Researching the use of ICT to teach mathematics: the case of mathematically able software

Kaye Stacey

The purpose of this paper is to present a broad survey of research questions, methods, and a few findings from over twenty years of research with various colleagues centred around the University of Melbourne and to suggest important issues for research. The paper will focus on questions specifically related to mathematics teaching and to the use of what we call "mathematically-able software". This is only a part of the ICT that mathematics teachers use, and indeed our projects extend beyond this focus (see for example Pierce & Stacey 2011; Price, Stacey, Steinle & Gvozdenko 2013; Stacey & Wiliam 2013). However, the focus on mathematically-able software is a critical one because this is the software that is most challenging to mathematics. This is responsible for ICT being one of what I see as the two major drivers for change in mathematics curriculum in our time (the other being the growth in the percentage of students attending secondary schooling around the world).

Because ICT is now such a major force in mathematics education, it is evident that studies of ICT in mathematics encompass very many aspects of curriculum, teaching and assessment. Consequently, they must draw on a diverse range of questions and theories, all motivated by the opportunities ICT offers to improve mathematics outcomes for students.

ICT in Australian schools

The work that I report has been carried out in the context of Australian schools. As a general rule, Australian people like the idea of using up-to-date technology and this is reflected by national expectation, supported in the Australian Curriculum and in government funding policies, that ICT should be used in schools (Government of Australia, 2013). In recent years, the major thrust has been to use ICT across the school in all subjects, and so this has strongly promoted the use of internet resources for research, digital textbooks and other learning resources, software for student presentations, collaboration tools, and learning management systems. ICT of this nature is principally a communications infrastructure for schools. This paper focusses on ICT forming a computational infrastructure for school mathematics.

Kaye Stacey, University of Melbourne

Within mathematics, Australian curricula and examinations (the 8 states and territories have somewhat separate systems) have used four function calculators in primary schools and scientific calculators in secondary schools since the early 1980s. Secondary school mathematics now generally uses complex calculators, including in university entrance examinations. In my state of Victoria, graphics calculators have been widely used and permitted in examinations since about 1995 and CAS calculators have been phased in for Year 12 examinations progressively since 2002. More precise details as well as a summary of research into these initiatives are reported in Stacey (2016) and in many of the references to this article (e.g. Leigh-Lancaster, 2010). Teachers are encouraged to use software such as spreadsheets and dynamic geometry in class. Of course, there remains some difficulties with resources (e.g. it is often a very difficult task to set up a data projector in a room) and not all teachers know about available resources or have up-to-date skills.

Mathematically-able software (MAS)

As noted above, this paper focusses on research into the use of mathematically-able software. These are open tools, where the user (generally in this case a student) inputs ”questions” in mathematical language to which the software provides answers. The classic MAS tools can be used in life outside school. Examples are calculators of all sorts, computer algebra systems (Mathematica, Maple, etc.) abbreviated here to CAS, statistics packages, and spreadsheets (e.g. Excel). However, we also include some software unlikely to be used beyond school such as dynamic geometry (e.g. Cabri, Geometer’s Sketchpad) and some applets with an open mathematical capability even if the topic is limited such as some of those from the National Library of Virtual Manipulatives from the Utah State University. Unlike educational software that directs what the user will do (for example, by presenting a series of questions to answer, or actions to undertake in a game), the teacher or student decides what to do with the MAS.

Because they have a role beyond school, MAS challenges the goals of education and what techniques are taught to students, whilst providing opportunities for learning. Much of our Melbourne research has been inspired by these challenges and opportunities. Since 1990, we have conducted research projects on most of these software tools.

