Orchestration of mathematical discussions drawing on students' computer-based work

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Research points out the importance of following up students' work on computer-based tasks with whole-class discussions in which students play a central role. However, at the same time, research highlights the challenge for teachers in orchestrating such follow-up discussions. This paper examines whether an established model developed as guidance for teachers to orchestrate mathematical whole-class discussions (Stein, Engle, Smith & Hughes, 2008) could be useful in this educational setting. Students' written responses to two different tasks are the main data used to examine the model. The results indicate that the model has great potential to guide these follow-up discussions.

Despite the importance of following up students' mathematical work with computers in whole-class discussions (Bartolini Bussi & Mariotti, 2008; Kieran, Tanguay & Solares, 2012), few researchers have studied teachers' implementation of such follow-up discussions. Kieran, Tanguay, and Solares (2012) demonstrate the challenge for teachers to orchestrate follow-up discussions which take students' computer-based work into account. However, there is a large body of research focusing on teachers' orchestration of productive whole-class discussions within non-technology environments (Franke, Kazemi & Battey, 2007; Stein et al., 2008). For example, researchers have investigated ways of enhancing student engagement in whole-class discussions, i.e. make them more student-centred. Based on an extensive research review, Stein et al. (2008) propose a model consisting of five practices to support teachers in the orchestration of whole-class discussions using students' problem solving strategies as point of departure. The model aims to go beyond "show and tell" by supporting teachers to align student strategies to key mathematical ideas. In Sweden, this model has been taken up in a significant way through the government CPD initiative for teachers of mathematics, and it has been used as a conceptual framework in studies focusing on teachers' orchestration of productive whole-class discussions (Larsson, 2015).

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This paper draws on a design research project, conducted by the authors of this paper in collaboration with four upper-secondary school teachers. The project aimed to investigate the implementation of short teaching units consisting of a researcher-designed computer-based lesson and a follow-up whole-class lesson devised by the class teacher. During the first lesson in each unit, students worked in pairs on task sequences designed for a piece of dynamic mathema-tics software, in this case *GeoGebra*. Besides developing students' proficiency with relevant GeoGebra tools, the tasks were designed to foster student reasoning concerning functions, particularly exponential functions (Brunström & Fahlgren, 2015). Findings concerning the teachers' orchestration of the follow-up lessons indicate that they typically were too teacher-centred to provide strong support for the development of student reasoning (Fahlgren, 2015). These findings encouraged us to consider how to support teachers in their planning and implementation of productive follow-up lessons.

Accordingly, the aim of this paper is to examine whether the model suggested by Stein et al. (2008) could be useful as guidance for teachers in orchestrating whole-class student-centered discussions based on students' previous computer-based work.

Theoretical background

There is a growing interest among researchers in studying ways of creating mathematical whole-class discussions in which students play a central role (Franke et al., 2007). Often, the purpose is to follow up students' previous work in pairs or in small groups and use it as a basis for a collective mathematical discussion in order to develop students' mathematical understanding (Stein et al., 2008). However, engaging students in mathematical discussions in which each student is given an opportunity to participate actively while simultaneously making certain that the intended mathematical direction of the lesson is followed is not an easy undertaking for teachers (Franke et al., 2007; Stein et al., 2008).

To support teachers in the orchestration of whole-class discussions while using students' work as a departure, Stein et al. (2008, p. 321) propose a model of five practices as a tool. These practices are:

(1) anticipating likely student responses to cognitively demanding mathematical tasks,

(2) monitoring students' responses to the tasks during the explore phase,

(3) selecting particular students to present their mathematical responses during the discuss-and-summarize phase,

(4) purposefully sequencing the student responses that will be displayed, and

(5) helping the class make mathematical connections between different students' responses and between students' responses and the key ideas.

