Conceptualizing a local instruction theory in design research: report from a symposium

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This is a report on the discussions (and post-reflections) of the MADIF10 symposium "Conceptualizing a local instructional theory in design research". Linking the discussion to Koeno Gravemeijer's keynote at MADIF9 and drawing on different ongoing research projects, the aim of the symposium was to discuss [examples of] the operationalization of design principles in order to deepen the understanding of some theoretical concepts in design research. The contribution of the symposium is the interpretation of how local instruction theory interrelates with other concepts in design research, for instance, the hypothetical learning trajectory. The role of the concepts as both design tools and as outcomes was presented and discussed.

Nowadays, *Educational design research* (i.e. "design research") is fairly often used as a methodological stance in doctoral and licentiate projects conducted in mathematics education research in Sweden. The reason for this may, on the one hand, depend on the typical professional background of the doctoral students, that is, in-service teaching. Hence, the research questions draw on a practice-driven realisation of the need to conduct studies aiming at improving teaching and learning in mathematics. On the other hand, research in mathematics education has also been criticized in the field for not producing useful instruction for teachers on how to design their teaching to improve learning (van den Akker, Keursten & Plomp, 1992; Reeves, 2006; Plomp, 2013), and educational design research is suggested as one way to develop such instruction. To this end, several research groups in Sweden apply design research methodology.

Reform mathematical pedagogy stresses inquiry and problematizing. This implies a change of perspective: from ready-made expert knowledge as a starting point for design to imagining students' elaborating and refining their current way of knowing (e.g. describing the hypothetical learning trajectory?). What is needed is an instructional design that supports students in developing their reasoning towards more sophisticated mathematical reasoning (Gravemeijer,

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2004). Design research can be described as the development of a teaching intervention to solve a complex educational problem while generating advanced theoretical knowledge about these interventions and processes (Plomp, 2013). This is in contrast to comparing studies in certain contexts and coming to the conclusion that method A is better than method B (Reeves, 2006). Design research instead allows researchers to generate and test theories in classroom contexts.

There are two strands of studies in design research: validation studies and development studies (Plomp, 2013). Plomp describes development studies as studies with the purpose to develop research-based solutions to an educational problem, and, hence advancing the scholarly knowledge about the characteristics of the designed and evaluated interventions. Validation studies, on the other hand, aim to develop or validate theories on educational interventions. The studies presented at the symposium contain both these strands in their problem formulation. The features put forward in defining design research is the way it addresses practice-driven problems, and, moreover, the development of instructions based on local theories (Cobb, Confrey, Lehrer & Schauble, 2003).

The idea of the symposium developed through discussions among young researchers who plan to apply, or have applied, design research methodology in their studies. The discussion in the symposium drew on different projects in order to deepen the understanding of the concept local instruction theory (Gravemeijer & Cobb, 2013; Gravemeijer, 2015). We will therefore briefly describe the content of the presentations in the symposium in the next section.

The presentations

At the symposium, Mellroth presented her on-going doctoral project to develop and evaluate a professional development program for in-service teachers on how to teach and challenge mathematically highly able students in the ordinary classroom. Inclusive education and equity in student learning are important aspects to consider in the professional development program. To develop and evaluate the program design, research methodology is used and the expected results are two-folded, that is: the formulation of a research based local instruction theory for teacher learning in a professional development program, as well as a framework on which to base teachers' hypothetical learning trajectory when teaching mathematically highly able students in inclusive settings.

In planning a possible extension of a study within his doctoral project, Olsson has considered to use design research and he presented his plans in the symposium. The starting-point of his project is earlier studies (e.g. Granberg & Olsson, 2015) in which he has investigated task design aiming at engaging students in productive problem-solving and reasoning. The tasks were solved with the support of the dynamic software-program GeoGebra. Implementing the results

in regular teaching would require further knowledge concerning, for instance, curricula, social norms, mathematical norms, and teaching preferences. At the symposium, Olsson explored if a possible way to implement the results in regular teaching could be through a design research project, that is, discussing possible endpoints, starting points, and an initial local instruction theory.

Liljekvist used design research methodology in her doctoral project where she and her colleagues developed and evaluated mathematical tasks (see Liljekvist, 2014). In the symposium, she problematized the process of transforming research results into classroom settings. The results of earlier studies on a micro level in a design cycle cannot be used without careful considerations. The need for an ecologically valid local instruction theory becomes evident, as we know, for instance, that teacher agency is an important aspect of successful interventions, and the importance to move beyond inefficient linear teaching instruction. She hence raised the issue of how to engage teachers in design research.

The report now continues with the theoretical concepts presented and discussed at the symposium. Then we describe how the developmental and validation aspects of design research is negotiated and established in the studies presented, that is, the different approaches to establishing a local instruction theory, including the theoretical grounds.