Structuring the research program

The diagram in figure 1 shows that we have structured our research program on MAS around three central themes. First, as noted above, the widespread accessibility of MAS outside school presents a challenge to the content and the goals of the school curriculum. Second, a major concern for educational systems in adopting MAS in schools is related to assessment. In our state, this concern has principally been directed at the end-of-school examinations for Year 12 students.These examinations are set and marked by the state authority. They are high-stakes for students and teachers, because the certification relates to school completion and performance is the most important factor in selection into university courses. This has been the major driver of adoption of CAS in secondary schools often from Year 9 up. The third theme relates to the ”pedagogical opportunities” that are supported when MAS is available for teachers and/or students. Curriculum change and assessment change are principally influenced by what we call calculation use of MAS (e.g. finding an integral, multiplying two multi-digit numbers, inverting a matrix, calculating a regression equation, or graphing a function). However, since the very beginning of affordable digital technology, it has been recognised that it can also be used in order to improve learning of mathematics. We call this ”pedagogical use”. For example, Etlinger (1974) noted how a calculator could be used to illustrate the subproducts that are involved in the long multiplication algorithm. At the time, very long multiplication was still regarded as a valuable skill because the calculator could only display about 6 digits. He also noted the pedagogical benefits of discussing anomalies such as $(1 \div 3) \times 3 = 0.9999999$ which occurred on the calculators of the day. Behind all these themes of use of MAS is the practical need to investigate questions of teacher professional development.

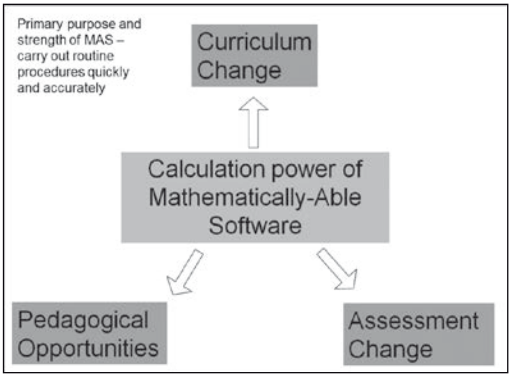

Figure 1. *Three themes of research on the use of MAS software*

In the next sections, I will examine each of the three parts of the research project in turn briefly and suggest areas where research is still needed. In these sections, it becomes evident that each issue is multi-faceted and so research into each inevitably draws on a wide range of research methods and frameworks. The relevance to ICT in mathematics teaching cuts across many concerns and so it cannot be pigeonholed as just one strand of research.

Curriculum change – content and goals

It has been clear for nearly five decades that the advent of digital technology has profound implications for the mathematics curriculum – its content and its goals. Of course, different people assess these implications in different ways. Whereas there appears to be almost universal agreement that ICT should be used "to teach better" (so that students to have better understanding, more confidence etc.), there is less agreement about what topics are now obsolete or of low priority, what new topics should replace them, and how the presence of ICT should change the goals of mathematics (see for example Ball & Stacey, 2001, 2005). For myself, I want the process of gradual pruning of obsolete topics to continue (it has been going on in Australia since at least the 1980s). I feel that considerably more attention should be given to new topics and especially to more consistently shifting to ICT-aware methods (e.g. using spreadsheets). Most of all, I would like ICT to assist us to more strongly emphasise the, often stated, goal of students becoming better problem solvers, especially being better able to formulate real world problems mathematically, better able to conduct mathematical investigations and creating a school mathematics that is less dominated by routine procedural work.

Although there is important work on each of these goals already, the questions below are still in need of research:

(a) Can we better illuminate the links between achieving competent use and full understanding of a topic and practising its procedures? Is the strength of the link different for different topics? For example, I hear no-one now decrying serious consequences of the lack of practice of the pen-and-paper algorithm for calculating square roots, which was widely taught until about 50 years ago; and there appears to be no real concern in the statistics education literature about the use of statistics packages. On the other hand, there seem to be endless debates on arithmetic and algebra. Reading Etlinger (1974) puts this is context.

(b) Mathematics with ICT is a different subject – can this be elaborated? What is the nature of that mathematics? How does using MAS change students' understanding of a topic? There is substantial knowledge of this only in a few areas (e.g. graphing, dynamic geometry dragging).

(c) What curriculum will really equip students for the ICT future, and how can large education systems make the changes needed to get there?