According to Stein et al. (2008), the teacher has to be aware of as many different solution strategies as possible to be able to anticipate various student responses to a problem. Furthermore, they argue, it is important to predict incorrect solutions as well as correct. The first practice serves as a preparation for the second one, in which the teacher observes and (preferably) makes notes about students' work to get an overview of the various solution strategies in use (Stein et al., 2008). This overview, then, serves as guidance in the subsequent practice, where the teacher should select particular students or student groups to present their responses in the class. When selecting particular responses it is important to make sure that these responses are useful to illustrate and discuss relevant mathematical ideas (Stein et al., 2008). It is also important to bring common misconceptions to the fore as a basis for discussion. The fourth practice, then, concerns sequencing the student responses so that ”teachers can maximize the chances that their mathematical goals for the discussion will be achieved” (Stein et al., 2008, p. 329). In the last practice, the role of the teacher is to orchestrate a whole-class discussion where ”mathematical ideas are surfaced, contradictions exposed, and understandings developed or consolidated” (Stein et al., 2008, p. 333).

Method

As mentioned in the Introduction, this study is part of a broader research project investigating the implementation of short teaching units consisting of a computer-based lesson and a follow-up whole-class discussion. The project was conducted together with four upper-secondary school teachers, who were responsible for the classroom implementation of three teaching units in one class each. In total 85 first year students (age 15–17) participated in the project. Central to the main project was the design of task sequences, one for each computer-based lesson, in the format of worksheets intended for course *Mathematics 1b*.

The overall aim of the task sequences was to foster student reasoning concerning exponential functions. In the first task sequence, students are introduced to a real world context, a sunflower that grows 30 % each week. In the subsequent task sequences, students gradually meet more generic situations that require more abstract mathematical reasoning. The researchers were responsible for designing a draft of the worksheets before they were deliberately discussed and revised at joint meetings with the teachers before then implementing each teaching unit. This paper focuses on how to approach follow-up discussions in the last teaching unit.

As a basis for evaluating the suitability of the Stein et al. model for this type of follow-up lesson, empirical findings derived from students' written responses on the worksheets in the main project are used. These responses had been inductively analysed to identify different categories of student strategies. It is the results from this empirical analysis that provide the basis for the further pedagogical analysis constituting this study. Bearing in mind specific elements of the Stein et al. model, for it to be applicable to follow-up discussion of tasks, it is necessary for: (a) students' computer-based work to result in various student strategies appropriate to discuss and compare and possible to align to key mathematical ideas, (b) the sequencing of the strategies to be of importance, and (c) it to be possible to anticipate several strategies used by the students.

Student responses to two different types of task embedded in the last task sequence are used: description/explanation tasks and prediction tasks. In description/explanation tasks students are expected to describe/explain the outcome of a particular investigation. In prediction tasks students are supposed to first predict an outcome before testing it and then reflecting on the outcome in relation to the prediction. Responses from one class on one example of each type are used to examine the five practices in the model.

Furthermore, audio-recordings collected at the joint meetings with teachers provided some information regarding anticipations about possible student responses and behaviors. While examining the five practices, we will assume that teachers only can get information on student responses during the students' work on the tasks. Although the researchers observed all the teaching units, the focus during the computer-based lessons was not on how the teachers were monitoring students' responses to the tasks, i.e. the second practice in the Stein et al. model.

Results and discussion

For each example the categories of student responses identified in the main study are introduced followed by a discussion in the light of the five practices.

Example 1. Description/explanation tasks

Figure 1 introduces the first task selected for analysis. The analysis of student responses resulted in four categories. These categories are presented and illustrated by student quotations below. The ordering is not of importance at this stage.

(i) Responses focusing on the visual experience of movement in the x-direction.
”When you change the value of slider C the graph will move to the right or to the left”

Now we will leave the example with the sunflowers and instead we will study the general exponential function, $f(x) = C \cdot a^x$, and how its graph is depending on the values of C and a.

Set the slider a at 2.

