

Foundational number sense: A framework for analysing early number-related teaching

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In this paper, by means of an extensive review of the literature, we discuss the development of a framework for analysing the opportunities, both implicit and explicit, that grade one students receive for acquiring those number-related understandings necessary for later mathematical achievement but which do not occur without formal instruction. The framework, which we have called foundational number sense, currently comprises seven interrelated components, although additional components may exist. Each component, as warranted by earlier research, is known to underpin later mathematical understanding and, when viewed collectively, addresses a definitional gap in the literature.

Introduction

In an earlier paper (Back, et al., 2013) we introduced and evaluated the efficacy of a framework for identifying the learning opportunities, both implicit and explicit, pupils receive for acquiring foundational number sense. Derived from the literature, this tentatively proposed framework was not only able to identify opportunities linked to those basic number competences thought to be necessary for successful mathematical learning but was sensitive to culturally different teaching traditions. In this theoretical paper we focus offer an extended account of the derivation of this framework.

Described as a “traditional emphasis in early childhood classrooms” (Casey et al. 2004: 169), children’s acquisition of number sense is acknowledged as a key objective of many early years’ mathematics curricula (Howell & Kemp 2005; Yang & Li, 2008). It is not only a predictor of later mathematical success, both in the short (Aubrey & Godfrey, 2003; Aunio & Niemivirta, 2010) and the longer term (Aubrey et al., 2006; Aunola et al., 2004), but brings numbers to life and enhances our relationships with them (Robinson et al., 2002).

While number sense “is considered internationally to be an important ingredient in mathematics teaching and learning” (Yang & Li 2008, p.443), there is evidence that it is gender-determined, with boys typically outperforming girls on standard measures at ages five and six, a difference compounded by parental education levels – the more highly educated the parents the better boys perform (Melhuish et al., 2008; Penner & Paret, 2008). On the other hand, evidence shows that number sense is gender-independent, although there are cultural

differences, with, for example, Chinese children exhibiting higher levels of counting skills than Finnish students, irrespective of age (Aunio et al., 2006). Of course, such research inconsistency may be a consequence of differences in the measures used. Where research seems to be consistent is in the influence of various components of the socio-economic status of a child's family (Melhuish et al., 2008; Starkey et al., 2004). Indeed, without appropriate intervention, which research shows can be effective (Van Luit & Schopman, 2000), children who start school with limited number sense are likely to remain low achievers throughout their schooling (Aubrey et al., 2006).

What is number sense?

While it is important to understand the consequences of poorly or inappropriately developed number sense, it is equally important that we have a clear understanding of what is meant by the term. In this respect, the National Council for Teachers of Mathematics has written, somewhat vaguely, that it is "an intuition about numbers that is drawn from all varied meanings of number" (NCTM, 1989, p.39). Others have been equally imprecise, as with, for example, the definitions of Case (1998), Griffin (2004) and McIntosh, Reys and Reys (1992). Indeed, despite its apparent importance, "no two researchers have defined number sense in precisely the same fashion" (Gersten et al., 2005, p.296), which would clearly make the development of classroom interventions problematic.

Interestingly, Berch (2005) has argued that such ambiguities are compounded by the fact that psychologists and mathematics educators work to different definitions, a dichotomisation exacerbated by our interpretation of the former literature, whereby researchers differ according to whether they work in the fields of general cognition or learning disabilities. Irrespective of such research traditions, our reading of the literature reveals two distinct perspectives on number sense. The first, which we have labelled foundational number sense, concerns the number-related understandings children develop during the first years of formal instruction. The second, which we have labelled applied number sense and which incorporates the first, concerns the number-related understanding necessary for people to function effectively in society. Students with a well-developed applied number sense

will look at a problem holistically before confronting details, look for relationships among numbers and operations and will consider the context in which a question is posed; choose or invent a method that takes advantage of his or her own understanding of the relationships between numbers or between numbers and operations and will seek the most efficient representation for the given task; use benchmarks to judge number magnitude; and recognize unreasonable results for calculations in the normal process of reflecting on answers (Reys, 1994, p. 115).

Such behaviours underpin what is known as adaptive expertise (Hatano & Inagaki, 1986). Adaptive experts have the flexible understanding, structured by the principles of the discipline (Pandy et al., 2004), necessary for solving non-routine problems. They not only modify or invent procedures (Hatano & Inagaki, 1986) but self-regulate their learning as a dynamic rather than static entity (Martin et al., 2005; Verschaffel et al., 2009). Adaptive expertise requires an appropriately deep conceptual knowledge to give meaning to the procedures taught (Hatano, 1982). In this paper, while mindful of the form and function of applied number sense, we focus on foundational number sense as the basis for much later teaching.

Defining foundational number sense

Foundational number sense is to the development of mathematical competence what phonic awareness is to reading (Gersten & Chard, 1999), in that early deficits tend to lead to later difficulties (Jordan et al., 2007; Mazzocco & Thompson, 2005). Significantly, it has been shown to be a more robust predictor of later mathematical success than almost any other factor (Aunio & Niemivirta, 2010; Byrnes & Waski, 2009).