(d) Are there new MAS that will impact on school curricula soon? For example, when will we have a tool which makes three-dimensional mathematics very feasible and what will we do with it?

Research on these questions, and research on curriculum changes that has already occurred requires a range of methods and frameworks, including drawing on theoretical mathematical and document analysis, theories of adoption of innovation, studies of attitudes, and empirical studies of learning.

Assessment change

As noted above, most of our research on assessment change has largely been related to accommodating the calculation power of MAS, especially that of graphics calculators and CAS calculators. A diverse group of researchers has examined the evolving practices and data from Victorian Certificate of Education through the introduction of graphics calculators (1995+) and CAS calculators (2002+) and now computer software. The findings are outlined in Stacey (2016) and a wide range of references are given there. This is summative assessment in a high stakes environment. School educational authorities in several parts of the world beyond Australia (e.g. Denmark, Scotland) and many university departments have also faced this issue. However, the question of designing assessment for a technologically-rich environment is broader. For example, the OECD's PISA 2012 survey (OECD 2013) had an optional computer-based assessment of mathematics (CBAM) that included some items which assessed unchanged mathematics in a way that was enhanced by the computer-based format and other items where students could use some computational power of the computer. Within the Assessment Change strand, research methods include empirical studies (natural and designed experiments, comparisons and surveys), document analysis and mathematical-didactic analysis, and theoretical studies especially related to values. Many studies have a local focus with international input and some generalizability.

We have organised our research and development work on assessment under three guiding principles (Stacey & Flynn, 2007). The first is the ”Mathematics principle”, referring to the imperative to assess mathematics that is important for students to learn. Much research on this has been reported (see for example Flynn & McCrae, 2001; Stacey & Wiliam, 2013) so space here permits only a few sample results. Our work under this principle has established that creating good assessment items for the new computational environment requires a new set of skills, because many items that were formerly testing significant mathematics now test something different. Routine, procedural, questions are the most affected because students can generally just ”type in” the question, so it is tempting to consider removing these relatively easy items from examinations in order to make space for items where students can use the technology to support more substantial problem solving. However, care has to be taken with this approach to ensure that examinations with technology do not become inappropriately difficult for students.

Along with others (e.g. Brown, 2010) we have found that evolution away from a largely procedurally-based examination is very slow with only weak drivers for change. Teachers mostly want to ”teach better” rather than to extend the expectations for students, so there is a tendency for CAS to be used to compensate for weaker skills rather than amplify what students can do. Moreover, in common with many other countries, our examination system now includes an ”ICT-forbidden” component. This political compromise removes pressure to update the curriculum.

The second principle for assessment is the ”Learning principle” – giving consideration to assessment actions that promote good effects in the classroom from both the teacher’s and the students’ perspectives. Assessment in our settings is the most powerful driver of what happens in classrooms, so it is important that assessment design promotes good classroom practices. For example, it seems important that assessment should, as far as possible assess mathematical thinking rather than technical proficiency with button pushing. Perhaps the most important work related to this principle has been to compare the performance of students who learned mathematics with CAS to those who learned with a graphics calculator. Students were permitted to use the technology they learned with, when answering the examination questions. A series of studies from 2006 to 2009 demonstrated that the students using CAS always performed slightly better than those with graphics calculators. This was the case in the components with and without ICT, and when controlled for factors related to general ability. In general, a similar percentage of students in both groups did very well, but fewer CAS students did poorly, and the average score of the middle students was slightly higher. The results are summarised in detail by Stacey (2016) along with several possible explanations.

A third series of studies have examined the ”Equity principle” – ensuring that the assessment is fair to all students, regardless of the hardware or software which they use. These studies have included investigations into: the effect of allowing technology on groups of students (e.g. low socio-economic status, girls); whether some brands and models confer an advantage (and procedures to ensure that this does not happen); empirical studies of effects on individual questions; and comparison of the use of computers with hand held devices. An important observation is that the ICT skills of teachers seems to be an important equity factor for their students, and an important design question is to consider what conditions would enable ICT to open up opportunities for more students to participate in advanced mathematics. The studies are also summarised in Stacey (2016).