3. Drag the slider C so that the value of C varies. Describe in your own words how to see the value of C in the graph.

Figure 1. *Example of a description task*

(ii) Responses focusing on the visual experience of movement in the *y*-direction.
"You can see that the value varies because the point *A* [referring to a point on the graph] moves with the graph, *x* is the same but the value of *y* changes"

(iii) Responses focusing on the connection between the parameter *C* and the value on the *y*-axis (in the point of intersection).
"The value of *C* becomes the graph's value on the *y*-axis"

(iv) Responses focusing on the connection between the parameter *C* and the value on the *y*-axis (in the point of intersection), but at the same time claiming that the value of *C* does not affect the rate of change.
"*C* decides the starting value, that is, where the graph intersects the *y*-axis. No matter what the value of *C* is, the graph will increase at the same rate"

On reflection, there are three aspects that emerge from the categorization: local view (ii, iii, and iv) or global view (i and iv), framing in terms of vertical movement (ii) or horizontal movement (i), and attention to rate of change (iv).

Discussion of the first example in the light of the five practices

(1) Anticipating likely student responses to cognitively demanding mathematical tasks

While preparing this task at the planning meeting, responses belonging to category (ii) and (iii) were discussed. Concerning the responses belonging to category (i), we argue that it should not be too difficult to anticipate if the teachers "put themselves in the position of their students while doing the task" (Stein et al., 2008, p. 323). The only responses that we regard as surprising, and hence hard to predict, are the responses claiming that the value of *C* does not affect the rate of change. However, as Stein et al. (2008) point out, once the task has been used in class the teacher can add unexpected responses to their list of anticipations.

(2) Monitoring students' responses to the tasks during the explore phase

The students are supposed to work and discuss in pairs and to express their responses in writing. Hence, it might be possible for the teacher to identify ideas present in the lesson to set an agenda for later discussions both by listening to student discussions and by reading their written responses.

(3) Selecting particular students to present their mathematical responses during the discuss-and-summarize phase

We suggest selecting at least one representative of each of the four categories. Categories (i) and (ii) are important to discuss and compare because they provide examples showing how different visual focuses could result in different experiences, i.e. horizontal or vertical movements.

The responses belonging to category (iii) are important to emphasize and discuss since these responses pinpoint the main characteristic of the parameter C. Further, it might be worth displaying several student responses from category (iii) to discuss the choice of wording. Some students were expressing themselves in terms of "starting value", and some were expressing that the value of C can be seen on the y-axis. There were also two responses using the word "intersects", which could be used by the teacher to introduce the term *point of intersection*. The possibility to introduce important mathematical notations in follow-up lessons is emphasized in the literature (Bartolini Bussi & Mariotti, 2008; Kieran et al., 2012).

Moreover, it is important to display responses belonging to category (iv), making the misconception concerning the rate of change visible and discussed. This could initiate a discussion on how the value of C affects the way the value of the function changes. This could be an ideal opportunity to clarify the difference between rate of change and relative rate of change, i.e. that the relative rate of change remains the same while the rate of change is affected by the value of C.

When selecting particular students to call on as candidate to present each idea in the whole-class discussion, teachers can try to ensure that different students are given the opportunity to present their ideas from time to time (Stein et al., 2008).

(4) Purposefully sequencing the student responses that will be displayed

We claim that it is preferable to first display and compare responses belonging to categories (i) and (ii). This allows for discussions about how local and global views might affect the dynamical visual experiences. This discussion then provides a base for an elaboration on the more focused question "how to see the value of C in the graph". At this stage, it is appropriate to discuss responses from category (iii) since they are based on a local view focusing on the intersection with the y-axis. Also, we find it important to display category (iii) before category (iv) and in this way focus on and finish the ”intersection discussion” before entering the discussion on how the value of C affects the rate of change. Hence, we argue that the sequencing of student responses is important in this example.

