Number, Reasoning and Representations. The Design and Theory Of An Intervention Program For Preschool Class In Sweden

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We describe the design process for an intervention program in Swedish preschool class. Initial research and theory based design principles are described and related to the construction of the intervention material. Field test feedback and its affect on the material are presented. Consequences of the validity of initial design principles are discussed and new principles are suggested.

Introduction

In this paper we will discuss an intervention study in the Swedish pre school class. Preschool class in Sweden belongs to the school system, is non obligatory but in practice reaches all six year old children. In the study, the effect of the intervention is measured carefully, using state of the art statistical methods. The performance of the pupils on a set of standardized tests are measured three times over time and related to the performance of pupils in an active control group, that is, pupils that also get an intervention that is similar in style, but different in content, organization and not math focused. The analysis of this data that is underway and show positive results, but we will not discuss that in this article. Instead we will concentrate on the aspect of *design*. The purpose of this paper is hence to present how several cycles of field testing and teacher feedback shaped the material and also led to theoretical insights. Methodologically, the paper hence falls under the design research paradigm. Edelson (2002) argues that design research is a form of educational research because:

> (a) design offers opportunities to learn unique lessons,
> (b) design research yields practical lessons that can be directly applied, and
> (c) design research engages researchers in the direct improvement of educational practice. (p. 105)

[1] I alfabetisk ordning

Cobb, Confrey, diSessa, Lehrer and Schauble (2003) argue that design experiments are supposed to generate theory. Edelson (2002) list three types of theory types that design research can lead to, domain theories, design frameworks and design methodologies. What we want to discuss in this paper most closely resemble a ”design framework” in Edelsons terminology.

> **Design frameworks** - describe the characteristics that a designed artifact must have to achieve a particular set of goals in a particular context (prescriptive). A design framework is a collection of coherent design guidelines for a particular class of design challenge. (Edelson, 2002, p.114)

In what follows, we will describe four design-feedback cycles and give three examples illustrating how we connected feedback to theory and subsequent changes in the material. We will end the paper by a discussion about some more fundamental theoretical insights gained by the process.

Background

Children´s experiences of using mathematics during the preschool years are highly predictive of their later success in mathematics in compulsory school (Duncan et al., 2007; Nunes, Bryant, Sylva & Barros, 2009). Some children lack important experiences from using and reasoning about mathematics during preschool years and therefore start school with a weak mathematical knowledge that tend to cause further mathematical difficulties in a downward spiral (Case & Okamoto, 1996; Clements & Sarama, 2009; Morgan, Farkas & Wu, 2009; Geary, 2011).

In recent years there has been a growing interest in early intervention in mathematics. Due to different theoretical definitions of "number sense" targeted interventions vary a lot regarding the mathematical content such as: *Preparatory arithmetic skills* e.g. seriation, classification and conservation of numbers (Malabonga, Pasnak, Hendricks, Southar & Lacey,1995), *one-to-one correspondence, efficient counting strategies, decomposition of numbers, add 1 to a number, and solving simple addition and subtraction story problems* (Clark et al., 2011), *number line estimation* (Ramani & Siegler, 2008). There are a few studies explicitly focusing on *number sense related to reasoning about numbers* (e.g. Nunes et al, 2007; Aunio, Hautamäki, Sajaniemi, and Van Luit, 2010). Math-oriented early childhood curricula have been developed in collaboration between researchers and teachers, e.g. *Number Worlds* (Griffin, 2003; 2007) focusing on the central conceptual structure of whole numbers developed by Case and Okamoto (1996) and *Building Blocks* (Clements & Sarama, 2007; 2009; 2011). The program *Building Blocks* focuses both on numbers and geometry and a particular feature of this program is that each domain is structured along a research-based hypothesized hierarchical learning trajectory. The theory of hypothetical learning trajectories (HLTs) is usually connected to developmental and cognitive psychology and, more recently, developmental neuroscience (Consortium for Policy Research in Education, 2011; Simon 1995). Typically, learning trajectories connects a theoretical idea about a particular learning process leading up to some learning goal, as well as practical activities designed to take the learner through the process. One of the early proponents of learning trajectories define them as “made up of three components: the learning goal that defines the direction, the learning activities, and the hypothetical learning processes—a prediction of how the students’ thinking and understanding will evolve in the context of the learning activities” (Simon, 1995, p. 136). Instruction and instructional programs based on learning trajectories have often proved successful and in particular several intervention programs for preschool builds on learning trajectories (see Clements and Sarama, 2011, for an overview).