Pedagogical Opportunities

Our work on the pedagogical opportunities offered by ICT has been structured by the pedagogical opportunities map, shown in figure 2 (Pierce & Stacey, 2009;

2010). We have used the map to guide our research program, and as a tool for tracking changes in teachers' practice in our studies on teacher-learning. We also use it in our work with in-service teachers to present them with a smorgasbord of ways in which they may choose to enrich their practice. It is important to note that all the opportunities arise because MAS carries out computation quickly and accurately and displays can be shared.

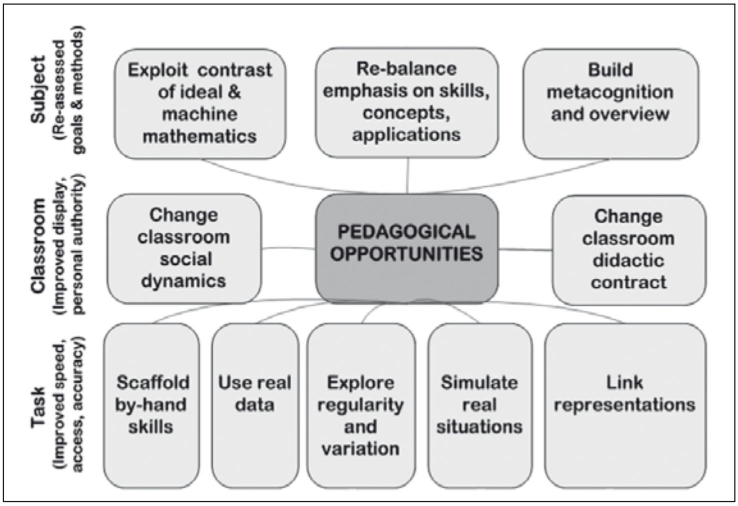

Figure 2. *The map of pedagogical opportunities for MAS*

There are three levels in the map of pedagogical opportunities. ICT can be used to support new types of tasks and to facilitate student work on tasks that were previously impractical. The bottom row, the "Task" level, identifies general ways in which teachers can use MAS to enhance teaching. The first box there draws attention to ICT being used to support students' learning of standard pen-and-paper skills or for developing concepts. For example, CAS can be used to demonstrate the usefulness of equation solving before students can solve equations independently, and later to check solutions to equations, or share part of the cognitive load as they learn the new procedures. Other boxes highlight how computational power supports students in their exploration of situations as they look for patterns, study the impact of variations, or use simulations. A much heralded possibility in the research literature is to give students very easy access to multiple representations of mathematical phenomena. This is in the rightmost box and is discussed below.

The second row highlights two ways in which teachers may use ICT, in particular MAS at the classroom level. MAS provides an opportunity to change the social dynamics of the classroom especially through the use of shared screens and easy display of student work for enhanced discussion and collaboration. There is also an opportunity to change the didactic contract, which is that part of the complex set of relationships of obligations between teacher and student in a classroom that is specific to the mathematical knowledge. The opportunities to change the didactic contract can arise in several ways. For example, ICT/MAS supports an "explosion of methods"; the number of methods available to solve problems increases sharply beyond the number of methods that are practical in a pen-and-paper environment. Students therefore may have more to contribute to a class discussion, and being able to check their work with CAS can make them more confident to make these contributions (Pierce, Stacey & Wander, 2010). On the other hand, introducing MAS can lead to a mismatch between students' and teachers' understanding of the didactical status of knowledge within classrooms, part of the didactic contract. For example, the study by Pierce, et al. (2010) found 77 % of students but only 1 of 6 teachers identified learning to use ICT as a main point of a certain lesson that they all taught. In the classroom some students wanted to have very small details of their ICT use confirmed by the teachers, behaviour that teachers saw as wasting time and drawing attention away from mathematics. This was a consequence of the break in the didactic contract and the teachers were later able to address it.