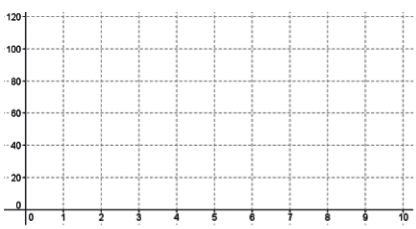
(5) Helping the class make mathematical connections between different students' responses and between students' responses and the key ideas

Although there was no request for an explanation in this task, the explanations concerning the key mathematical ideas, i.e. the intersection property and how the value of C affects the rate of change, are appropriate to discuss in the follow-up discussion. Preferably, the projected computer screen could serve as joint reference for students when presenting their various ideas.

Example 2. Prediction tasks

Figure 2 introduces the second task selected for analysis. Before this task, students had investigated how the parameter a affects the graph when $a > 1$ and $a = 1$ respectively. All students realized that the graph should include the point (0, 80) and that it should be decreasing, even if the shape of their decreasing graphs varied. The analysis of student responses resulted in four categories. These categories are presented and illustrated by student quotations below.

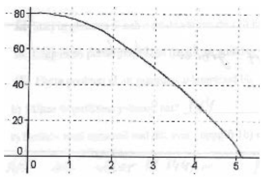
6 a) Guess (without using GeoGebra) what the graph of the function $f(x) = 80 \cdot 0{,}5^x$ will look like. Make a sketch in the coordinate system below.



b) Explain the thoughts behind your guess.
c) Compare your guess with the graph obtained in GeoGebra. Explain any differences that may occur!

Figure 2. *Example of a prediction task*

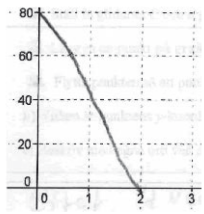
(i) Sketches showing a concave decreasing graph followed by an explanation comparing with the cases $a > 1$ and $a = 1$. Minor differences are expressed, when comparing with the graph obtained in GeoGebra.

a) 

b) ”Since 1 goes straight and 1.5 goes up, 0.5 must go down”

c) ”It lands at 8, but I thought that it should land at 5”

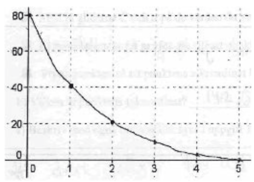
(ii) Sketches showing a decreasing straight line, followed by an explanation referring to the repeated halving or 50 % decrease of the value. Expressing that the guess is steeper, when comparing with the graph obtained in GeoGebra.

a) 

b) ”It will decrease by 50 % each week”

c) ”Ours is much steeper”

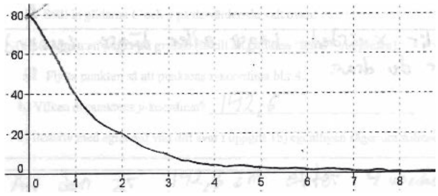
(iii) Sketches showing a convex decreasing graph crossing the x-axis, followed by an explanation referring to the repeated halving or 50 % decrease of the value. Expressing that their guess was consistent with the one obtained in GeoGebra.

a) 

b) ”The value is halved every time”

c) ”They look exactly the same”

(iv) Sketches showing a convex decreasing graph not crossing the x-axis, followed by an explanation referring to the repeated halving or 50 % decrease of the value. Expressing that their guess was consistent with the one obtained in GeoGebra.

a) 

b) ”Since it decreases by 0.5 the whole time from its new value”

c) ”Just right”

Besides the kind of explanations described above one student, in this class, claimed that ”the higher the value of x is the less the decrease should be”. To summarize, three of the four categories demonstrate student misconceptions (i, ii, and iii). These misconceptions concern the shape of the graph (i and ii) and/or intersection with the *x*-axis (i, ii, and iii). The explanations provided by students on task b) are of two types: comparing with the cases $a > 1$ and $a = 1$ (i) or referring to the ”repeated halving or 50 % decrease” (ii, iii, and iv). Concerning task c), none of the participating students provided any explanation.

