When the mathematics gets lost in didactics

Judy Sayers
Stockholm University

Abstract: This study shows how six elementary teachers, construed locally as effective, interpreted and were observed to enact the same curricular and didactical language very differently. One group of three provided high-level cognitively challenging tasks to engage children in mathematics. A second group of three, ensuring their children enjoy mathematics, subordinated mathematical learning to an emphasis on didactics. The actions of this second group made mathematics invisible.

Introduction

The theme of this conference, asking us to reflect upon the development of mathematics teaching through design, scale and effect, led me to reanalyse data from a larger recent study of English elementary teachers' beliefs and practice. The study looked in-depth not only at how six teachers taught mathematics but, more importantly, justified the ways in which they presented the subject. The results were both surprising and cautionary. All teachers espoused a rich problem-solving environment, but in reality, the manifestations of their beliefs varied greatly. Why? This paper reports on how the mathematical learning intentions of a teacher can get lost in the attention paid to didactics.

The paper that follows is a consequence of a constant comparative analysis that yielded five categories of mathematics-related didactics common to all six teachers. However, despite their explicit mathematical focus, the extent to which mathematical learning was evident in their practice varied considerably. These categories were: *the role of prior knowledge; the creation of mathematical connections; mathematical vocabulary; mathematical reasoning and rich mathematical tasks*. Drawing on the traditions of grounded theory from which the constant comparison process derives (Glaser and Strauss, 1967), the following discusses briefly, and atypically for a research paper, the literature pertaining to these categories prior to a more lengthy discussion of the methodology adopted in the study.

Prior Knowledge

The activation of students' prior knowledge has long been associated with constructivist perspectives on learning whereby "information is retained and understood through elaboration and construction of connections between prior knowledge and new knowledge" (Kramarski et al 2001: 298). Indeed, there is evidence that the more effective teachers are in their activation of students' prior knowledge, the more profound the student learning (Kramarski et al 2001). Research has also shown that while a student's prior knowledge is a strong precursor of new learning, when combined with student interest, the effect was greater (Tobias, 1994). The ability of teachers to activate students' prior knowledge is a strong indicator of the quality of a teacher pedagogical content knowledge (Baumert et al., 2010).

Mathematical connections

There is increasing evidence that where teachers make an appropriate and explicit *connection* between the mathematical concepts and procedures they teach, students acquire a more profound understanding of the subject and are able to solve more complex problems (Askew et al., 1997; Schneider & Stern, 2010). That is, where teachers encourage a relational view of mathematics - an understanding of structural relationships within and between concepts - rather than an instrumental view - rules characterised by mechanical steps - learning is deeper and made applicable (Skemp, 1987). However, if connections are encouraged inappropriately then the intended mathematics may not emerge (Van Zoest & Bohl, 2005).

Vocabulary

Being mathematically proficient means that one must acquire, understand and use effectively an appropriate vocabulary (Barwell, 2005). However, the acquisition of such a vocabulary is complex. As Steele (1999) notes,

> "Children develop language through their experiences. They develop, clarify, and generalize meanings of words by learning the words as symbols of experienced concepts, using the words, and having the people around them react to their word use. (Steele, 1999: 39)

This need to react to students' word use creates problems for teachers (Watson & Mason, 2007) not least because inducting students into an appropriately understood and operational mathematical vocabulary is typically a consequence of a guided interplay between formal and informal language, Leung (2005).

Mathematical reasoning

The development of students' mathematical reasoning is key objective of mathematics education (Hill et al, 2008). However, traditional teaching typically fails to encourage long term gains due to emphases on superficial memorisation strategies rather than the mathematical properties under scrutiny (Lithner, 2000). Indeed, a didactical emphasis on worked examples is inferior to the encouragement of metacognitive training in facilitating students' mathematical reasoning. (Mevarech & Kramarski, 2003). Such matters are strongly linked to notions of teachers' mathematical knowledge for teaching, not least because mathematical reasoning is a much higher order activity than the conceptual and procedural knowledge dominant in most classrooms (Rowland & Ruthven, 2011).