The third "Subject" level of pedagogical opportunities points to ways in which mathematics as a subject to learn can be altered by using MAS. For example, a very early research study (Heid, 1988) demonstrated how a calculus course could put primary emphasis on concepts and applications, rather than giving priority to the initial development of skills for differentiation. We have worked with teachers who use CAS to provide an overview of the topic and where it leads to enhance students' understanding of why they are learning mathematics – what one of our teacher colleagues calls "teaching the ends of a topic" (Garner, McNamara & Moya, 2003). And as Etlinger (1974) demonstrated right at the beginning of the technological revolution with his suggested investigation of why calculating 1/3 x 3 did not give the exact answer of 1, there are many possibilities to highlight mathematical thinking by observing the limitations of technology or the contrast between ideal mathematics in the head and enacted on a machine.

Investigating how to teach with multiple representations

Our research on pedagogical opportunities has used the pedagogical map of figure 2 in two ways. We have used it as a tool to map teachers' practice (see for example Pierce & Stacey, 2010) and we have also used it as the basis of our program, to better understand how to teach with ICT. In one instance we were invited to conduct a two year professional development program at a school that was introducing use of Texas Instruments TI-Nspire from Year 9 to 12 (Pierce & Stacey, 2009; 2013). TI-Nspire CAS software has symbolic algebra (e.g. solve an algebraic equation, including with parameters); graphing, dynamic geometry, tables of values, statistical functions and a document facility for multi-page investigations. All of the capabilities are linked.

At one time, we were invited to help the teachers design a ”cap-stone” lesson for a unit on quadratic functions for Year 10 students, taking about 100 minutes of class time. Teachers wanted to learn about the pedagogical opportunities and also the capabilities of the device. The students had handheld machines and the teacher used the parallel computer display on an electronic white board. With the teachers we designed a lesson, which was taught multiple times in a lesson-study context. The lesson was rich in representations of the central structure: geometric, symbolic, graphic. There is strong support for using multiple representations in the research literature, especially because conceptual knowledge has rich connections, because thinking mathematically involves exploring mathematical ideas from several perspectives, and because seeing a mathematical structure in different representations highlights different features. The lesson is described by Wander and Pierce (2009) and student material is available from RITEMATHS website (not dated). A large body of research on graphics calculators shows benefit of working with symbol-graph-(table) representations of functions. TI-Nspire CAS software provided many more possibilities. The lesson poses a series of questions about a fish shaped sign of fixed length, to be installed above Marina’s fish shop (see figure 3). One main question relates to minimising the area of the fish by adjusting the length of the body of the fish. These students had not studied calculus. Figure 3 shows possible fish shapes, the formula for area as a function of body length, the graph of the area

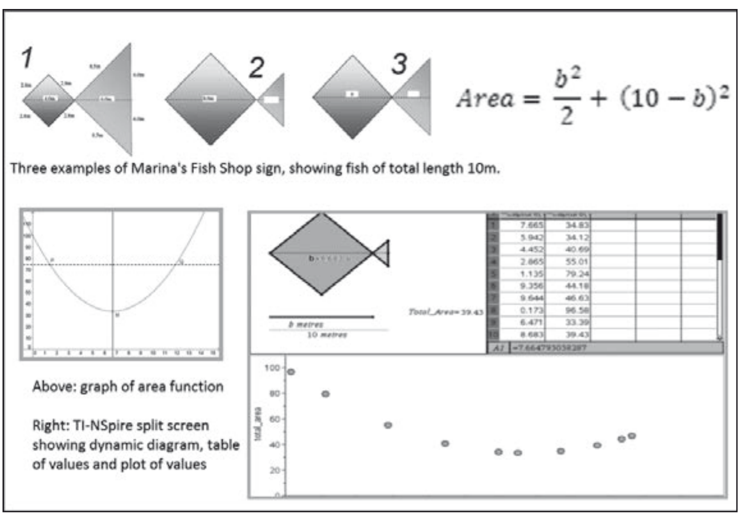

Figure 3. *Images from Marina’s fish shop lesson*

function and the triple screen from TI-Nspire showing the manipulable fish shape, the table of values and the plotted points used for quadratic regression.

