

FROM NATURAL NUMBERS TO INTEGERS (FROM $N \rightarrow Z$) – A LEARNING STUDY ABOUT THE IMPORTANCE OF IDENTIFYING CRITICAL ASPECTS, TO ENHANCE PUPILS' LEARNING.

Anna Lövström

School of Education and Communication, Jönköping University

The thesis is focused on producing knowledge about which distinctions are necessary for pupils to do, in order to extend their number domain from natural numbers (N) to integers (Z). The thesis adopts the theoretical framework Theory of Variation (Pang & Marton, 2003), which makes it possible to analyse the relation between teaching and learning in commensurable terms. Data is collected from two Learning Studies. Participating pupils are eight to nine years old. Preliminary results indicate, for example, that pupils tend to use subtraction according to the commutative law of addition. They don't seem to discern the difference between 3-2 and 2-3. That makes it difficult for pupils to realize the need for negative numbers.

Introduction

According to Ball (1993) and Kilhamn (2011), the understanding of negative numbers is difficult both for teachers and pupils. In Swedish schools negative numbers are taught in middle school, but Ball and Kilhamn see advantages in introducing negative numbers earlier. In this study we introduce negative numbers for pupils aged eight and nine.

Aim of the Study

The aim of the study is to produce knowledge about which distinctions are necessary for pupils to do, in order to extend their number domain from natural numbers (N) to integers (Z). The research questions are formulated as follows: Which aspects are critical for developing pupils' understanding of integers? Which aspects are critical for developing an understanding of negative numbers? Which teaching design is effective to make it possible for pupils, ages eight and

nine, to develop an understanding of negative numbers? The aim of the presentation is to share and discuss preliminary findings so far.

Methodology

Data is collected from two Learning Studies. The first one consists of ten lessons, 68 pre- and post-tests, fifteen repeated post-tests and interviews with fifteen pupils. The second one consists of five lessons, 61 pre- and post-tests. Participating pupils are eight to nine years old. Learning Study, which is guided by the Theory of Variation (Pang & Marton, 2003), is a model for improving teaching and learning in classroom settings. In the Learning Study cycle the teacher group try to find out what is critical for understanding the intended learning object.

Preliminary Results

To understand why negative numbers are useful, pupils need to discern the difference between subtractions of this type: $3-2=$ and $2-3=$. After the first lesson in the second Learning Study, some pupils were very convinced that subtractions of the type mentioned above always resulted in the same answer. In the teacher group we tried to create a design that made it possible for the pupils to understand why you can't treat those subtractions as similar. To extend pupils' number domain it seemed helpful to use simultaneous presentation of opposites, for example: subtraction versus addition, go to the left versus go to the right, tasks $4-2$ versus $2-4$ and opposite numbers.

References

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