Rich mathematical tasks

Mathematical tasks play a key role in facilitating understanding and discussion (Stein et al, 2014). The more 'complex', 'worthwhile' and 'intellectually-challenging the task, the more likely students are to acquire not only higher order knowledge and skills but also positive dispositions towards the subject (Silver et al., 2013). Teachers' use of rich tasks is typically construed as reflecting high expectations for student learning (Kazzemi & Franke, 2004).

Methodology & Methods

Case study allows us to explore in-depth how and why teachers teach in the ways they do (Silver, 2013). To this end, a multiple exploratory case study (Stake, 2002) was undertaken to examine six elementary teachers' perspectives on, and justifications for, the mathematics they expect their children to learn. Each teacher, who had specialised in mathematics during training, was well-qualified, considered locally to be effective and, importantly, an ambassador for the subject. This purpose sampling (Denzin & Lincoln, 2011) was intended to avoid the dichotomisations typically found when generalists are compared with specialists, particularly from the perspective of confidence (Goulding et al. 2002; Peker & Erekin, 2011).

Three approaches to data collection were employed to optimise the likelihood of unravelling the relationship between espoused belief and enacted practice. Initial interviews explored teachers' perspectives on the nature of mathematics and its teaching; video-recordings of random lessons, typically four per teacher, yielded evidence of patterns of practice and highlighted teachers' mathematical emphases; stimulated recall interviews (SRI) conducted shortly after each lesson elicited teacher's espoused intentions and justifications for their actions.

As with most case study investigations, much data was collected and, as is explained below, existing theoretical and analytical frameworks proved inadequate for meaningful interpretation. For example, a comprehensive framework for analysing teachers' didactical practices and inferable learning outcomes, used in an earlier comparative video study (Andrews, 2007), was unable to capture the complexity of the belief-practice relationship. Other frameworks, for example, Askew et al.'s (1997) categorisation of teacher types or Kipatrick et al.'s (2002) strands of mathematical proficiency, while able to support elements of the analysis, proved too lacking in specificity to be useful, even when employed in combination, highlighting the elusive and unpredictable nature of the belief-practice relationship (Skott, 2004).

These difficulties led me to adopt the constant comparison analytical approach of grounded theory (Strauss & Corbin, 1998), which is commonly used in case study (Yin, 2009), as it facilitates the thick description expected of case study analyses of complex educational settings (Merriam, 1998). In brief, constant comparison in this context entailed a repeated reading of the data from the first case to identify categories of belief and practice. As each was identified, the case data were reread to see if had been missed earlier. On completion of this first pass a second case was read for evidence of both the earlier categories and new ones. As each new category was identified, all previous case material was scrutinised again. Categorical definitions constantly refined as incidents were compared and contrasted (Denzin & Lincoln, 2011). This process of continual comparison and refinement, which facilitates the integration of categories into a coherent explanatory model (Taylor & Bogdan, 1998), led to the identification of the five categories introduced above.

Results & Analysis

In the following I show how these five categories played out in the beliefs and practices of the six case study teachers, given here the pseudonyms of Caz, Ellie, Fiona, Gary Louise and Sarah. In their various interviews, all six teachers made strong reference to each of the five categories although, as I show, the manifestation of those beliefs varied considerably.

The role of prior knowledge

Although all teachers were seen to emphasise *prior mathematical knowledge* at the beginning of each lesson, there were differences in their justifications for so doing. For Sarah, Fiona and Gary, the first step of every lesson was to bring to mind what the children had been learning previously. Where children failed to respond to direct questions they reminded them about activities they had undertaken together, e.g. Gary said '*remember when we had that polling booth in the classroom for the American elections*?' All three asked closed and tightly focused questions expecting a single correct answer. Interviews revealed that all three saw this as a linear 'stepped-process' within their lesson structure.

In contrast Caz, Louise and Ellie gave their children time to think and talk to partners about what they remembered or what they knew about the question asked. Moreover, this giving of time occurred, whenever an issue or idea appropriate for discussion arose. All three offered precise reasons related to giving children time to think mathematically. Caz based her understanding of child psychology training in how both children's understanding and mathematical concepts are built upon previous material.