The design research highlighted four aspects of teaching with multiple representations (Pierce, Stacey, Wander & Ball, 2011). The first was to highlight the need to avoid too much cognitive load when working with multiple representations. The lesson had verbal descriptions, pictures, a dynamic diagram with and without measurements, and later the symbols and the two sorts of graphs. Too many! This is a strong message from the HCI literature that has not been prominent in the mathematics education literature. However, as MAS technology adds functionality, it will become more prominent. Prepared screens (e.g. a movable fish in dynamic geometry) save time and keep focus on mathematics rather than ICT but effort is required for students to appreciate even simplest screens and small changes in representation. It is easy to underestimate this.

A second issue concerned student motivation. We began with the belief that examining the same phenomenon in different representations would lead to good learning. Whilst this may be true when the representations are new to students, in our first iteration of the lesson, students became bored when they had to find the minimum value empirically and from the graph. We had more success if each representation was used to inform a different part of the total investigation.

Another issue was to select the focus of the lesson and discard other possibilities. This arose because so many functionalities were available. The ”explosion of methods” meant that there were many ways of using the technology to solve a problem such as finding the minimum area. However, this created the need to choose carefully what we intended to be the focus of the lesson. Because teaching operates on a time economy, many good opportunities have to be passed up in any one lesson.

A fourth issue highlighted differences between ideal mathematics and mathematics within a device. In mathematics-in-the-head, there can be one variable ”the body length of the fish” which is used in drawing fish, in the algebraic formula, to label the columns of the table of values, to label the slider length on the dynamic diagram, and to report the quadratic regression. However, mathematics within a device must keep all of these occurrences of the body-length variable separate, with different names. Consultations with programming experts confirmed that this difference is inherent and not readily smoothed over. With multiple representations, variable naming can cause a semiotic storm!

Learning to teach with technology

Another section of our work has related to learning to teach with MAS technology. This change has required major investment by educational authorities and schools to help teachers develop the new technical and pedagogical skills that they need. The mandated change to assessment at Year 12 has been the major driver of the change, but schools have also been keen that students in Years 10 and 11 use the same device as they will use in the final examinations at Year 12. In several of our studies, we have found that learning to teach with MAS can be very hard for many of the teachers who are outside the small percentage of self-motivated early adopters. For example, even after 2 years supported use, some teachers in the professional development program described above, were still learning to use MAS to a level they see as adequate for teaching. Their confidence was still growing, along with their trouble shooting skills, and recognition of possibilities. Other studies have wider data, e.g. Pierce and Ball (2009) and Pierce and Stacey (2004). Most teachers find it easiest to use MAS just to calculate, rather than as pedagogical tool. The pedagogical opportunity of exploring regularities seems to be the first to be adopted. Time for learning is an important barrier for teachers, and regular updates of technology make the task a continuous challenge.

Reflections on ICT-related research

Using ICT, even considering only mathematically-able software, affects all aspects of teaching mathematics: the content and goals, the assessment, the classroom environment and the tasks on which students work. Taking advantage of the new opportunities and addressing the new challenges is a long term journey for mathematics teaching. It began at the start of the digital era when four-function calculators became sufficiently portable and affordable that nearly everyone could own one, simultaneously displacing the log tables and slide rules used by professionals. Since then, the unabated increase in mathematical and other capabilities of ICT ensures that there is no steady state just around the corner. Since all aspects of mathematics teaching are affected, the research methods, research questions and insight-delivering theoretical perspectives required are extremely diverse. There are basic insights related to technology, such as the notion of distributed cognition (Pea, 1987) that stresses the fundamental importance of studying the capabilities of the person plus the tool as one unit. Beyond such basic insights, research into ICT stretches across the concerns of mathematics education.