Discussion of the second example in the light of the five practices

(1) Anticipating likely student responses to cognitively demanding mathematical tasks

While discussing this task at the planning meeting, the teachers anticipated that most students would realize that the graph is decreasing, but that they would display sketches with different level of accuracy. The teachers anticipated that some students will make rough sketches based on a comparison with the cases $a > 1$ and $a = 1$, while others will insert points obtained by successively halving the starting value. When looking at the different kinds of graphs displayed by the students we claim that they all should be possible to anticipate, even if category (i) and category (ii) probably are the most difficult to predict. However, responses in category (i) are not that surprising, since these graphs could be obtained by reflecting a graph with $a > 1$ in the line obtained when $a = 1$. Concerning category (ii), research shows that students tend to revert to linear functions and that a common misconception is that if you have two points, a third point should lie on their linear extension (Kasmer & Kim, 2012). Category (iii) and category (iv) were the two dominant ones. However, it is not always easy to determine if a student response belongs to category (iii) or to category (iv). Therefore, it is extra important that these two categories are anticipated so that the teacher pays attention to the difference between them. The two types of explanation were anticipated at the planning meeting.

(2) Monitoring students' responses to the tasks during the explore phase

Concerning this practice, we refer to the discussion in relation to example 1.

(3) Selecting particular students to present their mathematical responses during the discuss-and-summarize phase

It became obvious from the student responses on this task that there is a need to clarify several things during the whole-class discussion. Responses belonging to category (i), (ii), and (iii) all contain serious misconceptions and it is clear from the responses on task c) that these misconceptions were not sorted out. Hence, we argue that it would be appropriate to select students so that category (i), (ii), and (iii) all are represented. Furthermore, (at least) one proper response belonging to category (iv) should be selected. Also, it could be instructive to select the explanation claiming that ”the higher the value of x, the lower the decrease should be”, since it pinpoints an important property of the graph.

(4) Purposefully sequencing the student responses that will be displayed

We suggest selecting a student response belonging to category (i) first. The reason for this is to first focus the discussion on the comparison between the various values of a, i.e. $a > 1$, $a = 1$ and $0 < a < 1$, and in this way focus on if the graph is increasing, constant or decreasing. In this way it will be possible to highlight the good thought behind this type of student response, before digging deeper into the shape of the graph. Next, a student response belonging to category (ii) might be a first step towards a more precise graph, since this will highlight the repeated halving or 50% decrease of the value. The linear graph in category (ii) could be contrasted by the student statement ”the higher the value of x is the less should the decrease be”. Perhaps the student expressing this utterance also can explain why it is true. To make this clear, a student response using repeated halving or 50% decrease to get values to be able to plot points in the coordinate system, like the one in category (iii), could be displayed. Finally, to focus the discussion on the question if the value becomes zero or not, a graph belonging to category (iii) could be contrasted by a graph belonging to category (iv). To summarise, we claim that it is important in which order the identified categories are displayed.

(5) Helping the class make mathematical connections between different students' responses and between students' responses and the key ideas

The variation in student responses makes it possible to pinpoint and discuss key ideas on the basis of some of these responses, not least responses revealing misconceptions. Important key ideas are: how the value of a determines if the graph is increasing, constant or decreasing, the difference between linearly decreasing and exponentially decreasing functions and that an exponentially decreasing function never intersects the x-axis.

Conclusion

Reflection on an observational study showing that whole-class discussions following up students' computer-based work tend to be teacher-centered (Fahlgren, 2015) raised the question of how to improve these discussions by making them more student-centered and tied to key mathematical ideas. The aim of this paper is to examine if the Stein et al. (2008) model for orchestrating productive mathematical whole-class discussions could be useful in following up students' computer-based work. Even if only two examples have been used to evaluate the model, these examples indicate that there is a great potential in the model also in this educational setting. In both examples we find all five practices in the model useful as guidance for teachers. However, more research is needed to examine the model further. Primarily the model has to be examined by observing classrooms where teachers are following a lesson plan based on the five practices.

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