Connections

Caz and Louise seemed to fit the description of a connectionist teacher (Askew et al.1997) well. They made explicit connections between different elements or concepts of mathematics. E.g. Caz was observed to hold a marked (counting) stick horizontally to model a number line before turning it through 90° and describing it as a scale. She believed that such representations *help children read scales...like... on a thermometer... particularly when the scale on the 'Y' axis does not represent one* unit. She was aware too, of avoiding colluding in the construction of children's misconceptions. She commented that a *common mistake children make is assuming each line up the y-axis is one, so I do not always count in ones on the counting stick.* In such actions Caz responded to both her perception of the children's needs and her ambition to *take them a little further on and make that connection.* Louise too would make explicit connections, often drawing illustrations on the board to model her explanations.

Ellie rarely made explicit connections, although she provided opportunities for children to make them for themselves. On one occasion a boy described a quadrilateral as a '*truncated triangle'*. Ellie later explained that this particular boy had been exploring solids the week before and, having spotted a truncated cone, wanted to know what it was called. Ellie encouraged children to develop both enthusiasm and a sense of enquiry. Interestingly, Ellie offers an alternative view on Askew et al.'s (1997) connectionist teachers, as she did not prompt explicit connections but did so implicitly.

Fiona made no explicit connections between areas of mathematics, but exploited concrete materials to illustrate concepts; for example plastic linking cubes were used to illustrate the partitioning of two digit numbers. However, observations highlighted some confused children as her vocabulary of *big ones* (tens) *and little ones* conflicted with the place value cards (20 and 5 for 25), she had used earlier. During interview she stated that, for her, it was not an issue, having 'told' her children how the concrete materials were connected to the concept of place value, so she would just repeat this learning again in the term. This particular event seemed indicative of a lack of awareness of the impact of her actions on her children's understanding of place value. Interestingly, Fiona consistently emphasised her role of *telling* of concepts to children.

Throughout their lessons, both Sarah and Gary made explicit connections between activities, rather than the mathematical concepts embedded in them. For example, Gary spoke about the ways in which his class collected data during a mock poll related to the US Presidential vote but not about the data themselves. Sarah used many manipulatives e.g. use of coloured cards, making explicit connections between the use of the cards rather than the concept being taught. That is, in the mind of these teachers the connection was made to the mathematics, but explicitly the connection was made to the activity or context and not the mathematics (Van Zoest & Bohl, 2005).

Vocabulary

Louise, Ellie and Caz frequently used games to encourage children's use of new and unfamiliar mathematical vocabulary. Sarah, Gary and Fiona provided lists of words, expecting children to use them in response to closed questions and would frequently read out these words during their lessons. Such practices, it seems, highlight von Glaserfeld's (1991) distinction between *teaching* children and *training* children. He adds that teachers have a better chance to modify children's' conceptual structures if a model informs interventions, such as the opportunity to use new vocabulary naturally, such as in a game.

Mathematical Reasoning

Expectations that children would think mathematically and engage in *reasoning* were consistently observed throughout Louise, Ellie and Caz's lessons. Caz encouraged children to 'argue' with her if they were confused or disagreed with anything she said. Often evoking such argumentation purposefully. During one fractions-related episode she had failed to notice an ambiguity in her presentation of a problem. It went, *if there were two cakes and six people*, *how many pieces would each person have?* One child, William, said that they will have one sixth of one bar and one sixth of the other before concluding, that each person would have two sixths altogether. Another child, Holly, pointed out that it should be two-twelfths not two sixths. This created a lengthy discussion amongst the class and although some children had accepted Holly's explanation, Caz explained later that she was eager to discuss the cognitive conflict to demonstrate how fractions can be confusing. Unpacking the problem as it arose (critical incident) and working with the children in reconciling the two perspectives was very much Caz's reflection of the incident.