A particular challenge for ICT-related researchers is timeliness. Research is most useful if it has something to say to practitioners when it is needed, and this demands working at the technological forefront. However, it is not possible to conduct extensive research related to a technology which is not yet easy to use in schools. Studies that try to work too far ahead often deflect towards issues of implementation (such as access to equipment which may quickly or slowly change) and these issues can overwhelm the mathematical, didactic and pedagogical findings that are required to guide practice in the longer term.

A second aspect for ICT-related researchers arises from the necessity to use devices that are engineered and marketed in particular ways. Again, the best research for mathematics education will focus on the fundamental characteristics of the device for teaching mathematics. This also means that concepts from engineering and product design such as work on user-experience and the adoption of innovations, may be profitably brought into mathematics education research more than is currently seen.

In regard to the three aspects of broad aspects of research on the impact of MAS technologies, it seems that there is still much to be done to build expertise in lesson design which captures the pedagogical opportunities of MAS. For assessment change, especially in regard to summative examinations, a large body of experience has accumulated in certain geographically-diverse countries related especially to algebra and functions, and statistics (see for example Drijvers, 2009). For other settings, there is little research specifically related to the use of MAS – most assessment research (see for example Stacey & Wiliam, 2010) relates to using the capacity of ICT for item presentation, selection, scoring and reporting etc. rather than MAS issues. The role of MAS in assessment is particularly important because it can drive both teachers' and students' actions. Behind all of this sits judgements and values about the way in which MAS can and should change the content and goals of school mathematics. I think that a reasonable judgement is that there has generally been only very slow adaptation to our new technological environment. With all of these exciting but challenging developments, I judge that the elusive goals of deeper and better mathematics remain elusive, but maybe not quite as elusive now we do have these technologies to help us.

Acknowledgement

The research drawn upon in this paper has been undertaken in collaboration with many colleagues who appear in the reference list. I greatly appreciate their contributions.

References

Ball, L. & Stacey, K. (2001). New literacies for mathematics: a new view of solving equations. *The Mathematics Educator*, 6(1), 55–62.

Ball, L. & Stacey, K. (2005). Teaching strategies for developing judicious technology use. In W. J. Masalski (Ed.), *Technology-supported mathematics learning environments (NCTM 67th Yearbook)* (pp. 3–15). Reston: NCTM.

Brown, R. (2010). Does the introduction of the graphics calculator into system-wide examinations lead to change in the types of mathematical skills tested? *Educational Studies in Mathematics*, 73 (2), 181–203.

Drijvers, P. (2009). Tools and tests: technology in national final mathematics examinations. In C. Winslow (Ed.), *Nordic research on mathematics education: Proceedings from NORMA08* (pp. 225–236). Rotterdam: Sense Publishers.

Etlinger, L. (1974). The electronic calculator: a new trend in school mathematics. *Educational Technology*, XIV (12), 43–45.

Flynn, P. & McCrae, B. (2001). Issues in assessing the impact of CAS on mathematics examinations. In J. Bobis, B. Perry & M. Mitchelmore (Eds.), *Numeracy and beyond (Proceedings of the 24th annual conference of the Mathematics Education Research Group of Australasia)* (pp. 222–230). Sydney: MERGA.

Garner, S., McNamara, A. & Moya, F. (2003, December). *CAS: the SAFE approach*. Paper presented at the 2003 MAV conference. Retrieved from https://www.mav.vic.edu.au/files/conferences/2003/Garner_et_al_paper.pdf

Government of Australia. Digital Education Advisory Group. (2013). *Beyond the classroom: a new digital education for young Australians in the 21st century*. Retrieved from http://apo.org.au/research/beyond-classroom-new-digital-education-young-australians-21st-century

Heid, M.K. (1988). Resequencing skills and concepts in applied calculus using the computer as a tool. *Journal for Research in Mathematics Education*, 19, 3–25.

Leigh-Lancaster, D. (2010). The case of technology in senior secondary mathematics: Curriculum and assessment congruence? In C. Glascodine & K.-A. Hoad (Eds.), *Proceedings of 2010 ACER research conference* (pp. 43–46). Retrieved from http://research.acer.edu.au/cgi/viewcontent.cgi?article=1094&context=research_conference

OECD (2013). A profile of students' performance in mathematics (Chapter 2). In *PISA 2012 Results: what students know and can do – student performance in mathematics, reading and science* (Volume I) (pp. 31–144). OECD Publishing.