Design process

In the case of the present intervention program, the design process involved iterative methods in four cycles before the full scale intervention started. Below we first describe some fundamental ideas of the initial design in some depth, then we give examples of feedback collected in subsequent cycles.

The participants in the initial design process were researchers in psychology, mathematics and mathematics education and other experts on mathematics education. The initial design was developed to cover adequate knowledge and concepts of numbers described in the research review on early mathematics. It includes the use of research based teaching principles claimed to be effective for children at risk for mathematical difficulties e.g. explicit and structured instructions (Kroesbergen & Van Luit, 2003; Gersten et al, 2008). A general principle for the material is hence that the content is organized thoroughly in strands where similar content is revisited several times.

The mathematical activities were organized in four themes designed to be carried out by teachers over ten weeks: *Sorting, classifying and patterns, Numbers, counting and patterns, Part-part-whole and Number line*, presented in a "teacher´s guide" (Sterner, Wallby & Helenius, in press). The themes comprise a theoretical background and the purpose of the series of activities, as well as explicit descriptions of the activities themselves. Each theme involves around ten sets of activities grouped in sessions. The mathematics sessions were organized in a structure with six phases:

Counting rhymes: A lesson starts with children and teacher gathering in a circle on the floor, counting in chorus up and down on the counting string. When a child, standing in the middle of the circle, pointing rhythmically at each child while all count together, the circle that children and teacher form is the very representation of the counting. Each of every child´s one hand and two hands respectively, forms the representations when counting in fives and tens, their eyes or ears form the representations when counting in twos etc.

Initial activity: The teacher introduces the current task and the work is done collectively in class. To provide visual support the teacher and the children use blocks, sticks, buttons, dices, numerals etc. to represent ideas and concepts.

Partner work: Children next work with partners or in small groups on similar and extended activities as they did earlier in class, using different representations. The teacher moves around among the children trying to capture thoughts and ideas that may be subject to further discussions.

Whole-class discussion: Children and teachers come together to a joint monitoring and discussion of pair work.

Children´s documentation: Children create drawings as documentations of what they did so far. The drawings are new representations that form the basis for future collective activities and discussions with teachers and peers in the next phase.

*Follow-up activity: C*hildren´s drawings are the starting point for further reasoning about the concepts they have worked on and connections, differences and similarities among the representations of those concepts.

In the initial design, each session was based on the Concrete - Representational – Abstract (CRA) model, a linear model where teacher and pupils start working with manipulatives and gradually advances to the use of visual representations and further on to abstract symbols (Witzel; Mercer & Miller, 2003). This means that our general design closely resembled a learning trajectory based program, where each session was designed to take children from a concrete manipulation stage through several phases of representations and towards some form of symbolic or abstract reasoning with for example dots, squares and other icons or symbols like written numerals.

In addition to the CRA-model, this sequence was a way to operationalize several other design principles. One important principal that guided the development of the structure of the sessions was a particular view of *activity.* Freudenthal (1991) was opposed to traditional teaching which he meant often take its starting point in the results of mathematical activities that someone else has found out, while teaching and learning is about the activity itself and on its effect. From this point of view, it is the activity as a process that is at hand and this process can be expanded and deepened in different ways over time. Activity is a key concept also within Vygotsky’s theory and children´s use of cultural signs, like language, art, counting, reading, writing and drawing help them master their mental processes. In Vygotsky's theory, language is viewed both as a cultural tool to develop and share knowledge within a social community, and as a psychological tool to structure the processes and content of one's own thinking

(Vygotsky, 1978). All development in the child appears twice, first on a social and then at an individual level:

> An interpersonal process is transformed into an intrapersonal one. Every function in the child´s cultural development appears twice: first, on the social level, and later, on the individual level; first between people (interpsychological) and then inside the child (intrapsychological). This applies equally to voluntary attention, to logical memory, and to the formation of concepts. All the higher functions originate as actual relations between human individuals (p. 57).