Fiona did not emphasise reasoning or thinking in any discussion we had. Her focus was on her didactic approaches to her lessons rather than the mathematical learning. This was an interesting observation as her explanations, like Sarah's, were to focus on the '*how to do...*' something rather than what it is connected to, or *why* they were learning this element of mathematics, other than it was an *assessment target*. In similar vein, Gary's focus was the acquisition of knowledge necessary for passing statutory tests the following year, which, were manifested in his frequent use of mathematical memorising exercises of facts. Indeed, Gary was adamant with respect to the importance of such practices in mathematical learning, often emphasising the role of tricks, practising and the memorising of facts, just as when he was a child at school.

Rich Mathematical Tasks

Louise, Caz and Ellie demonstrated an understanding of where the concepts they were teaching would lead and chose specific tasks as a consequence. These were not always planned for, and often a consequence of critical incidents (Cooney, 1987). For the remaining three teachers, activities were drawn from a series of photocopiable teacher resource books, or a snapshot of different concepts jumbled into one lesson, with little emphasis on related learning, concepts or mathematical intent. For example, although both Caz and Gary discussed real-life tasks during interview, the ways in which these were presented differed starkly. Caz tended to draw on her *children's* real life experiences to illustrate or reinforce a concept. E.g. she emphasised the irregularities in people's abilities to reference the passing of time by asking children to identify aspects of their lives related to the notion of five minutes. This led to her commenting, in interview, that *for Latia it was about mathematics in dancing, for Josh it was about swimming the length of a pool and for Tom it was about scoring a goal*. Caz used such serendipitous moments and real-life experiences to encourage children to think mathematically. In contrast, Gary also referred to real-life situations but *directed* his children to specific events like the American presidential elections they had previously modelled in class. His justifications were similar to those of Caz, drawing on the importance of real-life situations, but the difference was that Gary provided both content and context. Thus, he made all the thinking, and connections.

Discussion

All six teachers were aware of the relevance of the five components to mathematical learning. However, the classroom manifestations of similarly espoused beliefs tended to dichotomise. One the one hand was a group, Caz, Ellie and Louise, whose beliefs and practices were commensurate in their explicit focus on children's learning of mathematics. On the other hand was a group, Sarah, Fiona and Gary, whose beliefs, while clearly located in the same vocabulary as the first group, focused on issues independently of the mathematics they may or may not have taught; a group whose practices subordinated mathematics to activities. For example, while all focused on the activation of prior knowledge at the start of their lessons, these three believed and behaved as though it were a ritual element of all lessons - *talk about what we did last time and then move on*.

One group talked about the activities they employed, independently of the mathematic they taught, while the latter focused explicitly on mathematical ideas. That is, the one group referred to the enjoyment of learning, irrespective of mathematics, while the other referred to the challenge that is mathematics (Moyer, 2001). In other words, one group seemed focused on *training* children, while the other on *teaching* children (Von Glasersfeld, 1991). This notion of training was clearly reflected in Gary's encouraging his children to use a vocabulary list to answer his questions. The mathematics also appeared to get lost in Sarah and Fiona's class, as they both focussed on the '*how to do...*' something as the means of addressing their next *assessment target*. In sum, Gary, Sarah and Fiona's practice presented very few opportunities for children to engage meaningfully in mathematical reasoning. They believed they did, but observations indicated that this was subordinated to enjoyment. For them, mathematics was about how they taught; it was not about the cognitive engagement of children in mathematics.

Conclusions

When I started this study, such differences in experienced specialist teachers' mathematical objectives were unexpected, as all were well qualified, and acknowledged locally as effective. Yet, only three of the six teachers provided consistent opportunities for children to think and explore collectively while making connections with and for each other individually. Explicit collective construction of new mathematical knowledge was privileged, by means of rich tasks, individual enquiry, argumentation and justification supported by an expectation of appropriate mathematical language. Their three colleagues consistently attended to how rather than what they taught - their attention was on activities, manipulatives, incremental steps and amount of mathematics covered. For group the mathematics was transparent and for the other it was opaque, warranting the question, *Where is the mathematics?*

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