Pea, R. (1987). Practices of distributed intelligence and designs for education. In G. Salomon (Ed.), *Distributed cognitions: psychological and educational considerations* (p. 47–8. Cambridge University Press.

Pierce, R. & Ball, L. (2009). Perceptions that may affect teachers' intention to use technology in secondary mathematics classes. *Educational Studies in Mathematics*, 71 (3), 299–317.

Pierce, R. & Stacey, K. (2004). Learning to use CAS: voices from a classroom. In M. Johnsen Hoines & A. B. Fuglestad (Eds.), *Proceedings the 28th conference of the International Group for the Psychology of Matematics Education* (pp. 25–32). Bergen University College.

Pierce, R. & Stacey, K. (2009). Lesson study with a twist: researching lesson design by studying classroom implementation. In M. Tzekaki, M. Kaldrimidou & C. Sakonidis (Eds.), *Proceedings of the 33rd conference of the International Group for the Psychology of Mathematics Education* (pp. 369–376). Thessaloniki: PME.

Pierce, R. & Stacey, K. (2010). Mapping pedagogical opportunities provided by mathematics analysis software. *International Journal of Computers for Mathematical Learning*, 15 (1), 1–20.

Pierce, R. & Stacey, K. (2013). Teaching with new technology: four "early majority" teachers. *Journal of Mathematics Teacher Education*, 16 (5), 323–347.

Pierce, R. & Stacey, K. (2009). Researching principles of lesson design to realise the pedagogical opportunities of mathematics analysis software. *Teaching Mathematics and Its Applications*, 28 (4), 228–233. doi:10.1093/teamat/hrp023

Pierce, R. & Stacey, K. (2011). Using dynamic geometry to bring the real world into the classroom. In L. Bu & R. Schoen (Eds.), *Model-centered learning: pathways to mathematical understanding using GeoGebra* (pp. 41–55). Rotterdam: Sense.

Pierce, R., Stacey, K., Wander, R. & Ball, L. (2011). The design of lessons using mathematics analysis software to support multiple representations in secondary school mathematics. *Technology Pedagogy and Education*, 20 (1), 95–112. doi:10.1080/1475939X.2010.534869

Pierce, R., Stacey, K., Wander, R. (2010). Examining the didactic contract when handheld technology is permitted in the mathematics classroom, *ZDM*, 42 (7), 683–695. doi: 10.1007/s11858-010-0271-8

Price, B., Stacey, K., Steinle, V. & Gvozdenko, E. (2013). Smart online assessments for teaching mathematics. *Mathematics Teaching*, 235 (July), 10–15.

RITEMATHS (not dated). *Marina's fish shop*. Retrieved from https://extranet.education.unimelb.edu.au/DSME/RITEMATHS/general_access/curriculum_resources/quadratic_functions/index.shtml#Marina

Stacey, K. & Flynn, P. (2007). Principles to guide assessment with technology. In N. Idris (Ed.), *Classroom assessment in mathematics education* (pp. 1–18) Selangor: McGraw-Hill.

Stacey, K. & Wiliam, D. (2013). Technology and assessment in mathematics. In M. A. Clements, A. Bishop, C. Keitel, J. Kilpatrick & F. Leung (Eds.), *Third international handbook of mathematics education* (pp. 721–752). Dordrecht: Springer.

Stacey, K. (2016). Mathematics curriculum, assessment, and teaching for living in the digital world: computational tools in high stakes assessment. In M. Bates & Z. Usiskin (Eds.), *Digital curricula in school mathematics* (pp. 251–270). Charlotte: Information Age Publishing.

Wander, R. & Pierce, R. (2009). Marina's fish shop: a mathematically- and technologically-rich lesson. *Australian Mathematics Teacher*, 65, 6–12.