In our design, whole class collaboration and partner work function as activities on the social level while children´s drawing at one point function on an individual level An underlying assumption here is that drawing facilitates children´s reflection on the mathematical content they previously worked on in collaboration with teacher and peers, but from a different perspective, and that the interaction between the collective and the individual, contributes to the development of thinking. Children's drawings are creative representations that connect back to the collective reality they were previously engaged in. In the follow-up activity their drawings once again turn into an activity on the social level. This is also why we in the instructions to the teachers put emphasis on reasoning. Instead of presenting reasoning in the traditional mathematical way as a process of logical deduction to arrive at a conclusion we follow Devlin (1991) and present reasoning as a process of gathering information in order to reach a decision. Our definition encompasses the traditional view since a conclusion is a form of decision and a logical deduction is a way of using, or refining, information. The definition also allows other types of reasoning. Moreover, the definition allows reasoning to be a basically individual mental process but also communicative group work building on verbal information as well as information from other forms of communication and from concrete actions.

First testing cycle.

In the first stage of the iterative stage of the design process, sessions and themes were tested out by six preschool class teachers and the children in their classes. One thing we learned from the collaboration with the teachers in the first cycle was to carefully choose the manipulatives to be used in the activities. In one activity the children are expected to investigate and reason about how to move soft toys between delimited quantities in order to make those quantities equivalent. The teacher experienced that the activity did not work at all since children´s attention was drawn to the soft toys that everyone wanted as many as possible and they forgot all about solving the number problem. We later found this phenomenon described in the research literature (DeLoache, 2000). The more children are attracted to the physical attributes of the representation the harder it seems to be to see the symbolic information and to stick with that.

Second and third testing cycle

In the second, and later also in the third, cycle six new teachers were recruited to the team. In both cycles, a researcher, the second author of this paper, and the teachers met at six seminars during a period of five months each where the mathematical content and the teaching learning strategies were discussed. In the time between those seminars the teacher tried out the activities in their classes and documented their experiences. Teacher´s documentation then became the basis for in-depth discussions at the next seminar.

A problem that emerged during the second cycle was difficulties to promote all children to participate in the discussions, to express their views and suggest solutions. The teachers felt uncertain on how to pose open questions that would take the discussions and children´s thinking further. We decided to complement the material with examples on open questions such as: How do we know that..? What is similar and what is different in these solutions? How do we know that we have found all solutions? What will happen if we change...? How do you think Thomas thought when he made this pattern? More importantly, we also introduced a puppet into the pedagogy that sometimes came and asked questions and contributed to the reasoning in the group. The puppet has at least three equal important functions:

1. Children's ability to imagine the puppet as a "real" person help to bring out the playfulness in mathematics and "trick" them to teach the puppet and express their own views.
2. The puppet asks questions and makes statements that triggers the children's desire to reason about concepts and relationships between concepts, come up with hypothesis, provide explanations and propose solutions.
3. Using the puppet´s questions and statements, the teacher can help children turn their attention to certain mathematical aspects and phenomenon.

In Vygotsky´s view of activity, play, fantasy and imagination are central features (Vygotsky, 2004). Within play children break out of the immediate situation from what is taken for granted and they can draw attention to phenomenon which they would not otherwise notice. An important point is that by using the puppet the teacher can create what Mercer and Howe (2012) call "socio-cognitive conflict", that is to highlight perceptions of mathematical phenomenon that the teacher knows will collide with opinions that some of the children have. An example is when the puppet states that if you move the objects in a set such that they are spread out over a larger area, then there will be more of them. This statement will collide with many of the children´s opinions because they know it is not true. When they try to explain to the puppet to justify their own opinions then these opinions in next turn will collide with the opinions that the puppet expresses and that some of the children also hold. By exploring and reasoning about the phenomenon teacher and children create a room for collaborative learning.

Stage 4 analysis

In the fourth cycle eight teachers participated. Seminars were conducted in a similar manner as in stage 2 and 3. It was not until now it became apparent to us that teachers felt frustrated and uncertain of how to proceed with a subsequent session when all children did not reach what the teachers perceived as the learning goals of the present session. For example, when children documented their experiences from the work on part-part-whole relations of number seven, some children visualized the combinations by making drawings of concrete objects in two colors in different combinations. Other children drew the combinations by using dot number patterns and still others used mathematical symbols to represent different combinations like 7 0, 6 1 with an empty space between the numerals within each combination to symbolize the sum.

