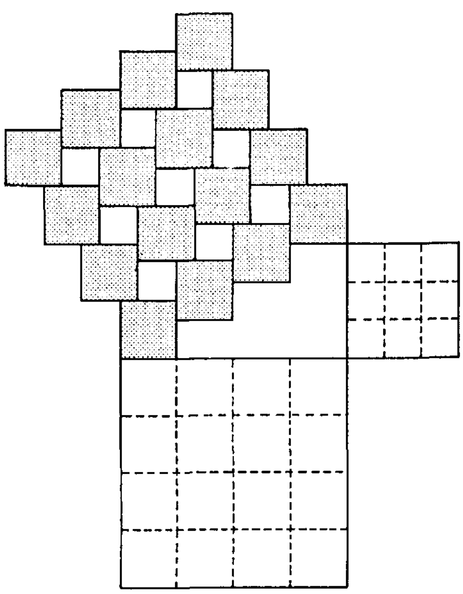

Let I and II be two arbitrary squares. We dissect I into 9 little congruent squares, and II into 16 congruent squares, and join the 25 pieces together as in the figure.

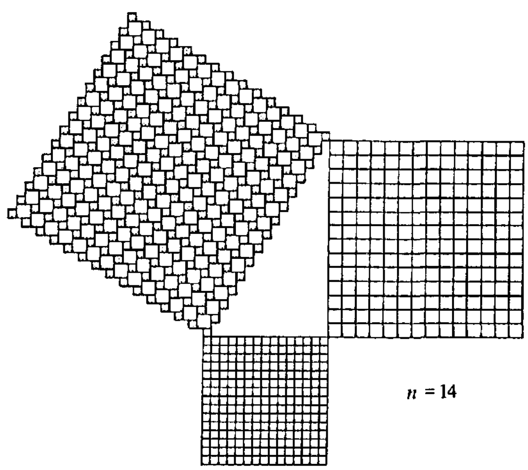
The thus obtained toothed square III is equal in area to the sum of the real squares I and II; III = I + II. As the toothed square is easily transformed into a real square IV of the same area,




we arrive at IV = I + II, i.e. the Pythagorean proposition in all its generality.

Instead of dissecting I and II into 9 and 16 subsquares, it is possible to dissect them into $n^2$ and $(n+1)^2$ congruent subsquares for each other value of $n$ ($n \in N$). Figure 9 illustrates the case $n = 14$.

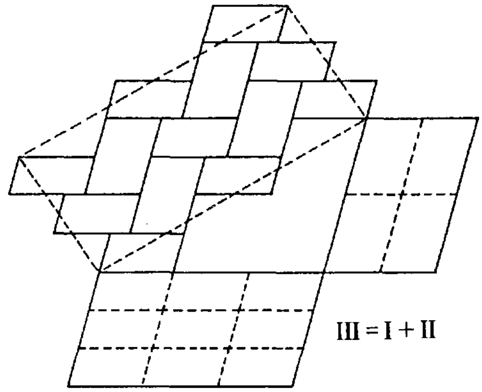




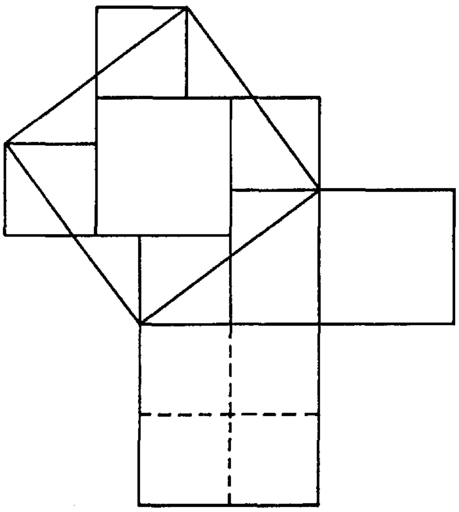
To each value of n there corresponds a proof of the Pythagorean proposition. In other words, Loomis was right: ". . . the end is not yet", for there exists an *infinity* of demonstrations of this famous theorem.

For relatively high values of n, the truth of the Pythagorean proposition is almost immediately visible.

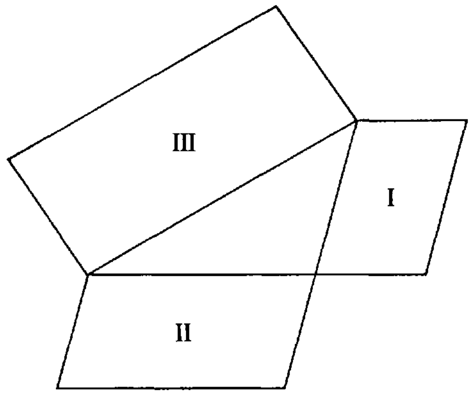
For $n = 1$, one obtains a very short, easy understandable proof.



III = I + II



Analogously, the generalization of the Pythagorean theorem for parallellogrammes can be proved in infinitely many ways.




**References:**

1. Elisha Scott Loomis, *The Pythagorean Proposition,* 1940, 1972.
2. Paulus Gerdes, *Zum erwachenden geometrischen Denken,* Dresden, Maputo, 1985.