Developing a local instructional theory

Design research aims at understanding more of the interrelatedness between teaching and learning in order to improve teaching. Let us therefore link the symposium at MADIF10 to the constructivist stance explained by Koeno Gravemeijer in his keynote at MADIF9 in Umeå, 2014 (Gravemeijer, 2015). He raised the important questions (for researchers as well as for teachers) of what mathematics we want the students to construct, and, consequently, how we may design teaching that promotes students' construction of such mathematical knowledge, which is a main issue in both theory-building and practice development (see e.g. Cobb et al., 2003; Gravemeijer, 2004; Liljekvist, 2014; Ruthven & Goodchild, 2008).

> [...] if we want students to reinvent mathematics by doing mathematics, teachers have to adapt to how their students reason and help them build on their own thinking. To do so they need a framework of reference to base their HLTs on. We may offer them such frameworks in the form of "local instruction theories" – and corresponding resources. A local instruction theory consists of theories about both the process of learning a specific topic and the means to support that learning. (Gravemeijer, 2015, p. 1)

In his keynote Gravemeijer pinpointed the specific kind of design research discussed here, that is, research for development of local instruction theories (Gravemeiljer, 2015). Such design research projects have three phases; 1) preparing for the experiment, 2) experimenting in the classroom, and 3) conducting retrospective analyses.

Preparing for the experiment means to formulate a local instruction theory that can be elaborated while conducting the design experiment (Gravemeijer & Cobb, 2014). The first step is to clarify endpoints formulated as mathematical learning goals, and thereafter consider a starting point. In this step, the researchers need to take into consideration the results of earlier instructions and problematize them beyond the purely mathematical goal. For instance, what is the need for changing classrooms norms and expectations of mathematical teaching? The local instruction theory hence includes conjectures of possible learning processes and possible means of supporting these learning processes.

In Olsson's study, this preparation phase is based on earlier research investigating mathematical reasoning and reasoning supported by dynamic software (see e.g. Granberg & Olsson, 2015, Lithner, 2008). For Mellroth, however, the preparation is slightly different as she needs to consider two-folded endpoints, that is, both on the teacher professional development level and the student level in classrooms. Gravemeijer & Cobb (2013) points out that theories emerging from design research are developed at various levels. At the level of the instructional activities *micro theories* are developed, *local instruction theories* are developed at the level of the teaching sequence (in the professional development course or in the classroom), and *domain specific theory* is developed as the umbrella of the two others. In Mellroth's study a micro theory can describe a specific activity, for example, teachers' role-play when solving a task, and in Olsson's study it may be a specific task aiming to be solved with the support of a dynamic software.

When endpoints, starting points, and the preliminary local instruction theory are formulated, the experimenting in the classroom or in the professional development course can start. One characteristic of a design experiment is the cyclic process of designing, testing and re-designing instructions. The researcher(s) conduct a thought experiment by envisioning how the proposed instructional activities might be realized, then analyse the actual process, and, finally consider refining specific aspects of the design before the next thought experiment, and so on. The retrospective analysis aims at contributing to the development of local instructions and more encompassing theories (Cobb, 2003).

Drawing on Simon (1995), Gravemeijer shows the need for researchers, as well as teachers, to have an idea of a possible path through instruction activities, that is, a *hypothetical learning trajectory* that supports the envisioning of students' thinking and learning (see e.g. Gravemeijer, 2015; Simon & Tzur, 2004). Based on to what extent the actual learning trajectory corresponds to

the hypothetical one, new instructions and revised learning trajectories can be designed. Gravemeijer (2004) points out that neither teachers, nor researchers can rely on fixed teaching sequences, since a teacher continuously has to adapt to the actual thinking and learning of her students.

Instead, a teacher can be offered a framework with exemplary instructional activities as a source of inspiration, which was the aim of all the studies presented at the symposium. Creating hypothetical learning trajectories should be supported by local instruction theories as rationale for the description of envisioned learning. Local instruction theories will bridge between more general theories and the practice of how to create hypothetical learning trajectories. This is evident in Mellroth's study as she makes conjectures from hypothetical learning trajectories in the professional development course, as well as develop hypothetical learning trajectories *with* the teachers on their teaching in their classrooms.

Even if the design of local instructions is emphasized, there is a need to understand and conceptualize local preferences. Focus on local theories often means that design researchers develop frameworks that explain local circumstances where grand theories, such as Piaget´s development theories or Vygotskian theories of learning, are too general for clarifying local phenomena (Cobb et al., 2003; Liljekvist, 2014). The development of such intermediate frameworks may lead to proposals of alternative conceptions of the existing understanding of a domain, and this must be specified in terms of endpoints and possible trajectories for learning (Ruthven, Laborde, Leach & Tiberghien, 2009). Ruthven et al. suggest that focus on design tools provides an effective mechanism for developing teaching activities. In the next section we will hence give an overview of the operationalization of design principles presented at the symposium.