Teachers had an idea of the group moving through the representations together in an attempt to make sure that each child in the end reached the abstract phase in the CRA pedagogy. A seemingly simple solution was to make it clear the end of one session was not to be seen as a place to evaluate particular learning outcomes but instead as a place to reason about all representation that existed in the group concerning the activity they had been working with. So instead of making sure all the pupils reached a particular goal, it was emphasized that the role of the teacher was to make sure each child got opportunity to present their own representation of the activity, and have it and it's relation to other children's representations reasoned about in the group

Schematically, we decided to replace the linear view on how to advance from concrete to abstract representations with a circular teaching-learning model for reasoning about representations.

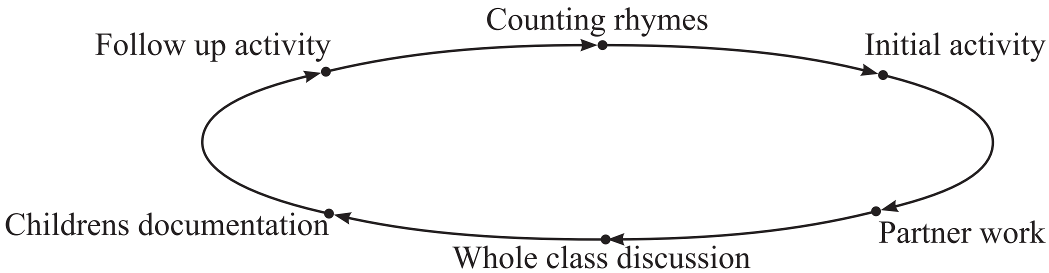

Figure 1. A circular teaching-learning structure

As shown in Figure 1 the same six phases that include collaboration in whole class, small groups/pairs, and also children´s individual work mathematics are now organized into a circular structure.

From the point of view of our initial design principles, this change effectively means that we discard, or at least reinterpret the CRA model as well as the idea of a learning trajectory based design. Our conclusion was that there seems to be a practical conflict between on one hand the Vygotsky and Freudenthal inspired view of activity and the social aspect of learning of mathematics that we designed our teaching sequence around, and on the other hand the fundamental idea of learning trajectories, where a particular target for learning is chosen and then activities are developed that are supposed to help the learner reach that target. Obviously, we still keep defined goals for the intervention program as a whole and, in similar vein as successful interventions that are based on learning trajectories, we also keep our explicitly described activities, designed to be carried out in a particular sequence.

Discussion

Designing research based materials for teaching is not a process of simply deriving the concrete design from theories or results. Rather it is about making choices among the many different ideas and principles that have proven themselves worthwhile in previous research. From documented experience of others, you can construct your own coherent design. But the consequences of choices among principles only become fully apparent when the actions that represent these principles get scrutinized by practice. In this paper we presented our initial ideas and principles for forming a research based intervention in preschool class. We exemplify three cases where field testing by teachers and the feedback generated led us to change certain aspects of the material. It is interesting to consider the different nature of these three cases.

The first case was on the level of activity design and concerned re-design of a class of activities where children where to meet certain aspects of number by means of numerosities of concrete objects.

The second case concerned a central part of the pedagogy, namely the idea that the teacher should be able to promote all children to share their ideas. By means of introducing a new element in this pedagogy, the puppet, we managed to give the teachers a new tool that could help them with their job. This idea is also documented in earlier research and, as discussed above, it fits well with the general views of learning that lies behind the program.

Both these cases can be thought about as changes made to improve the function of ideas already built into the program. In the third case the feedback from teachers, rather led us to reconsider some of our basic principles of design. The goal of taking each child through the concrete-representational-abstract phases were discarded and replaced with a goal to have all individuals representations reasoned about in the group. Rather than changing the content of any of the activities, we changed how the general intention with the activities was described to the teachers.

At this stage, we are left with a design that can be described in terms of structured activities, where children are to meet, use, develop and reason about different representations of the number concept in groups and as individuals. The focus for teachers is that the group advances through the series of activities so that individuals will get the intended experiences. This focus means that the design is fundamentally different from a learning trajectory based program in the sense that individual activities do not come paired with goals for learning outcomes. The goal for the teacher is instead that each student in each session can have their representations of the work reasoned about in the group. If the evaluation of the program continues to show good results and in the future also prove to be scalable, we believe this design offers interesting alternatives to existing intervention programs.

This design is the result of work both from researchers, designers as teachers well as teachers and their children. The effectiveness in terms of overall student outcomes is currently quantitatively analysed. It would be an interesting exercise for future design work and research to examine in what sense these design principles are transferable to other contexts, like other areas of mathematics or other ages of students.

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