So, what are the characteristics of foundational number sense? Broadly speaking it is the ability to operate flexibly with number and quantity (Aunio et al., 2006; Clarke & Shinn, 2004; Gersten & Chard 1999) and can be expressed as attributes like “awareness, intuition, recognition, knowledge, skill, ability, desire, feel, expectation, process, conceptual structure, or mental number line” (Berch 2005, p. 333). In particular, there is evidence that elements of number sense are innate to all humans and independent of instruction. This *preverbal* (Ivrendi, 2011; Lipton & Spelke, 2005) component comprises an understanding of small numbers in ways that allow for comparison. For example, Feigenson et al. (2004, p. 307) found that “6-month-olds can discriminate numerosities with a 1:2 but not a 2:3 ratio, whereas 10-month-old infants also succeed with the latter”, adding that “adults can discriminate ratios as small as 7:8”. Thus, as Lipton and Spelke (2005, p.978) observe, “numerical discrimination becomes more precise during infancy... but remains less precise than that of adults”. This preverbal number sense is independent of formal instruction, developing in the early years as an innate consequence of human, and other species’, evolution (Dehaene, 2001; Feigenson et al., 2004).

Later, but still before the start of formal school, and frequently dependent on individual family circumstances, children as young as three can undertake, without instruction, addition- and subtraction-related tasks with confidence and accuracy (Zur & Gelman, 2004). By age four children have normally acquired initial counting skills and an awareness of quantity that allows them to respond to questions about ‘more’ and ‘less’ (Aunio et al., 2006). At about the time they

start school children typically acquire a sense of a mental number line, including “knowledge of number words, the ability to point to objects when counting, and knowledge of cardinal set values” Aunio et al., 2006, p.484). However, although there are indicators of a typical developmental trajectory, the properties of foundational number sense remains vague. In this paper our interest lies not with the preverbal number sense described above but the number sense that requires instruction (Ivrendi, 2011). Our reading of the literature leads us to conclude that there are seven, although there may be more, interrelated components, which are:

1. Foundational number sense involves number recognition, its vocabulary and meaning (Malofeeva et al., 2004). It entails being able to both identify a particular number symbol from a collection of number symbols and name a number when shown that symbol, typically up to twenty (Clarke and Shinn, 2004; Van de Rijt et al., 1999; Gurganus, 2004; Yang & Li, 2008). Significantly, children who experience difficulty with number recognition tend to experience later mathematical problems generally (Lemke & Foegen, 2009) and particularly with subitising, a key process of early arithmetic (Koontz & Berch, 1996; Stock et al. 2010).

2. Foundational number sense incorporates systematic counting (Berch, 2005; Clarke & Shinn, 2004; Van de Rijt et al., 1999; Griffin, 2004). It includes notions of ordinality and cardinality (Ivrendi, 2011; Jordan et al., 2006; Jordan & Levine 2009; LeFevre et al., 2006; Malofeeva et al., 2004) and, in particular, the learning of the order of the various number names (Van Luit & Schopman, 2000). Typically, children can count to twenty and back or count upwards and backwards from an arbitrary starting point (Lipton & Spelke, 2005), knowing that each number occupies a fixed position in the sequence of all numbers (Griffin et al., 2004). Significantly, unsophisticated counters may have later difficulties developing adaptive solution strategies for the various arithmetical problems they encounter (Gersten et al., 2005; Stock et al., 2010).

3. Foundational number sense includes an awareness of the relationship between number and quantity. In particular, children understand not only the one-to-one correspondence between a number’s name and the quantity it represents but also that the last number in a count represents the total number of objects (Malofeeva et al., 2004; Van Luit & Schopman, 2000). It incorporates quantity discrimination, whereby children recognise that eight represents a quantity that is bigger than six but smaller than ten (Gurganus, 2004; Lemke & Foegen, 2009). Importantly, quantity discrimination is a predictor of a child’s later mathematics achievement (Kroesbergen et al., 2009).

4. At the foundational level, number sense includes awareness of magnitude and of comparisons between different magnitudes (Case, 1998; Clarke & Shinn, 2004; Griffin, 2004; Gurganus, 2004; Ivrendi, 2011; Jordan et al., 2006; Jordan & Levine 2009; Yang & Li, 2008) and deploys language like ‘bigger than’ or

‘smaller than’ (Gersten et al., 2005). In particular, children who are magnitude aware have moved beyond counting as “a memorized list and a mechanical routine, without attaching any sense of numerical magnitudes to the words” (Lipton & Spelke, 2005, p. 979). Moreover, being magnitude aware supports the development of other mathematical skills, particularly subitising (Aunio & Niemivirta, 2010; Nan et al., 2006; Stock et al., 2010).