Operationalization of design principles

To develop instructions and design principles, there is a need of insights into how and what the students learn during a teaching sequence, that is, their learning trajectories. The local instruction theory constitutes a framework for developing hypothetical learning trajectories describing possible learning through activities.

Even if there is a lack of useful instructions on how to design teaching, there are at least some design principles that could be used as a basis for further research. Gravemeijer described in his Keynote at MADIF9 how he and his colleagues used Freudenthal's Theory on realistic mathematics education (RME) as a base for design (Gravemeijer, 2015). Ruthven et al. (2009) suggest that Brousseau´s Theory of didactical situations (TDS) will provide a tool for designing teaching sequences. TDS has been used by Liljekvist (2014) as a

starting-point for task design principles, and Olsson is also planning his design research based on this theory. TDS refers to problem-solving tasks and teaching environment designed to put students in learning situations where they will construct new knowledge through adapting existing knowledge. This includes a process of devolution, that is, it is the student's responsibility to solve the problem although supported by relevant feedback on her actions (see e.g. Brousseau, 1997).

In Mellroth's study, the interventions are based on developed and researched practical examples from Australia (UNSW, 2014), for example, and Germany (Benölken, 2015). The design principles are hence based on knowledge from literature and practice where the components are linked to each other in a consistent way, which ensures validity regarding how to teach and challenges mathematically highly able students in the ordinary classroom. Inclusive education and equity in student learning are important principles for designing the teachers' hypothetical learning trajectories. In this study, the practicability was ensured because the participating teachers choose themselves to join the professional development program and were supported by their principals. Another way to ensure the feasibility connection is its connection to research-based practice used in other countries aiming to support mathematically highly able students (Fuchs & Käpnick, 2009; Nolte, 2012; UCONN, 2015).

> [C]oupling the creation of scholarly knowledge within the practice of researching with the creation of craft knowledge within the practice of teaching makes possible approaches to collaboration between researchers and teachers which can contribute to building a more systematic knowledge-base for teaching. (Ruthven & Goodchild, 2008, p. 584)

In design research, one can choose to involve teachers' craft knowledge more or less in a study. However, the design research methodology invites researchers to engage in how to link the practice of research with teachers' practice of teaching, thus contributing to a teaching knowledge-base with resulting local instruction theories. In the operationalization of the design principles in the studies discussed in the symposium, teachers' craft knowledge is considered in different ways. In Mellroth's study the teacher is both a "student" and a partner in constructing the local instruction theory. In Olsson's and Liljekvist's forthcoming studies, the teacher is the carrier of craft knowledge within the practice of teaching to calibrate the hypothetical learning trajectories.

Summary

The discussion in the symposium was drawn on different design research projects in order to deepen our understanding of the concept of *local instructional theory*, and, consequently, its interrelatedness to a *hypothetical learning trajectory.*

The idea of this symposium was developed through discussions among young researchers who plan to apply, or have applied, design research methodology in their studies. One argument for using design research as methodology in mathematics education research is to increase the relevance of research for practice and for educational policy (van den Akker, Gravemeijer, McKenney & Nieveen, 2006). In design research, there is scholarly knowledge in both theory and practice, because theoretical insights and practical solutions in real world context are developed simultaneously (McKenney & Reeves, 2013).

The symposium focused on one key concept connected to design research in mathematics education: the local instruction theory (Gravemeijer & Cobb, 2013, Gravemeijer, 2015). The aim of the symposium was to share our understanding of the concept, discuss our arguments for the conceptualization, and how to operationalize it in each study.

The research studies that formed the basis of this symposium are in different work-in-progress stages: from being in the planning stage to already completed design cycles. The symposium was hence an opportunity to look in-depth into parts of the design research methodology through the different studies.

The contribution of the symposium is the interpretation of how local instruction theory interrelates with other concepts in design research, for instance, the hypothetical learning trajectory. The role of the concepts as both design tools and as outcomes was presented and discussed.

In light of the discussion on the MADIF9 keynote by Koeno Gravemeier (2015), it was a suitable topic for a MADIF10 symposium. We hope that the fruitful and critical discussions in the symposium will continue in the MADIF community in order to deepen our understanding of design research methodology, and to further the discussion of design research studies to develop local instruction theories in the Swedish mathematics education context.

Acknowledgement

Thanks to all the participants in the MADIF10 symposium who contributed to the discussion.

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