5. A foundational number sense aware child is able to estimate, whether it be the size of a set (Berch, 2005; Gersten et al., 2005; Jordan et al., 2006, 2007; Malofeeva et al., 2004; Van de Rijdt et al., 1999;) or an object (Ivrendi, 2011). Estimation involves moving between representations - sometimes the same, sometimes different - of number, for example, placing a number on an empty number (Booth and Siegler, 2006). However, the skills of estimation are dependent on the skills of a child to count (Lipton and Spelke, 2005).

6. A foundational number sense aware child will be able to perform simple arithmetical operations (Ivrendi, 2011; Jordan & Levine 2009; Yang & Li, 2008); skills which underpin later arithmetical and mathematical fluency (Berch, 2005; Dehaene, 2001; Jordan et al., 2007). Indeed, simple arithmetical competence, which Jordan and Levine (2009) describe as the transformation of small sets through addition and subtraction, has been found to be, at grade one, a stronger predictor of later mathematical success than measures of general intelligence (Geary et al., 2009).

7. Foundational number sense includes awareness of number patterns and, in particular, being able to identify a missing number (Berch, 2005; Case, 1998; Clarke & Shinn, 2004; Gersten et al., 2005; Gray & Tall, 1994; Jordan et al., 2006, 2007). Such skills reinforce the skills of counting and facilitate later arithmetical operations (Van Luit & Schopman 2000). Importantly, failure to identify a missing number in a sequence is a strong indicator of later mathematical difficulties (Chard et al., 2005; Clarke & Shinn, 2004; Gersten et al., 2005; Lemke & Foegen, 2009).

The development of foundational number sense

How a child comes to acquire number sense is complex and, in some ways, circular. For example, Malofeeva et al. (2004) argue that counting and knowledge of numerical symbols underpin the development of number sense concepts, and yet these are themselves components of number sense. That being said, “there is general agreement that number sense is a construct that children acquire or attain, rather than simply possess” (Robinson et al., 2002, p. 85); therefore, it would seem sensible to assume it does not occur by chance but “requires a conscious, coordinated effort to build connections and meaning on the part of the teacher” (Reys, 1994, p. 115). In general, this means that teachers should, *inter alia*, encourage children to work with concrete materials and

familiar ideas; discuss and share solutions and discoveries; compose and recompose different representations of numbers; explore number patterns and number relationships; create alternative methods of calculation and estimation (Griffin, 2004; Tsao & Lin, 2012). Such invocations resonate well with the characteristics described above. Moreover, in the light of evidence that young children from high-socioeconomic status (SES) backgrounds are five times as successful as children from low SES backgrounds on tasks like, ‘which number is bigger, 5 or 4?’ (Griffin, Case, & Siegler, 1994), the case for intervention seems clear, particularly as “aspects of number sense development may be linked to the amount of informal instruction that students receive at home on number concepts” (Gersten et al., 2005, p. 297). Importantly, “number sense develops gradually over time as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms” (Sood & Jitendra, 2007, p. 146).

Issues in foundational number sense

The consequences of poor number sense are significant. For example, basic counting and enumerations skills have been found to be predictive of later arithmetical competence in England, Finland, Flanders, USA, Canada and Taiwan respectively (Aubrey & Godfrey, 2003; Aunola et al., 2004; Desoete et al., 2009; Jordan et al., 2007; LeFevre et al., 2006; Yang & Li, 2008). In other words, there is an international consensus that poorly developed number sense underlies later mathematical failures (Jordan et al., 2009; Gersten et al., 2005; Malofeeva et al., 2004). There is also evidence that teachers may have contributed to their children’s difficulties. For example, while children’s counting competence increases with age, their tolerance of unusual counts, that is counting procedures that do not progress naturally from left to right, diminish, leading to the conclusion that the ways in which they are typically taught may be counter-productive in terms of establishing an awareness that the order of a count is an inessential element of the process (LeFevre et al., 2006). Moreover, as Wagner & Davis (2010, p. 40) note, an emphasis on an understanding of quantity in the early years of schooling is so eclipsed by later expectations of computational competence that “students become numb to the meaning of the numeric symbols they learn to manipulate”.

Conclusions

In this essentially theoretical paper we have explicated a framework for analysing classroom activity in the early years of mathematics teaching. This is a novel undertaking as earlier evaluative studies have focused on children’s competence and not the opportunities teachers provide for them. Importantly, the framework has been piloted on two lessons, one from Hungary and one from

England (Back et al, 2013), and found effective in identifying the number sense opportunities presented by the teachers concerned. However, the reader is reminded that the validity of each of these seven components can be found in the literature from which it derives. That is, the importance of each component in the subsequent development of children's mathematical competence has been warranted by the research that informed its inclusion here. What has yet to be done, beyond the initial trial discussed above, is an assessment of the framework's efficacy as a tool for analysing the opportunities teachers offer their students, to be followed by an analysis of different ways in which they do this.

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